SWIM 2016
Summer Workshop on Interval Methods

ENS de Lyon, France
June 19-22, 2016

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Sunday, June 19, 2016
1 place de l'École

10:00 - 11:45	Tutorial *Computer-assisted proofs in dynamics* Daniel Wilczak
12:00 - 13:30	Lunch on your own we have booked a table at *Le Ninkasi, 267 rue Mérieux, 69007 Lyon*
13:45 - 18:00	Tutorial *Computer-assisted proofs in dynamics* Daniel Wilczak

Monday, June 20, 2016

Amphitheater B, 3rd floor, LIP - ENS de Lyon

08:30 - 09:00 *Welcome* by Luc Jaulin and Nacim Ramdani

09:00 - 09:30 *Rigorous Global Optimization with Interval Unions*
Tiago Montanher, Ferenc Domes, Hermann Schichl and Arnold Neumaier

09:30 - 10:00 *Bounding Nonlinear Functions by Combining Interval Arithmetic, Taylor Models, and Global Optimization*
Matthias Althoff and Dmitry Grebenyuk

10:00 - 10:30 Coffee and tea break, Salle Passerelle, 4th floor, ENS de Lyon

10:30 - 11:00 *An Interval Technique to Check the Performance of Control Laws Applied to Wind Turbines*
Nassim Loukkas, Nacim Meslem and John-Jairo Martinez-Molina

11:00 - 11:30 *Convergence Domain of Image-Based Visual Servoing with a Line-Scan Camera*
Vincent Drevelle

11:30 - 12:00 *Robust Output-Feedback Control for a Class of Interval Model: Application to a Piezoelectric Tube*
Mounir Hammouche, Micky Rakotondrabe and Philippe Lutz

12:00 - 12:30 *Interval Methods for Robust Variable-Structure Control with One- and Two-Sided State Constraints*
Andreas Rauh and Harald Aschemann

12:30 - 14:00 Lunch (offered), at the mensa

14:00 - 14:30 *Decision Making Under Twin Interval Uncertainty*
Barnabas Bede, Olga Kosheleva and Vladik Kreinovich

14:30 - 15:00 *Interval Arithmetic in GNU Octave*
Oliver Heimlich

15:00 - 15:30 *Certification of Roundoff Errors with SDP Relaxations and Formal Interval Methods*
Victor Magron

15:30 - 16:00 *Avoiding Fake Boundaries in Interval Analysis*
Guilherme Schvarcz Franco and Luc Jaulin

16:00 - 16:30 Coffee and tea break, Salle Passerelle, 4th floor, ENS de Lyon

16:30 - 17:00 *Interval Based Parallel Computing of the Viability Kernel*
Stéphane Le Ménec

17:00 - 17:30 *Guaranteed Confidence Region Characterization for Source Localization using LSCR*
Cheng-Yu Han, Alain Lambert and Michel Kieffer

17:30 - 18:00 *Image-Based Mobile Robot Localization using Interval Methods*
Ide-Flore Kenmogne and Vincent Drevelle

18:00 - 18:30 *Range-Only Multistatic Radar Detection of a Windfarm Based on Interval Analysis*
Eduard Codres, Waleed Al Mashhadani, Anthony Brown, Alexandru Stancu and Luc Jaulin

20:00 - 22:30 Dinner (at your own expenses) at La Gargotte, 15 rue Royale, 69001 Lyon

Tuesday, June 21, 2016

Amphitheater B, 3rd floor, LIP - ENS de Lyon

09:00 - 09:30 *A Parametric Kantorovich Theorem with Application to Tolerance Synthesis*
Alexandre Goldsztejn, Stéphane Caro and Gilles Chabert

09:30 - 10:00 *Interval Tools for Computing the Topology of Projected Curves*
Rémi Imbach, Guillaume Moroz and Marc Pouget

10:00 - 10:30 *Finding Zeros for Systems of Two Analytic Functions*
Dahne Joel

10:30 - 11:00 Coffee and tea break, Salle Passerelle, 4th floor, ENS de Lyon

11:00 - 11:30 *Validated Integration of Dissipative PDEs - Chaos in the Kuramoto-Sivashinsky Equations*
Daniel Wilczak and Piotr Zgliczyński

11:30 - 12:00 *Computing Attracting Ellipsoids for Nonlinear Systems using an Interval Lyapunov Equation*
Léopold Houdin, Alexandre Goldsztejn, Gilles Chabert and Frédéric Boyer

12:00 - 12:30 *Contraction, Propagation and Bisection on a Validated Simulation of ODE*
Julien Alexandre Dit Sandretto and Alexandre Chapoutot

12:30 - 14:00 Lunch (offered), at the mensa

14:00 - 15:00 *Set-Based Methods in Programs and Systems Verification*
Sylvie Putot and Éric Goubault

15:00 - 15:30 *Control of an Autonomous Underwater Vehicule under Robustness Constraints*
Juan Luis Rosendo, Dominique Monnet, Benoît Clément, Fabricio Garelli, Irvin Probst and Jordan Ninin

15:30 - 16:00 *Consensus Control: Fundamentals and some Recent Developments*
Alexandru Stancu and Zhengtao Ding

16:00 - 16:30 Coffee and tea break, Salle Passerelle, 4th floor, ENS de Lyon

16:30 - 17:00 *Robust Hybrid State Estimation using Interval Methods*
Moussa Maïga, Nacim Ramdani, Louise Travé-Massuyès

17:00 - 17:30 *An Interval Branch-and-Bound Algorithm for Parameter Estimation*
Bertrand Neveu, Martin de la Gorce and Gilles Trombettoni

17:30 - 18:00 *OMNE is a Maximum Likelihood Estimator*
Jeremy Nicola and Luc Jaulin

18:00 - 18:30 *Interval Analysis for the Representation of Phoneme Databases in Speech Recognition Systems: Fundamentals of a Computer-Based Assistance System in Speech Therapy*
Andreas Rauh, Susann Tiede and Cornelia Klenke

Fête de la Musique, Music Uninterrupted & Songs In the City

Wednesday, June 22, 2016
Amphitheater B, 3rd floor, LIP - ENS de Lyon

09:00 - 09:30	*Inner Approximation of a Capture Basin of a Dynamical System* Thomas Le Mézo, Luc Jaulin and Benoît Zerr
09:30 - 10:00	*Interval Trajectory Tracking with Flatness* Olivier Mullier and Estelle Courtial
cancelled	*Homotopy Perturbation Method for Solving Nonlinear Interval Differential Equations* Nisha Rani Mahato and Snehashish Chakraverty
10:00 - 10:30	Coffee and tea break, Salle Passerelle, 4th floor, ENS de Lyon
10:30 - 11:00	*Exact Solution to a Parametric Linear Programming Problem* Lubomir Kolev and Iwona Skalna
11:00 - 11:30	*An Implementation of a Posteriori Interval Analysis Technique and its Application to Linear Algebra Problems* Vladimir Glazachev
11:30 - 12:00	*A Concoction of Zonotope Abstraction and Constraint Programming for finding an Invariant* Bibek Kabi, Éric Goubault and Sylvie Putot
12:00 - 12:30	Closing
12:30 - 14:00	Lunch (offered), at the mensa

Abstracts
ordered in chronological order
(as in the program)

Monday, June 20, 2016

Rigorous global optimization with interval unions

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Keywords: Interval union arithmetic, Interval union Newton methods, Interval union branch and bound, Global optimization

Introduction

This talk introduces the interval union arithmetic, a new concept which extends the traditional interval arithmetic. An interval union is a finite set of closed and disjoint intervals where the bounds of the extremes can be $\pm\infty$. For example, $\boldsymbol{u} = ([-1, 0], [2, 3])$ and $\boldsymbol{v} = ([5, 6], [12, 13])$ are two interval unions and elementary operations between them are defined to be the union of the operator applied to all possible pairs of elements in $\boldsymbol{u}$ and $\boldsymbol{v}$. Intuitively we have for example

$$\boldsymbol{u} + \boldsymbol{v} = ([4, 6], [7, 9], [11, 13], [14, 16]).$$

Interval unions are designed to properly represent the division by intervals containing zero. If $a = [-1, 1]$ and $b = [5, 6]$, the extended division $\frac{b}{a}$ results in two disjoint intervals. Traditional interval arithmetic is not suited to cope with this situation nor in theoretical neither in computational aspects. On the other hand, if $\boldsymbol{a} = ([-1, 1])$ and $\boldsymbol{b} = ([5, 6])$ are interval unions then

$$\frac{\boldsymbol{b}}{\boldsymbol{a}} = ([-\infty, -5], [5, \infty])$$

is an interval union and all elementary operations are closed in the set of all interval unions. Some theoretical results of interval analysis remain valid when we are dealing with interval unions. That is the case of the fundamental theorem of interval arithmetic, and therefore the natural extension of real functions to interval unions. On the other hand, due to the lack of convexity some inclusion results like the interval mean value theorem do not hold even for the univariate case.

Rigorous global optimization

This work provides a framework based on interval union arithmetic for rigorous global optimization of unconstrained functions. We show that an interval union branch and bound tree is more efficient and economical than interval boxes storage. We also present an heuristic to avoid the exponential growth of interval union boxes which could affect the efficiency of the method. We combine the interval union versions of the Gauss-Seidel and Newton procedures with traditional implementations using intervals to accelerate the convergence of methods largely used in the global optimization literature.

An interval union linear system is a family of linear systems of form

$$Ax = b \quad (A \in \boldsymbol{\mathcal{A}}, b \in \boldsymbol{b}) \tag{1}$$

where $\boldsymbol{\mathcal{A}}$ and $\boldsymbol{b}$ are an interval union matrix and vector respectively. We extend the Gauss-Seidel procedure to enclose all solutions of (1) within an initial interval union box and show that the

new approach produces useful bounds even when the traditional Gauss-Seidel procedure fails. The method proposed also handles with intervals containing zero in the diagonal entries of $\boldsymbol{\mathcal{A}}$ in a natural way and do not requires any special treatment as in the interval Gauss-Seidel procedure.

We extend the Newton operator that, given an initial interval union box $\boldsymbol{x}_0$, produces a sequence $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots$ such that

$$\boldsymbol{x}_0 \supseteq \boldsymbol{x}_1 \supseteq \ldots \supseteq \mathcal{S}_{\boldsymbol{x}_0,x}$$

where

$$\mathcal{S}_{\boldsymbol{x}_0,x} := \{y \in \boldsymbol{x}_0 \mid \exists f^* \in \boldsymbol{f}(\boldsymbol{x}_0) \text{ and } g^* \in \boldsymbol{f}'(\boldsymbol{x}_0) \text{ such that } f^* + g^*(y-x) = 0\}.$$

We show that a special care is needed on the choice of the expansion point x and present some possible alternatives to evaluate it. The interval union Newton operator requires the solution of a linear system of each iteration and we show how the interval union Gauss-Seidel can be used to rigorously solve this problem.

Conclusion

Numerical experiments on 102 problems from the Princeton library of nonlinear models show that the proposed approach, combining interval and interval union Newton methods, accelerates the convergence rate of the branch and bound methods in up to 20% with no lack of rigor. The experiment also suggests that the interval union arithmetic could be used to improve the efficiency of constraint propagation methods that are commonly employed in rigorous optimization. This work do not consider constraints in the optimization problem though the generalization of our methods to this case is straightforward.

References

[**1**] T. Montanher and F. Domes and H. Schichl and A. Neumaier, Rigorous unconstrained global optimization with interval union arithmetic, *In preparation.*

[**2**] H. Schichl and F. Domes and T. Montanher and K. Kofler, Interval Unions, *In preparation.*

Bounding Nonlinear Functions by Combining Interval Arithmetic, Taylor Models, and Global Optimization

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Keywords: Interval arithmetic, Taylor models, global optimization, benchmarking.

Introduction

This work aims at providing tight bounds for the range of nonlinear functions when all variables are bounded by intervals. Interval arithmetic provides satisfying results when the ranges of uncertain variables are small or when the dependency of variables is negligible. However, when those conditions are not fulfilled, the results become excessively over-approximative. Global optimization techniques provide better bounds, but those technique have unfavorably computational complexity for high-dimensional problems. We bridge the gap between interval arithmetic and global optimization by investigating smart combinations of interval arithmetic, Taylor models, and techniques from global optimization.

Context and state of the art

Three major techniques for bounding the range of nonlinear functions exist: interval arithmetic [4], Taylor models [1], and global optimization [10]. Each of these techniques has unique features: Interval arithmetic is especially lightweight and efficient, but suffers from the dependency problem [5]. Taylor models are a good technique to fight the dependency problem, e.g. given the interval $[x] = [\underline{x}, \overline{x}]$, where $\underline{x}, \overline{x} \in \mathbb{R}$, $\underline{x} \leq \overline{x}$, $[x]-[x]$ is computed as $2[x]$ when using interval arithmetic, while the result is 0 for Taylor models. Since Taylor models per se do not provide ranges, one additionally requires techniques such as interval arithmetic to evaluate them. Finally, global optimization usually provides the tightest bounds at the cost of unfavorable complexity with respect to the number of variables.

Although a few papers exist that compare the previously mentioned techniques, there is very little work on combining them to obtain the best of all worlds. In [9], the advantages and disadvantages of interval arithmetic and Taylor models are compared. A comparison between Taylor models, boundary arithmetic, and ultra-arithmetic is performed in [6]. Taylor models and affine arithmetic are compared in [8]. The global optimization code *GlobSol* is evaluated against *COSY-GO* based on Taylor models in [7].

In this work, we obtain tight bounds by

- computing the intersection of pure interval arithmetic and Taylor models evaluated by interval arithmetic,
- evaluating Taylor models with techniques from global optimization,

and compare those results with each other and with existing techniques on a set of benchmarks that are derived from benchmarks in reachability analysis found in [2] and [3].

Conclusion

We present new combinations of interval arithmetic, Taylor models, and global optimization to bridge the gap between lightweight techniques and global optimization.

Acknowledgement

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An interval technique to check the performance of control laws applied to wind turbines

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Keywords: uncertain nonlinear systems, interval analysis, reachability analysis, control performance, set-membership tests, control validation methods

Introduction

In this work, we apply reachability analysis to design a set-membership algorithm to evaluate the performance of nominal controllers. The proposed algorithm allows to check the robustness of given controllers, designed from nominal models, against both nonlinearities and uncertainties of real systems. Three main tasks are carried out by this algorithm:

1. **Step 1**: Transform the desired control specifications as set-membership criteria.
2. **Step 2**: Compute an outer-approximation of the reachable set of the closed-loop system.
3. **Step 3**: Evaluate the set-membership criteria to check either the desired specifications are satisfied by all the possible behaviors of the closed-loop system.

Main results

Consider a controlled system described by the following differential inclusion

$$\dot{\mathbf{x}} \in \mathcal{F}\big(\mathbf{x}, \mathbf{p}, \mathbf{k}(\mathbf{x}_m, \mathbf{r})\big) \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^n$ stands for the state vector of the system. The initial value of this vector $\mathbf{x}(t_0)$ is supposed unknown but belongs into a bounded set $\mathcal{X}_0$. The system parameters gathered in the vector $\mathbf{p}$ are poorly known but they are assumed evolving in a bounded set $\mathcal{P} \subset \mathbb{R}^p$. The control law $\mathbf{k}(\mathbf{x}_m, \mathbf{r}) \in \mathbb{R}^m$ is computed from a nominal model of the real system. The vector $\mathbf{r}$ stands for the desired setpoint and $\mathbf{x}_m$ is the measurement of the state vector which is affected by bounded noises. It means that,

$$\mathbf{x}_m \in \mathbf{x} + \mathcal{E}$$

where $\mathcal{E}$ stands for the feasible domain of the measurement error. So, based on the reachability analysis [1, 2, 3] one can compute an over-approximation $[\mathcal{R}_x]([t_0, t_f], \mathcal{P}, \mathcal{E}, \mathcal{X}_0, t_0)$ of the reachable set of (2). Note that, this over-approximation must contain all possible solution $\mathbf{x}(t)$ to (2) over the time interval $[t_0, t_f]$ generated from the set of initial conditions $\mathcal{X}_0$ at the initial time t_0 and from all feasible parameter vectors $\mathbf{p} \in \mathcal{P}$ and $\mathbf{e} \in \mathcal{E}$. Before introducing our Set-Membership Control Evaluation (**SM-CE**) algorithm, let us give some useful definitions linked to the desired control performance expressed in the unknown but bounded error context.

- **Target set**: the desired behavior of the system at the steady state can be characterized by a set of state vector called target set and denoted by $\mathcal{T}_s$. The ultimate bound of the closed-loop system must be enclosed in the target set.

- **Reaching-time**: the rapidity of the system, or its reactivity, is measured by its reaching-time t_r, which is equivalent to the classical settling time. More formally, t_r is the time instant for which

$$[\mathcal{R}_x](t_r, \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_0) \subseteq \mathcal{T}_s \tag{3}$$

  and for all $t \geq t_r$ one get

$$[\mathcal{R}_x]([t_r, t], \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_r) \subseteq \mathcal{T}_s \tag{4}$$

- **Safety set**: this sub-set of the state space, denoted by $\mathcal{U}_x$, frames all admissible transient behaviors of the system. For instance, the safety set can be characterized by the state constraints and/or by authorized overshoot of the system outputs,.... So, the nominal controller must steer the reachable set of (2) such that following inclusion is satisfied.

$$[\mathcal{R}_x]([t_0, t_r], \mathscr{P}, \mathcal{X}_0, t_0) \subseteq \mathcal{U}_x \tag{5}$$

- **Feasible set**: contains all the feasible control values and denoted by $\mathcal{U}_u$. This reflects the fact that, in practice, actuators cannot follow always the theoretical control laws. Thus, the output of the nominal controller must satisfy the following constraint:

$$\mathbf{k}(\mathbf{x} + \mathscr{E}, \mathbf{r}) \subseteq \mathcal{U}_u \tag{6}$$

After the introduction of these definitions, the novelty of this work is stated in the below proposition.
Proposition: *A nominal state-feedback controller is said guaranteed if all the set-membership tests listed in the following* ***SM-CE*** *algorithm are satisfied.*

SM-CE algorithm: Inputs $(t_r, \mathbf{r}, \mathcal{T}_s, \mathcal{U}_x, \mathcal{U}_u)$

1. Compute $[\mathcal{R}_x]([t_0, t_r], \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_0)$
2. Check if $[\mathcal{R}_x](t_r, \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_0) \subseteq \mathcal{T}_s$
3. Check if for all $t \geq t_r$, $[\mathcal{R}_x]([t_r, t], \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_r) \subseteq \mathcal{T}_s$
4. Check if for all $t \in [t_0, t_r]$, $[\mathcal{R}_x]([t_0, t_r], \mathscr{P}, \mathscr{E}, \mathcal{X}_0, t_0) \subseteq \mathcal{U}_x$
5. Check if for all t, $\mathbf{k}(\mathbf{x} + \mathscr{E}, \mathbf{r}) \in \mathcal{U}_u$
6. If all the set-membership tests are true
   Output: $\mathbf{k}(\mathbf{x}_m, \mathbf{r})$ is efficient
7. Else
   Output: $\mathbf{k}(\mathbf{x}_m, \mathbf{r})$ is not efficient

To illustrate the interest of the proposed approach, the performance of a linear **LQR** controller applied to a small scale wind turbine, presented in Fig. 1, are evaluated.
Simulation and experimental results will be presented. Moreover, the robustness of the controller with respect to the uncertainty on measurements is evaluated. In fact, the maximal bound on the measurement error for which the nominal controller still conserves its performance is provided.

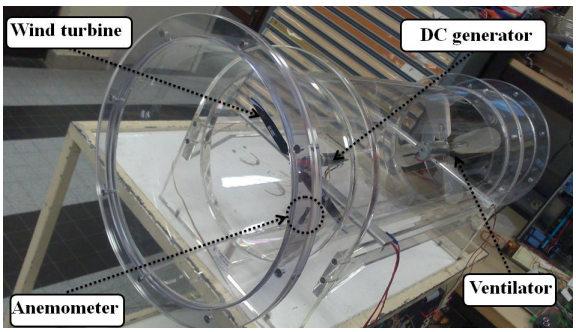

Figure 1: Small scale wind turbine (Picture of the test-bench of GIPSA-lab).

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Convergence domain of image-based visual servoing with a line-scan camera

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Keywords: interval analysis, robotics, visual servoing, stability

Introduction

Visual servoing (see [1] for an introduction to basic approaches) consists in controlling the motion of a robot by using computer vision data. Visual servoing schemes aim to minimize an error defined between a vector $\mathbf{s}$ of visual features derived from image measurements, and the vector $\mathbf{s}^*$ of the desired values of the features (which correspond to the reference position). A first classical visual servoing scheme is the *image-based visual servoing* (IBVS), that employs for $\mathbf{s}$ a set of features that are directly available in the image data. Another is position-based visual servoing, where $\mathbf{s}$ is a set of robot position parameters that have to be estimated from image data.

The classical IBVS approach is considered in the sequel. It consists in using the image coordinates of a set of points to define the feature vector $\mathbf{s}$. They are compared to their coordinates in a reference image taken at the desired camera position to control the robot motion. Stability and convergence of IBVS has been studied but remains challenging [2]. Visual servoing will be done in the so-called *eye-in-hand* configuration, in which the camera is mounted on the robot.

An holonomic 3 degrees-of-freedom robot is considered. Its configuration is given by its coordinates (x, y) in the plane and its heading θ. The robot is equipped with a line-scan camera (a camera that captures a single row of pixels, i.e an image line). For the sake of simplicity, the camera and the robot pose are assumed to be the same.

This work aims to compute the set of camera poses from which IBVS will converge to the reference pose (that corresponds to the reference image). Since classical IBVS is done by matching feature points between the current image and the reference image, we also need to check that the feature points always stay in the camera field of view.

Line-scan image-based visual servo control

A 3-DOF robot evolves in a planar world. It is equipped with a line-scan camera (1-D sensor) with pose (x, y, θ). The coordinates of a point $\mathbf{X}$ of the world in the camera body-frame are $\mathbf{X} = (X, Z)^\top$.

The projection x of a point $\mathbf{X}$ on the image line is given by the 1-D pinhole model (perspective projection):

$$x = \frac{X}{Z} = \frac{u - u_o}{f},$$

(where u is the point abscissa in pixel units, u_0 the principal point, and f the focal length). For IBVS, we take as feature x, the image line abscissa of the point.

The 3 DOF of the robot are directly controlled at the camera center, through the camera velocity vector $\mathbf{v}_c = (v_x, v_y, \omega)$, where v_x, v_y are instantaneous linear velocities and ω rotation speed. This leads to movement of points in the camera frame:

$$\dot{\mathbf{X}} = \begin{pmatrix} \dot{X} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} -v_x + \omega Z \\ -v_y - \omega X \end{pmatrix},$$

and thus to movement of their projection on the image line: $\dot{x} = \frac{-v_x + x v_y}{Z} + (1 + x^2)\omega$.

We then define the *interaction matrix* $\mathbf{L}_x$ such that $\dot{x} = \mathbf{L}_x v_c$ with $\mathbf{L}_x = \begin{pmatrix} \frac{-1}{Y} & \frac{x}{Y} & 1 + x^2 \end{pmatrix}$. Since at least three points are required to control the 3-DOF. The feature vector is thus $\mathbf{x} = (x_1, x_2, x_3)$ and the corresponding interaction matrix $L_{\mathbf{x}}$ is obtained by stacking interaction matrices for three points.

Control law is given by $\mathbf{v}_c = -\lambda \mathbf{L}^+_{x^*}(\mathbf{x} - \mathbf{x}^*)$, where $\mathbf{L}^+_{x^*}$ is the Moore-Penrose pseudo-inverse of the interaction matrix for the desired position.

Convergence domain computation

In addition to the differential equation derived from control law, additional constraints are implemented and have to be verified along the flow. They define the acceptable configurations domain, ensuring the the feature points stay in the camera field of view, and robot does not leave the working space.

We compute a guaranteed approximation of the convergence domain for the 3DOF IBVS task in two steps:

- First, we compute an attraction domain of the desired position using Lyapunov theory. (see [3] to compute a domain around an asymptotical stable point).
- Then, we employ guaranteed integration to iteratively increase the proven convergence domain, using a similar approach to [4]. Using subpavings and a branch and bound method, an inner and outer approximations of the convergence domain are computed. Boxes whose flow at a finite time t is a subset of the already proven convergence domain, and whose flow over $[0, t]$ stays in the acceptable configurations domain are added to the proven convergence domain. Boxes that are outside the acceptable configurations domain, or whose flow leave it, are eliminated.

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Robust output-feedback control for a class of interval Model: Application to a piezoelectric tube actuator

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Keywords: Robust output-feedback, interval models, SIVIA (Set Inversion via Interval Analysis), piezoelectric tube actuator.

Introduction

During the last decades, the problem of designing robust control laws for interval systems has gained much attention [1-3]. In fact, intervals permit to bound easily parametric uncertainties during the modeling which lead therefore to interval systems. Different controllers synthesis approaches have been developed for state-feedback architecture and cascade architecture. In this presentation, we present an output-feedback controller design for linear and time-invariant interval systems. We apply afterwards the results to a smart material based actuator. For that we propose to use the robust output-feedback with regional pole assignment to provide a guaranteed stability margin and a desired performance.

Problem formulation

In our work we address the problem of output-feedback controller with integral compensator for an interval state-space system with realization $([A],[B],[C],[D])$. The objective of the suggested output-feedback control is to find the controller matrix gain $[K]$ such that the state matrix of the closed-loop system possesses its eigenvalues within a given subregion $\Omega_{Desired\,region}$.

Main results

An algorithm based on Set Inversion Via Interval Analysis (SIVIA)[3] combined with interval eigenvalues computation [4-6] and eigenvalues clustering techniques [2] is proposed. This recursive SIVIA-based algorithm allows to approximate with subpaving the set solutions $[K]$ that satisfy the inclusion (eq.7):

$$eig\left[A_{aug\_cl}([A],[B],[C],[D],[K])\right] \subseteq \Omega_{Desired\,region} \tag{7}$$

where A_{aug_cl} is the augmented closed-loop matrix with integral compensator and $\Omega_{Desired\,region}$ is the desired subregion of eigenvalues.

The solution $[K]$ of the above algorithm guarantees the specifications for the closed-loop. To choose the optimal gain $[K_{opt}]$ that will ensure the best behaviors for the closed-loop among this solution $[K]$, we propose to apply a LQR synthesis for an output-feedback architecture with the aid of edge theorem [6].

Finally, the effectiveness of the proposed algorithm is illustrated by a real experimentation on a piezoelectric tube actuator.

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Interval Methods for Robust Variable-Structure Control with One- and Two-Sided State Constraints

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Keywords: Nonlinear control; Sliding mode; Variable-structure control; Dynamic systems

Abstract

As shown in previous work, interval techniques can be used for the implementation of real-time capable controllers. These controllers are either generalizations of model-predictive approaches or variants of variable-structure techniques [3, 6]. These variable-structure approaches are motivated by the general methodology of sliding mode control [10, 9], where the guaranteed asymptotic stability of the closed-loop system dynamics is proven in real time by firstly defining a suitable Lyapunov function candidate. Secondly, asymptotic stability has to be ensured by parameterizing the variable-structure gain in such a manner that the time derivative of this Lyapunov function candidate becomes negative definite along the current state trajectory. This property of negative definiteness has to be guaranteed despite any parameter or disturbance value within corresponding predefined intervals.

To prevent the violation of state constraints in a rigorous way, barrier Lyapunov function techniques can be implemented. They can be used to handle either one- or two-sided state constraints in a verified manner. Here, the state constraints either represent feasibility properties of controllers (e.g. efficiency and technological applicability of a control law) or safety constraints. The case of one-sided state constraints was investigated in detail in [4, 3]. There, the temperature control for a high-temperature fuel cell was considered, for which the maximum stack temperature has to be limited to an upper threshold value. This value has to be chosen in such a way that local overtemperatures are prevented which may lead to the risk of an accelerated aging of the fuel cell stack.

Moreover, the use of two-sided state constraints was introduced in [5] to make sure that neither *hard lower bounds* nor *hard upper bounds* are violated for selected state variables. As for the cases above, asymptotic stability has to be guaranteed by the application of interval methods in real time.

As it is well-known, classical sliding mode techniques consist firstly of a so-called equivalent control to enable tracking of sufficiently smooth reference trajectories. For states located exactly on the sliding surface (specifying the desired closed-loop dynamics) this part of the control signal is the only active component as long as no disturbances or uncertain parameters influence the dynamics. Secondly, model imperfections as well as state values which are not (exactly) located on this sliding surface are counteracted by using a variable-structure component. In many classical approaches, the amplitude of this variable-structure signal is selected to be constant. However, choosing a constant variable-structure gain may have the drawback of unnecessarily large chattering which needs to be reduced as far as possible to avoid non-advantageous actuator wear and energy consumption.

In contrast to classical sliding mode approaches, the fundamental idea of interval-based sliding mode control is the online adaptation of the control signal in terms of intervals for the uncertain system states and uncertain parameters [7]. This can be performed in real time by using software libraries for basic interval functionalities [2]. A further difference to many classical sliding mode

procedures is the fact that the interval-based counterparts can also be applied directly to system models which are not directly given in a nonlinear controller canonical form. As shown in [8], this property simplifies the controller design especially for the control of non-flat system outputs [1].

In this contribution, numerous control engineering examples are presented which visualize the parameterization and application of interval-based variable-structure controllers with both one- and two-sided state constraints. An outlook on future work will address first attempts on how to transfer these control procedures to a cascaded stabilization of uncertain dynamic processes by generalizing the backstepping control methodology to systems with interval uncertainty.

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Decision Making Under Twin Interval Uncertainty

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Keywords: decision making, twin intervals

Introduction

According to the decision theory, a reasonable person should select an alternative a for which an appropriate objective function $u(a)$ – called *utility* – attains its largest possible value. The utility function is usually selected in such a way that if for some action a, we know the probabilities p_i of different outcomes o_i, then the utility of a is equal to the expected value of the utilities: $u(a) = \sum_{i=1}^{n} p_i \cdot u(o_i)$. Such a utility function is determined uniquely modulo a linear transformation $u(a) \to u'(a) = k \cdot u(a) + \ell$, where $k > 0$.

For some actions, we have no information about the probabilities of different outcomes o_i. In this case, all we know about the expected utility $u(a)$ is that it is in the interval $[\underline{u}(a), \overline{u}(a)]$, where $\underline{u}(a) = \min_i u(o_i)$ and $\overline{u}(a) = \max_i u(o_i)$. To make decisions under such interval uncertainty, we must, in particular, we able to compare such actions with actions for which we know the expected utility $u(a)$. Thus, we need to be able to assign, to each interval $[\underline{u}(a), \overline{u}(a)]$, an equivalent utility value $u(a)$.

A way to assign such an equivalent utility value was proposed by a Nobel Prize winner Leo Hurwicz: $u(a) = \alpha \cdot \overline{u}(a) + (1-\alpha) \cdot \underline{u}(a)$, where $\alpha \in [0,1]$ describes the optimism level of the decision maker: $\alpha = 1$ means that the decision maker only takes into account the best-case scenario, $\alpha = 0$ means that only the worst-case scenario is taken into account, and $\alpha \in (0,1)$ means that both best-case and worst-case scenarios are taken into account. It turns out that the Hurwicz assignment is invariant relative to linear transformations of utility – and it is the *only* invariant assignment.

In practice, sometimes, we do not know the exact values of $\underline{u}(a)$ and $\overline{u}(a)$. For example, we may only know the bounds on each of these bounds: we know that $\underline{u}(a) \in [\underline{u}^-(a), \underline{u}^+(a)]$ and that $\overline{u}(a) \in [\overline{u}^-(a), \overline{u}^+(a)]$. Such a situation is known as a *twin interval*. How can we make decisions under such twin interval uncertainty?

General Idea

Our main idea is to use Hurwicz assignment several times. Specifically, for the lower bound $\underline{u}(a)$, all we know that it is in the interval $[\underline{u}^-(a), \underline{u}^+(a)]$. According to the Hurwicz assignment, this is equivalent to having $\underline{u}(a) = \alpha \cdot \underline{u}^+(a) + (1-\alpha) \cdot \underline{u}^-(a)$.

Similar, by applying the Hurwicz assignment to the interval $[\overline{u}^-(a), \overline{u}^+(a)]$, we conclude that the upper bound is equivalent to $\overline{u}(a) = \alpha \cdot \overline{u}^-(a) + (1-\alpha) \cdot \overline{u}^+(a)$.

Thus, the original twin interval is equivalent to the interval $[\underline{u}(a), \overline{u}(a)]$, for which the Hurwicz assignment produces an equivalent value

$$u(a) = \alpha \cdot \overline{u}(a) + (1-\alpha) \cdot \underline{u}(a) =$$
$$\alpha^2 \cdot \overline{u}^{+}(a) + \alpha \cdot (1-\alpha) \cdot \overline{u}^{-}(a) + \alpha \cdot (1-\alpha) \cdot \underline{u}^{+}(a) + (1-\alpha)^2 \cdot \underline{u}^{-}(a).$$

Alternatively, we can consider the situation differently: we do not the actual interval. The smallest possible interval – in terms of component-wise order – is $[\underline{u}^{-}(a), \overline{u}^{-}(a)]$. The largest possible interval is $[\underline{u}^{+}(a), \overline{u}^{+}(a)]$. For the smallest interval, Hurwicz's equivalent utility is

$$u^{-}(a) = \alpha \cdot \overline{u}^{-}(a) + (1-\alpha) \cdot \underline{u}^{-}(a).$$

For the largest interval, the equivalent utility is $u^{+}(a) = \alpha \cdot \overline{u}^{+}(a) + (1-\alpha) \cdot \underline{u}^{+}(a)$. Thus, possible values of utility form an interval $[u^{-}(a), u^{+}(a)]$. For this interval, the Hurwicz equivalent value is $\alpha \cdot u^{+}(a) + (1-\alpha) \cdot u^{-}(a)$, which is, as one can check, exactly equal to the above value.

Applications

Some physical quantities we can measure directly. However, in many practical situations, we are interested in a quantity y which is difficult (or even impossible) to measure directly. To estimate the values of such a quantity, a natural idea is to find auxiliary easier-to-measure quantities $x_1, \ldots, x_n$ which are related to y by a known dependence $y = f(x_1, \ldots, x_n)$, and then use the results $\widetilde{x}_i$ of measuring x_i to compute the estimate $\widetilde{y} = f(\widetilde{x}_1, \ldots, \widetilde{x}_n)$.

Often, the only information that we have about each measurement error $\Delta x_i \stackrel{\text{def}}{=} \widetilde{x}_i - x_i$ is the upper bound Δ_i on its absolute value: $|\Delta x_i| \le \Delta_i$. In this case, the only information that we have about the actual (unknown) value x_i is that x_i belongs to the interval $[\underline{x}_i, \overline{x}_i] = [\widetilde{x}_i - \Delta_i, \widetilde{x}_i + \Delta_i]$. Usually, we do not know the dependence between the values x_i (and we do not even know if there is a dependence).

The traditional interval approach to this situation is to conclude that y belongs to the range $\mathbf{y} \stackrel{\text{def}}{=} \{f(x_1, \ldots, x_n) : x_i \in [\underline{x}_i, \overline{x}_i]\}$. However, in reality, the range $[\underline{y}, \overline{y}]$ depends on the possible dependence between the variables x_i. In general, $\underline{y} = \inf\{f(x_1, \ldots, x_n) : (x_1, \ldots, x_n) \in S\}$ and $\overline{y} = \sup\{f(x_1, \ldots, x_n) : (x_1, \ldots, x_n) \in S\}$, where S is a set for which, for every i, the projection $\pi_i(S)$ on the i-th axis coincides with $[\underline{x}_i, \overline{x}_i]$.

For different sets S, we have different values $\underline{y}$ and $\overline{y}$. It is therefore desirable to compute the ranges $[\underline{y}^{-}, \underline{y}^{+}]$ and $[\overline{y}^{-}, \overline{y}^{+}]$ of the corresponding values – i.e., to compute the corresponding twin interval. Here, $\underline{y}^{-}$ and $\overline{y}^{+}$ are the endpoints of the range $\mathbf{y}$, which can computed by the usual interval techniques, so the question is how to compute $\underline{y}^{+}$ and $\overline{y}^{-}$.

In the talk, we show how to compute these bounds for the practically important case when quadratic and higher order terms in Δx_i can be safely ignored, and thus, $\Delta y = \widetilde{y} - y = \sum_{i=1}^{n} c_i \cdot \Delta x_i$, where $c_i = \dfrac{\partial f}{\partial x_i}(\widetilde{x}_1, \ldots, \widetilde{x}_n)$. In turns out that in this case,

$$\overline{y}^{-} = \widetilde{y} + 2 \max_i(|c_i| \cdot \Delta_i) - \sum_{i=1}^{n}(|c_i| \cdot \Delta_i) \text{ and } \underline{y}^{+} = \widetilde{y} - 2 \max_i(|c_i| \cdot \Delta_i) + \sum_{i=1}^{n}(|c_i| \cdot \Delta_i).$$

In particular, for arithmetic operations like $f(x_1, x_2) = x_1 + x_2$, this means that the sum, product, etc., of two intervals is now viewed as a twin interval.

We can then use the above formulas for decision making under twin interval uncertainty to make appropriate decisions.

Interval arithmetic in GNU Octave

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Keywords: IEEE Std 1788-2015, interval arithmetic, decorated intervals, programming, library, interpreted language, unit test suites, free software

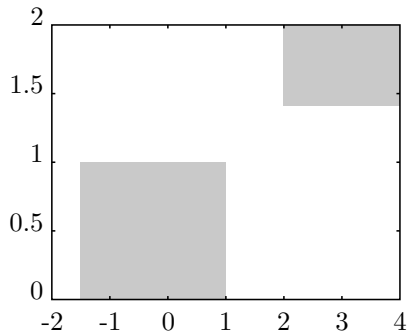
Introduction

The IEEE standard for interval arithmetic [1] has been released in 2015. Several interval arithmetic libraries have been created, which vary greatly in their philosophy, completeness and—most important—mathematical definition of certain functions and arithmetic evaluation. The standard grants support for several interval arithmetic flavors, but fights incompatibilities on many layers: interval arithmetic applications shall be portable, predictable, and reproducible. This is especially important since interval arithmetic shall lead to reliable results. Also, a common standard is necessary to accelerate the availability of fast interval operations in hardware.

The author has added support for interval arithmetic [2] into GNU Octave, a high-level programming language for numerical computations [3]. This has led to the first complete *standards conforming* library with various beneficial effects for the area, which shall be presented in detail.

Methods

Interval support for GNU Octave is implemented as a loadable package which adds an interval data type and interval operations. The package has been implemented from scratch following the IEEE standard for interval arithmetic. To keeps things simple, the implementation sticks to the set-based model of intervals, stored in inf-sup representation with boundaries in the `binary64` number format.




```
pkg load interval
intervaltoexact (infsup ("0.1"))
  ⇒ ans = [0X1.9999999999999P-4, 0X1.999999999999AP-4]
x = infsupdec ("-10??d [-6/4, 1] [2, 4]")
  ⇒ x = 1×3 interval vector

        [-Inf, -10]_dac   [-1.5, +1]_com   [2, 4]_com

y = sqrt (x)
  ⇒ y ⊂ 1×3 interval vector

        [Empty]_trv   [0, 1]_trv   [1.4142, 2]_com

plot (x, y)
```

Figure 2: Demonstration of functions with Octave. 1, a tight bare interval around the decimal number 0.1 whose boundaries are printed in hexadecimal. 2, a decorated interval vector where the entries use various forms of interval literals. 3, interval decorations carry information about the course of evaluation. 4, plotting of intervals with the result shown on the left.

Another early design choice is a tight integration into the Octave language, and thus interval vectors and interval matrices may be used as usual in this environment.

To produce valid and accurate results, GNU MPFR is the backbone of all arithmetic operations. In particular, this makes the implementation highly portable since it does not depend on rounding mode switches or platform-dependent arithmetic operations. It is known to produce correctly-rounded and identical results among the following architectures: x86-64, x86, ARM, PPC, MIPS, and s390x. A test suite of over 9000 unit tests has been derived from various other interval arithmetic libraries, which incorporates test cases from libieeep1788, MPFI, FI_LIB, and C-XSC.

Results

From tight integration into Octave comes an easy to use tool, see Figure 2. The interactive[1] programming environment quickens prototyping of interval algorithms and brings capabilities for post-processing of results. The interval package is also greatly compatible with the educational book *Introduction to interval analysis* [4], which has originally been written for MATLAB.

However, the major benefit is that researchers may use IEEE Std 1788-2015 for their research today. For example, this is one of the first implementations of decorated intervals and the total behavior of interval arithmetic functions is well defined by an international standard.

Beyond the required operations from the standard document there are several other features: plotting of intervals (in 2D and 3D), solvers and matrix operations, set inversion via interval analysis (SIVIA) and contractor programming. Combination with other packages for Octave, for example a symbolic computation package, yields further possibilities.

The author has compiled a portable test suite of interval arithmetic test cases, which can verify the correctness and standards conformance of other libraries as well, and plug-ins for several popular interval arithmetic libraries have been prepared at `https://github.com/oheim/ITF1788`. The author hopes that this will leverage the distribution and implementation of standards conforming interval arithmetic.

Acknowledgement

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[1] The interpreted Octave language uses a runtime "duck typing" approach in contrast to compile time static typing, which one might prefer in a validated computing scenario. However, the former makes it easier to try out new and existing algorithms in interval computations.

Certification of Roundoff Errors with SDP Relaxations and Formal Interval Methods

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Keywords: semidefinite programming, polynomial optimization, moment relaxations, sums of squares, floating-point arithmetic, proof assistant, roundoff error, sparsity pattern

Introduction

Roundoff errors cannot be avoided when implementing numerical programs with finite precision. The ability to reason about rounding is especially important if one wants to explore a range of potential representations, for instance in the world of FPGAs. This problem becomes challenging when the program does not employ solely linear operations as non-linearities are inherent to many computational problems in real-world applications.

Existing solutions to reasoning are limited in presence of nonlinear correlations between variables, leading to either imprecise bounds or high analysis time. Furthermore, while it is easy to implement a straightforward method such as interval arithmetic, sophisticated techniques are less straightforward to implement in a formal setting. Thus there is a need for methods which output certificates that can be validated inside a proof assistant.

Context and state of the art

Semidefinite programming (SDP) is relevant to a wide range of mathematical fields, including combinatorial optimization, control theory, matrix completion. In 2001, Lasserre introduced a hierarchy of SDP relaxations for approximating polynomial infima. Recent applications of this SDP hierarchy were developed in either applied mathematics or computer science. In real algebraic geometry, these hierarchies yield approximations as closely as desired of exact projections of semialgebraic sets. In nonlinear optimization, SDP hierarchies allow to compute Pareto curves (associated with multicriteria problems) as well as solutions of transcendental problems. These hierarchies can also be easily interleaved with computer assisted proofs.

Combining SDP relaxations and formal interval methods

We present a formal framework to provide upper and lower bounds of absolute roundoff errors.

The framework for **upper bounds** [3] is based on optimization techniques employing semidefinite programming (SDP) and sparse sums of squares certificates, which can be formally checked inside the Coq theorem prover. A common issue is that semidefinite programming use finite precision floating point numbers, thus the sums of square certificate is only correct up to a certain numerical error. We address this issue by using, again, finite precision floating point numbers, this time inside Coq. More precisely, we consider polynomials in Coq whose coefficients are intervals of floating point numbers and we use the Coq library of floating point intervals constructed by Guillaume Melquiond.

The framework for **lower bounds** is based on a new hierarchy of convergent robust SDP approximations for certain classes of semialgebraic optimization problems. This hierarchy yields a monotone

non-increasing sequence of lower bounds converging to the global maximum of a polynomial f over a simple semialgbraic set (e.g. box) $\mathbf{K} = \mathbf{X} \times \mathbf{E}$, when f has linear dependency on the variables in $\mathbf{E}$. By contrast with the converging sequence of bounds in [1], we prove that nonnegativity of f over $\mathbf{K}$ is equivalent to semidefinite positiveness of countably many uncertain moment matrices, with perturbations in $\mathbf{E}$. Each resulting robust program in this hierarchy can be exactly solved via SDP by using [2].

Our tool covers a wide range of nonlinear programs, including polynomials and transcendental operations as well as conditional statements. We illustrate the efficiency and precision of this tool on non-trivial programs coming from biology, optimization and space control.

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Avoiding fake boundaries in interval analysis

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Keywords: interval analysis, contractor programming, fake boundaries

Introduction

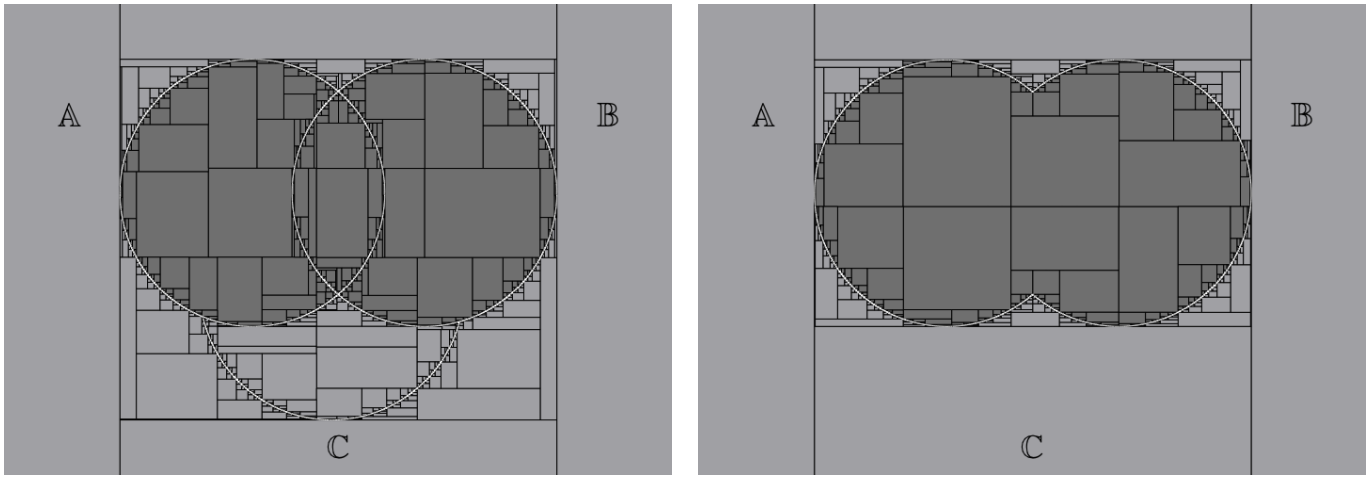
When combining a set of functions using unions, intersections and/or complements inside a paver, in a contractor programming [1] or separator programming [2] approach, we may get unwanted unclassified tiny boxes. For these boxes, that are clearly inside or outside the solution set, are named as *fake boundary.* To illustrate this phenomenon, consider three sets $\mathbb{A}$,$\mathbb{B}$ and$\mathbb{C}$ of $\mathbb{R}^2$ and define the set of operations:

$$\mathbb{X} = (\mathbb{A} \cup \mathbb{B} \cup \mathbb{C}) \cap \left(\mathbb{A} \cup \mathbb{B} \cup \overline{\mathbb{C}}\right)$$

Using the fundamental laws of set algebra and De Morgan's rules, we can describe its complementary set by:

$$\overline{\mathbb{X}} = \left(\overline{\mathbb{A}} \cup \overline{\mathbb{B}}\right) \cap \left(\overline{\mathbb{A}} \cup \mathbb{B}\right) \cap \left(\mathbb{A} \cup \overline{\mathbb{B}}\right)$$

From these two expressions, and using contractor algebra, an inner and an outer approximation for $\mathbb{X}$ generates fake boundaries, as illustrated by Figure 3 (a). Whereas, the desired paving is presented in Figure 3 (b).



(a) (b)

Figure 3: Paving with fake boundaries (a) and paving obtained with our method (b)

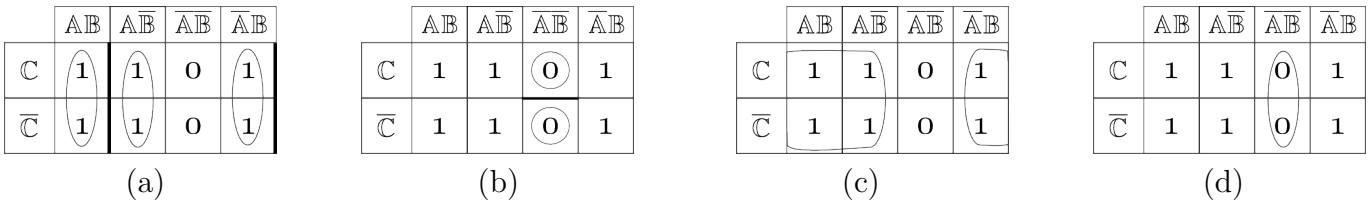


Figure 4: Karnaugh map for the initial expressions of $\mathbb{X}$ (a) and $\overline{\mathbb{X}}$ (b). Karnaugh map obtained after enlarging blocks the equation $\mathbb{X}$ (c) and $\overline{\mathbb{X}}$ (d).

Why such fake boundaries exists?

The algebraic expressions $\mathbb{X}$ and $\overline{\mathbb{X}}$ could be represented by a Karnaugh map, as shown in Figure 4. Each block corresponds to a term of the conjunctive form in the expression $\mathbb{X}$ and $\overline{\mathbb{X}}$. For instance, the first term $\left(\overline{\mathbb{A}} \cup \overline{\mathbb{B}}\right)$ of $\overline{\mathbb{X}}$ corresponds to vertical block of ones in the column $\mathbb{A}\mathbb{B}$ of the Figure 4 (a).

This block tells that $\mathbb{G} = (\mathbb{A} \cap \mathbb{B}) \cap \partial\mathbb{C}$, where $\partial\mathbb{C}$ denotes the boundary of $\mathbb{C}$ cannot be a fake boundary. Indeed, a tiny box on the boundary of $\mathbb{G}$ will be eliminated by the two contractors associated to $\mathbb{A}$ and $\mathbb{B}$. Then, $\mathbb{G}$ will be classified as inside of $\mathbb{X}$. Therefore, an inner or outer fake boundary exists when two ones or two zeros are neighbor but not in the same block. This is illustrated by the thick bars at the Karnaugh map in Figure 4.

Eliminating fake boundaries

In order to eliminate these fake boundaries, a algebric expression should merge neighboring groups with same value, zero or one. Fortunately, it is easy to be done once that a Karnaugh map is built. Thus, all blocks can be extended, resulting in following equation:

$$\mathbb{X} = \mathbb{A} \cup \mathbb{B} \quad \text{and} \quad \overline{\mathbb{X}} = \overline{\mathbb{A}} \cap \overline{\mathbb{B}}$$

The associated blocks of these expressions are given in Figure 4 (c) and (d) and the resulting paving, without undesired frontiers, is presented at Figure 3 (b). Note also that the complexity of both expressions were drastically reduced, which could represent less computation at running time.

Conclusion

Therefore, the existence of a fake boundary is directly related to the algebric expressions associated. In this sense, a simplification by the Karnaugh method could be utilized to not only eliminate the presence of undesired boxes, as also to reduce the number of bisections performed by a SIVIA approach [3].

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Interval Based Parallel Computing of the Viability Kernel

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Keywords: Interval based computation, Viability theory, parallel computing, many core architecture

Abstract

Viability theory provides a collection of concepts and algorithms to study continuous dynamical systems (stability, reachability, Validation and Verification). Interval based computation provides nice guaranteed numerical methods for set approximation (over and under approximations of sets) as those defined in viability theory. Refined interval techniques as contractor programming and guaranteed integration allow in particular to implement the viability kernel and the capture basin algorithms. A parallel computing architecture based on 256 processors has been used for performing dynamical system integration in interval context. Results based on the car on the hill benchmark will be presented before studying more complex differential game problems (kinematics with two players / controls as in pursuit evasion games). The main contribution of this study is to use a many core architecture that allows real time performances. The features of these algorithms and of this processing architecture are compatible with the various constraints that happen in embedded systems as light UAVs and as ground mobile robots.

Guaranteed confidence region characterization for source localization using LSCR

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Keywords: interval analysis, sivia, LSCR, localization, wireless sensor network, confident region

Introduction

In wireless sensor networks (WSN), localizing each sensor of the network is a fundamental issue, since locations are often required to process the collected information. It is also a challenging non-linear parameter estimation problem from noisy measurements. Localization is usually performed considering the different types of signals transmitted by some anchor nodes, see [2] for an overview.

This paper considers localization from Receiver Signal Strength (RSS) measurements, which has received significant attention for more than a decade. In most of the papers, the noise is assumed to be normal or log-normal, see [5], which allows to apply maximum likelihood (ML) or maximum *a posteriori* estimation techniques and asymptotic characterization of the estimator confidence region, evaluating, *e.g.*, its Cramér-Rao bound (CRB). Alternative bounded-error localization techniques have been proposed in [4] to get set estimates. Nevertheless, in practice, noise samples do not necessarily follow a Gaussian distribution, and the bounds considered in bounded-error estimation are either often violated because they are too small, or lead to huge unexploitable solution sets when the bounds are too large.

Recently, the *Leave-Out Sign-dominant Correlation Regions* (LSCR) [1] has been introduced to define non-asymptotic confidence regions (NACR) for estimators considering very mild assumptions on the noise samples corrupting the measurements. In [3], the characterization of confidence regions as defined by LSCR has been formulated as a set-inversion problem which can be solved using SIVIA.

The aim of this paper is to show that LSCR can be put at work to characterize NACR in the context of source localization from RSS measurements. Several tools borrowed from global optimization using interval analysis (contractors, monotonicity, mixed centred forms, etc.) have been analyzed to improve the efficiency of set inversion in this context. Promising results have been obtained compared to state-of-the-art techniques, however, much remains to be done.

Comparison of LSCR and ML

Fig. 1 compares the results of source localization from RSS using a classical ML approach with CRB evaluation and NACR evaluation defined by LSCR. A square region of 30m×30m is considered. $N_\mathrm{a} = 5$ anchor nodes, which location is represented by stars, have perfect knowledge of their location; $N = 32$ nodes of the WSN represented by '+' have to determine their location from RSS measurements

of the signals broadcast by the anchors. The Okumura-Hata model is used to describe the RSS as a function of the distance between an anchor and the receiving node

$$y_{a_k} = P_0 - 10 n_P \log_{10} \left(\| \boldsymbol{\theta}_0 - \boldsymbol{\theta}_{a_k} \| / d_0 \right) + \varepsilon_{a_k}, \tag{8}$$

where y_{a_k} is an RSS measurement of the signal emitted by the anchor a_k; P_0 is a known reference power measured at a distance d_0 of the anchor; n_P is the path loss exponent; ε represents the measurement noise (usually considered as log-normal). The location of the node of interest is $\boldsymbol{\theta}_0$, the location of the anchor a_k is $\boldsymbol{\theta}_{a_k}$. We assume that $\boldsymbol{\theta}_0$ and n_P are unknown and that P_0 as well as n_p are the same for all anchor-node pairs. The noise corrupting data is taken as Gaussian-Bernoulli-Gaussian (GBG), *i.e.*, it is $\mathcal{N}(0, \sigma_0^2)$ with a probability $1-p$ and $\mathcal{N}(0, \sigma_1^2)$ with a probability p. Here, $\sigma_0^2 = 2$, $\sigma_1^2 = 5$, and $p = 10\%$. The estimation of its location and of n_p is performed by each node considering $N_m = 10$ independent RSS measurements obtained from each anchor.

Fig. 1(a) shows the results obtained using the ML approach where Levenberg-Marquardt's algorithm has been used for minimization with random initialization. The small green ellipses represent the CRB corresponding to a 90% confidence region centered on the estimated location. The lines connecting the true to the estimated locations represent the localization error. Fig. 1(b) shows the projections of the approximations of the 90% confidence regions as defined by LSCR and characterized using SIVIA. The projection of the uncertain boxes is in yellow. The projection of the boxes which have been proved to be within the 90% confidence region are in green. The proposed approach is more robust to uncertainties on the noise corrupting the data than classical approaches.

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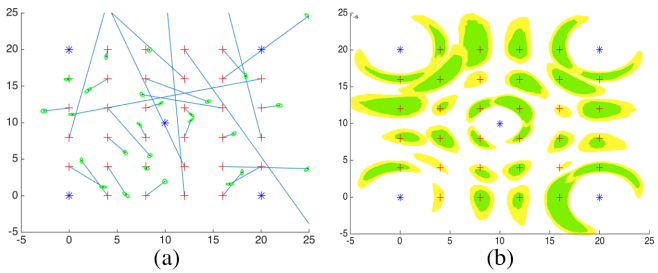

Figure 5: Node location (red crosses) estimates from RSS of waves emitted by anchors (blue stars). (a) Estimates and confidence regions from ML estimation (b) Confidence regions defined by LSCR

Image-based Mobile Robot localization using Interval Methods

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Keywords: Interval Analysis, Image-based pose estimation, Localization

Introduction

To navigate, robots need to locate themselves. In the case of unmanned aerial vehicles (UAVs), the standard solution consists in using GPS, INS and Compass measurements. Yet, this solution is not appropriate in difficult environment like indoors or close to large buildings; where GPS signals losses and erroneous orientation measurements from the compass are observed.

Our aim here is to provide a reliable pose confidence domain; a box in which we are sure the robot is situated. In other words, we wish to compute a "safety area" around the robot that should be considered by the controller in order to avoid collisions with eventual robots or objects present in the navigation environment.

GPS and compass unavailability can be overcome by using a camera in order to enhance robot localization. In computer vision, many solutions to pose estimation from a set of known landmarks (such as POSIT, PnP, etc. see [1] for a survey) exist but classically provide a punctual estimate of the robot location. Interval analysis is a powerful tool for rigorous uncertainty propagation (see [2] for a 3D vision application, and [3] for GPS position uncertainty domain computation). To quantify the robot pose uncertainty, we propose an interval-based set-membership approach, which computes over time a bounding box of the pose of the robot, taking image measurements and landmark positions uncertainties into account.

Method

We propose here an algorithm that tracks the robot trajectory using uncertain image measurements of landmarks that are in the UAV's camera field of view. Starting from an initial box (representing the initial search space), at each image acquisition epoch, we contract the robot pose box by using the camera projection equations (expressed as constraints) that give the correspondence between a point from the world to the image plane. This step can be seen as a correction step.

Between two image acquisitions, the robot evolution is predicted using velocities measurements acquired by embedded proprioceptive sensors.

Image-based pose estimation

Let $x^c$and X^wbe the vectors containing respectively the measurements in the image plane and the landmarks positions in the world frame (represented with homogeneous coordinates). Then, the

observation model is in the form of:

$$x^c = K \; \Pi \; {}^cT_w \; X^w \tag{9}$$

where,

- K is the camera intrinsic parameters matrix,
- Π is the perspective projection matrix (from meters to pixels),
- and ${}^cT_w$ an homogeneous matrix that permits to compute the frame transformation (projection from world to image). This matrix is a function of the robot pose (rotations using Euler angles representation ${}^cR_w$ and position ${}^ct_w$) in the world frame and is expressed as follow :

$$ {}^cT_w = \begin{pmatrix} {}^cR_w & {}^ct_w \\ 0_{3x1} & 1 \end{pmatrix} \tag{10}$$

The classical pose estimation methods (e.g. the POSIT algorithm) relies on finding the pose (best matrix) that minimizes the re-projection error.

In the proposed interval-based method, since the measurements have some bounded uncertainty, each x_i^c and X_i^w is an *interval vector*, such that $x_i^c \in [x_i^c]$ and $X^w \in [X_i^w]$. We then define a constraint satisfaction problem (CSP) using ${}^cR_w(\varphi, \theta, \psi)$ and ${}^ct_w(X, Y, Z)$ as *variables* and we define its *constraints* by applying Eq.(1) on each pair $i = 1 : N$ (N number of observed landmarks) of image-world points. We also add geometric constraints in our CSP considering the limitations and characteristics of some UAV parameters and movements feasibility. Afterwards, we build a contractor that contracts the initial box with respect to the whole system's constraints.

A test case will be presented to illustrate the method and quantify the pose uncertainty. As classical control strategies rely on a point position estimate, our method which provides rigorous bounds (but no point estimate) is later combined with POSIT (which gives a precise point estimate).

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Range Only Multistatic Radar Detection of a Windfarm Based on Interval Analysis

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Introduction

Multistatic Radar system is a radar network of spatially separated antennas of transmitters and receivers, where each pair of transmitter and receiver is referred to as a Bistatic radar pair [1].

Many research publications investigate multistatic radar target detection and tracking based on the mathematic of probability density function PDF such as, Kalman filter, Particle filter or Probability Hypotheses Density (PHD) filter [2]. However, interval analysis techniques show promising potential in target detection and tracking. Mostly, because of solving a problem with degree of uncertainty is adequately represented by intervals [3][4]. The solution value is assured within the interval and bounded by the margin of error as the lower bound and upper bound of the interval.

This study investigates the implementation of interval analysis approach in detecting a windfarm model of 3x3 turbines by a multistatic radar system of four receivers and two transmitters. The detection was based on range only measurements that are associated within a centralized data fusion unit.

Range only measurements

Range only detection technique calculates the target position based on the association of range measurements collected from multiple sensors for each target. (Figure 1) below illustrates a multistatic return from the 3x3 windfarm model at each receiver. The peak in power received translates into target detection at a constant range. Noticeably, the return signatures from the windfarm differ from one receiver to another, due to the difference in turbine orientation for each receiver point of view. For instance, the first receiver has one power peak per turbine while the fourth receiver registers multiple peaks for each turbine. This aspect created further challenges in data association and false detection elimination.

Each registered constant range forms an elliptical shape around the bistatic pair (Figure 2). Evidently, with the absence of angle of arrival knowledge, the target can be located at any point on the ellipsoid. Therefore, multiple associated measurements from different bistatic pairs are necessary to locate the target at the intersection area of those measurements.

Detection/Figures/001.png

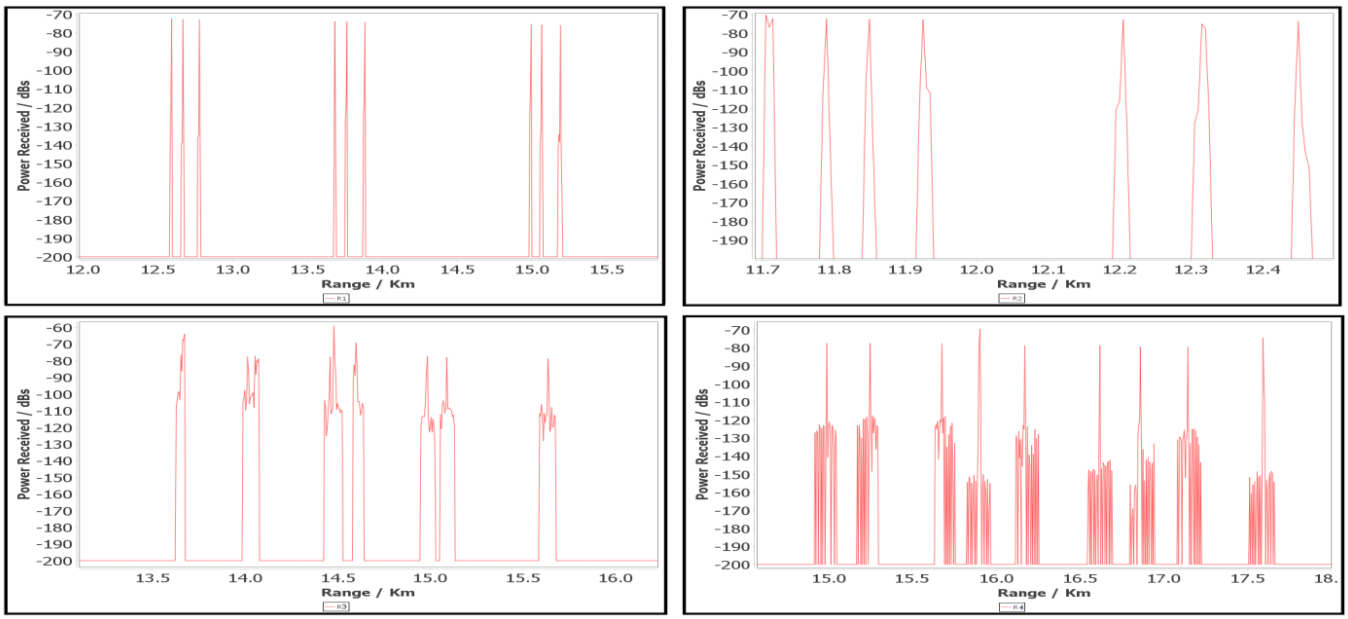

Figure 6: The 3x3 Windfarm radar return at each of the four receivers at a time step

Detection/Figures/002.png

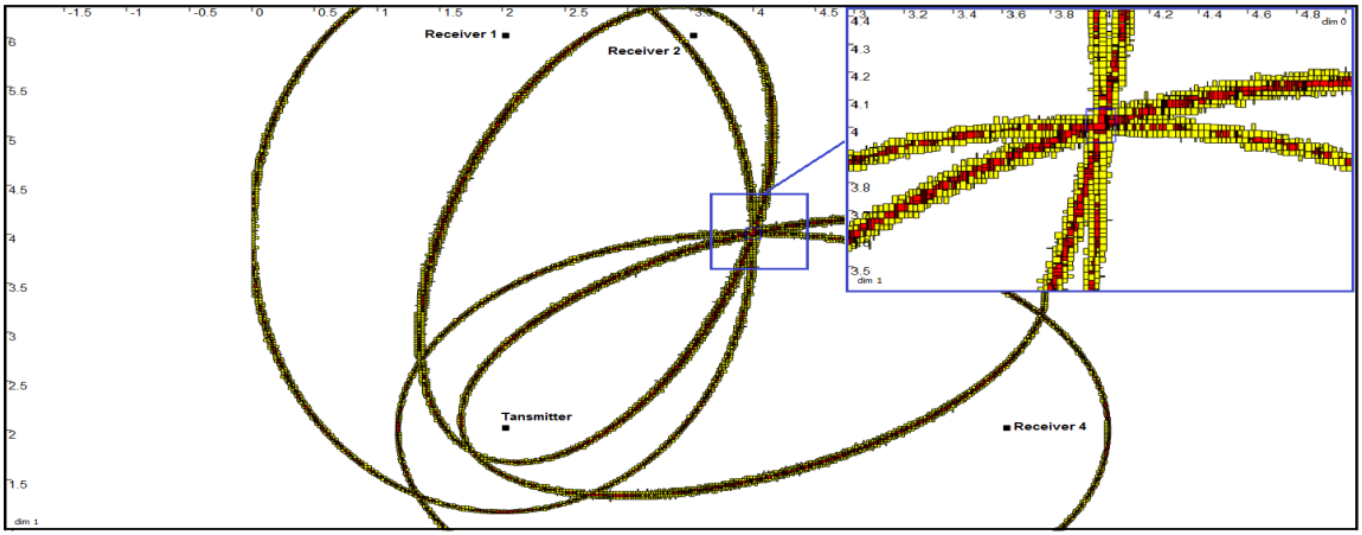

Figure 7: The range only measurements on a wind turbine

Interval Analysis implementation

The problem of finding the solution set for all the measurements from all receivers can be cast into a constraint satisfaction problem. The constraint for one measurement is defined by the following set of equations:

$$[m_{i,j}] = R_t + R_{ri}$$

$$R_t = \sqrt{(x - X_t)^2 + (y - Y_t)^2} \qquad R_{ri} = \sqrt{(x - X_{ri})^2 + (y - Y_{ri})^2}$$

where

$[m_{i,j}]$ – is the measurement with uncertainty. $i \in \{1, \ldots, N\}$, $j \in \{1, \ldots, M\}$, N is the number of receivers, M is the number of measurements for each receiver.
(X_t, Y_t) – is the position of the transmitter.
(X_r, Y_r) – is the position of the receiver $i \in \{1, \ldots, N\}$.

The solution set for one measurement is defined as:

$$\mathbb{S}_{i,j} = \{x \in \mathbb{R}, y \in \mathbb{R} | [m_{i,j}] = R_t + R_{ri}\}$$

The solution set for all the measurements from all receivers is:

$$\mathbb{S} = \bigcap_{i=1}^{N} \bigcup_{j=1}^{M} \mathbb{S}_{i,j}$$

The inner and outer approximation for the solution set described above can be found using interval methods techniques such as forward-backward contractors and separators with SIVIA(Set Inversion Via Interval Analysis).

Figure 2. above shows the separators result for four measuerments, where the ultimate solution of the turbine position is estimated from the intersection of all separators as discussed before.

Detection Results

The simulated result shown in Figure 3.a, shows the detection of 3x3 windfarm by a multistatic radar configuration of one transmitter and four receivers.All turbines were successfully detected. However, the close formation of the turbines array generates false detections elsewhere because of the possible false measuement association and intersection.

To addess this issue, an extra transmitter is added to the system. Results from each group of transmitter and receivers were matched (intersection) to eleminate ghost targets and improve the accuracy of the detection (as shown in Figure 3.b.).

Conclusion

Interval analysis shows promising results in target detection for multistatic radar system based on range only measurements input. The detection approach was implemented in the context of a small 3x3 wind farm model. Further experiments were carried to reduce the false detection by comparing multiple end results from multiple groups of transmitter and receivers.

The interval approach has shown stable results for the same set of targets on multiple runs, whereby the other PDF approaches might produce different results for the same target scenarios on multiple runs.

The future work is to simulate the interval analysis detection for a larger wind farm formation and to further study the optimal number of sensors for acceptable detection according to the number of targets.

Detection/Figures/003.png

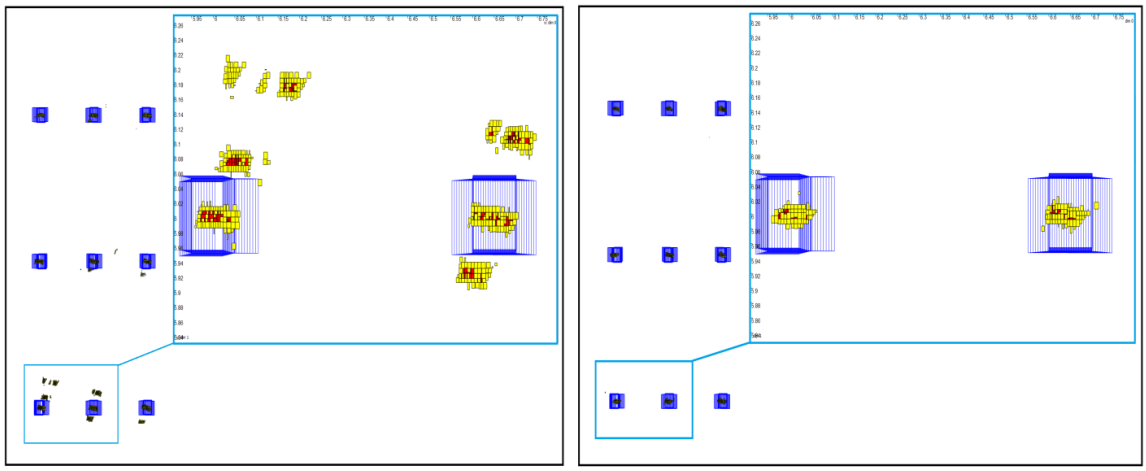

Figure 8: Figure 3. From left to right: a) Top view (XY) detection result for a 3x3 wind farm by 1 transmitter and 4 receivers. b) Top view (XY) detection result for a 3x3 wind farm by 2 transmitters and 4 receivers.

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A parametric Kantorovich theorem with application to tolerance synthesis

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Keywords: parametric Kantorovich theorem, global optimization, parallel robot sensitivity

1 Problem and contribution

We consider a system $f(x, q, p) = 0$ of n equations and n unknowns, denoted by $x \in \mathbb{R}^n$, with two kinds of parameters: $q \in \mathbb{R}^m$, interpreted as controlled parameters (called commands for short), and $p \in \mathbb{R}^q$, interpreted as design environment parameters with some uncertainties. For a fixed command value q, we call the solutions x_0 of $f(x_0, q, 0) = 0$ the *nominal solutions* corresponding to this command, while x satisfying $f(x, q, p) = 0$ for $p \neq 0$ is called a p-perturbed solution. The nominal solutions of interest are furthermore subject to given constraints $g(x, q) \leq 0$, possibly encoding domains for x and q. We aim at bounding rigorously the worst case distance from any nominal solution satisfying $g(x_0, q) \leq 0$ to its corresponding perturbed solution. This is done in two steps:

- Determining a uncertainty domain for which the correlation between nominal and perturbed solutions is non-ambiguous (a uniqueness condition on the existence of the perturbed solution inside a neighborhood of the nominal solution will be involved).
- Computing an upper-bound $\epsilon(e)$ on the distance between the nominal solution and its p-perturbed solution. Both a crude constant upper-bound and a sharp upper-bound depending on $\|e\|$ will be provided.

We propose a parametric Kantorovich theorem, which will achieve these two tasks. The idea is to compute worst case Kantorovich constants with respect to parameters q using nonlinear nonsmooth global optimization (a branch and bound algorithm and numerical constraint programming), and to use a rigorous first order model of the dependence with respect to parameters p.

2 Case study

The PRRP manipulator is modeled by the following equation:

$$(x - a + p_1)^2 + (q - b + p_2)^2 = (l + p_3)^2, \tag{1}$$

where x is the pose, q is the command, parameters values are $a = 1$, $b = 1$ and $l = 3$, and p_i are uncertainties acting on them. The constraints g are $2 \leq x \leq 3 \wedge 3 \leq q \leq 4$. The proposed parametric

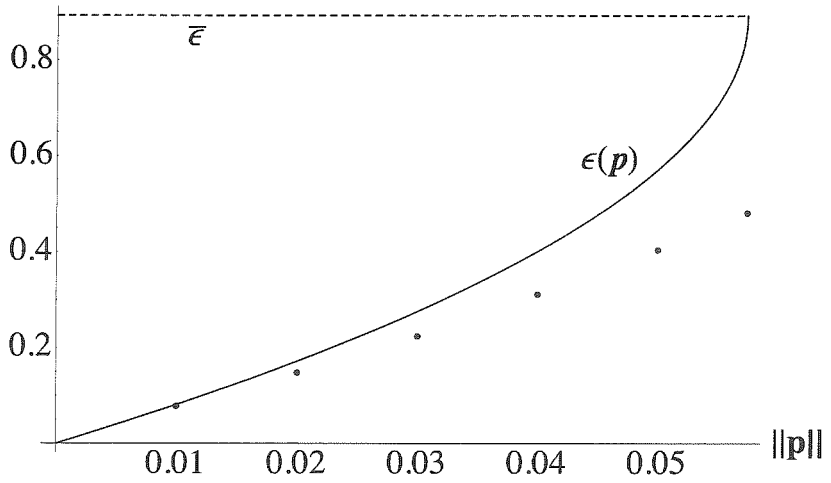

Figure 1: The upper bound $\epsilon(p)$ (dots represent exact maximal error, showing the overestimation of $\epsilon(p)$).

Kantorovich theorem requires solving the following three nonlinear optimization problems:

$$\kappa \geq \max_{\substack{\text{(1) and } g(x,q)\leq 0 \\ \|p\|\leq 0.1}} \left| (x-a-p_1)^2 + (q-b-p_2)^2 - (l+p_3)^2 \right| \tag{2}$$

$$\chi \geq \max_{\substack{\text{(1) and } g(x,q)\leq 0 \\ \|p\|\leq 0.1}} \frac{1}{2\,|x-a-p_1|} \tag{3}$$

$$\gamma \geq \max_{\substack{\text{(1) and } g(x,q)\leq 0 \\ \|p\|\leq 0.1}} \frac{|x-a-p_1| \;+\; |q-b-p_2| \;+\; |l+p_3|}{|x-a-p_1|}\,. \tag{4}$$

The upper bounds $\kappa = 1.48$, $\chi = 0.56$ and $\gamma = 7.75$ have been computed using the global solver IBEX[1] in less than 0.1 second on a standard laptop for each of them. Once upper bounds κ, χ and γ are computed, the proposed parametric Kantorovich theorem proves that every nominal pose has a unique corresponding perturbed pose for uncertainties satisfying $\|p\| \leq 0.057$, and that for these uncertainties the distance between the nominal pose and the perturbed pose is smaller than $\epsilon(p)$, with $\epsilon(p)$ shown on Figure 1.

Experiments on parallel manipulators up to 3 degrees of freedom, i.e., $n = 3$ and $m = 3$, have been successfully conducted so far and will be presented.

[1]Optimization problems to be solved are non-convex, non-smooth and equality and inequality constrained, and we require rigorous upper bounds. Up to our knowledge, IBEX, which implements interval based constraint programming within a standard branch and bound algorithm, is currently the only software available that is able to solve such problems.

Interval tools for computing the topology of projected curves

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Keywords: Interval methods, topology of projected curves, systems of large polynomials.

Introduction

In this talk we are interested in using certified numerical tools of interval analysis to compute the topology of the projection $\mathcal{B}$ in the (x, y)-plane of a space curve $\mathcal{C}$ defined as the intersection of two surfaces $P(x, y, z) = 0$ and $Q(x, y, z) = 0$. Even if $\mathcal{C}$ is smooth, $\mathcal{B}$ contains singular points and isolating these singularities is an important step to compute the topology of $\mathcal{B}$.

We briefly present how singularities can be characterized as solutions of a system that can be numerically solved and then describe a numerical solver dedicated to systems of large polynomials using interval bisection and adaptive multi-precision arithmetic.

Characterizing the singular points of a projected curve

State-of-the-art approaches to compute the topology of algebraic plane curves use symbolic calculus to determine its singularities. In the case of a projected curve, they suffer from the size of its implicit representation $R(x, y) = 0$: the polynomial R can be described as a resultant polynomial which size in terms of degree, number of monomials and bit-size of their coefficients grows roughly speaking as the product of the sizes of P and Q (see columns 1 and 2 of the table below).

In the algebraic case (*i.e.* when P and Q are polynomials), we propose to achieve numerical isolation of the singularities of $\mathcal{B}$ by finding the real solutions of a system $(\mathcal{S}_2)$ of two bi-variate polynomial equations that admits, at least in the generic case (*i.e.* when P and Q are generic), only regular solutions [1]. Equations of $(\mathcal{S}_2)$ are built from a sub-resultant chain and are still large polynomials (see column 3 of the table below). Using classical methods as homotopy requires to work with high arithmetic precision to find all the solutions. Moreover, the solving of $(\mathcal{S}_2)$ has to be certified to guarantee the topology of $\mathcal{B}$.

The table below shows sequential times in seconds to solve $(\mathcal{S}_2)$ with three solvers when P and Q are dense polynomials. `RS`[2] is a symbolic bivariate solver using rational univariate representations (see [2]), `Bertini`[3] is a homotopy solver using adaptive multi-precision and `Subdivision_solver`[4] is the solver using interval bisection and adaptive multi-precision that we designed (see [3]).

degree, bit-size of polynomials			time in s. for solving $(\mathcal{S}_2)$		
P and Q	R	eqs. of $(\mathcal{S}_2)$	`RS`	`Bertini`	`Subdivision_solver`
$6, 8$	$36, 112$	$25, 92$	24	1 600	2
$7, 8$	$49, 136$	$36, 115$	138	83 000	10
$8, 8$	$64, 157$	$49, 135$	725	380 000	43
$9, 8$	$81, 180$	$64, 158$	2 720	2 700 000	163

[2] `https://who.rocq.inria.fr/Fabrice.Rouillier/software.php`

[3] `http://bertini.nd.edu/`

[4] `http://www.loria.fr/~rimbach/software.html`

`Subdivision_solver`: a branch and bound approach to solve systems of large polynomials

We developed a numerical solver implementing a classical branch and bound method especially designed to deal with large polynomials involved in the system ($\mathcal{S}_2$). It basically proceeds as follows: solutions are sought in an initial box that is bisected in sub-boxes until it is possible to certify either the existence and the uniqueness of a solution in a sub-box, or the non-existence of a solution. Existence and uniqueness in a box is certified with the Krawczyk operator, and non-existence with interval evaluations.

A major obstacle regarding efficiency lies in the evaluation of large polynomials on intervals that has to be fast and limit over-estimation. We use the library `fast_polynomials` [4] that provides fast evaluation of multi-variate polynomials using Horner evaluation scheme to address the former point. As confirmed by experiments, using a centered evaluation at second order (*i.e.* Taylor expansion at order two) circumvents over-estimation. Indeed, even with the additional cost of evaluating derivatives, this strategy increases the efficiency of the overall process by reducing drastically the number of sub-boxes to be considered.

The arithmetic precision is adapted during the subdivision process thanks to a new heuristic criterion that detects when the Krawczyk operator can not certify the existence and the uniqueness of a solution due to numerical inaccuracy.

Our solver is described in [3] and is available as a `sage` package.

Conclusion and open questions

As a conclusion, we would like to submit the following questions to the community:

- Is it possible to improve thickness of interval evaluation of polynomials by considering different evaluation schemes?
- Is there a way to justify (in addition to numerical results) the use of centered evaluation at order two?
- Concerning arithmetic precision adaptation: How to improve our heuristic criterion? Can we prove the efficiency of this criterion?

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Finding Zeros for Systems of Two Analytic Functions

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Keywords: analytic functions, finding zeros, integration, two dimensions

Introduction

Let $D \subset \mathbb{C}^2$ be a box, a cross product of intervals. We wish to find enclosures of all the zeros for an analytic function $f : D \to \mathbb{C}^2$. Many methods for finding and verifying zeros of nonlinear functions exists, and are in many cases very effective. Hence the main problem lies in making sure we have found all zeros of the function. Here we make use of the properties of analytic functions; by employing a two dimensional version of the logarithmic integral we are able to calculate the exact number of zeros in the domain. This enables us to perform an exhaustive random search, until all zeros are found and validated.

Main Results

The method is divided into three parts. The first part is to determine the total number of zeros in the domain by calculating the logarithmic integral around the boundary. The second part is to generate a large amount of candidate zeros. The third and last part is to try to validate the candidate zeros until the known number of zeros is reached.

In the original form the logarithmic integral is defined using differential forms and can not easily be handled by computers[1]. Limiting the domains to boxes and rewriting the integral gives us a version that is more suited for rigorous computations. Given an interval enclosure of a function and its derivative, an enclosure of the integrand can be calculated, and with that an enclosure of the integral. Since we know that the value of the integral is an integer – the number of zeros – we only require that the width of the integral enclosure is sufficiently small so it identifies this integer.

When the amount of zeros is known, what remains is to find them. To do this we generate a large number of approximate zeros using a standard floating point Newton method. Then we take each of these points and create a small box around it: a set-valued Newton method is then used to validate the existence and uniqueness of a zero within the small box.

The performance of the algorithm, especially the integration which is the most critical part, will be discussed. Some simple examples and some applications in physics will be presented.

Acknowledgments

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Validated integration of dissipative PDEs — chaos in the Kuramoto-Sivashinsky equations.

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Keywords: validated integration of PDEs, periodic orbits, computer-assisted proofs, chaos

Introduction

In the study of nonlinear PDEs there is a huge gap between what we can observe in the numerical simulations and what we can prove rigorously. Interval analysis and computer-assisted proofs are an attempt to overcome this problem.

We propose an algorithm for continuous-time enclosures of trajectories of dissipative PDEs. This algorithm can be used to compute Poincaré maps for PDEs. Then, applying abstract results from the theory of discrete dynamical systems, we can prove some results regarding the Poincaré map. In consequence, we can conclude on the dynamics of the PDE.

The following theorem is the main computational result.

Theorem 1. *Consider the Kuramoto-Sivashinsky PDE on the line [1,2], which is given by*

$$u_t = -\nu u_{xxxx} - u_{xx} + (u^2)_x, \qquad \nu > 0,$$

where $x \in \mathbb{R}$, $u(t,x) \in \mathbb{R}$ *and we impose odd and periodic boundary conditions*

$$u(t,x) = -u(t,-x), \qquad u(t,x) = u(t,x+2\pi).$$

The above system, with the parameter $\nu = 0.1212$*, is chaotic in the following sense: a suitable Poincaré map* P *is semiconjugated to a subshift of finite type with positive topological entropy. The maximal invariant set for* P *contains countable infinity of periodic points of arbitrary large periods.*

Context and state of the art

In the last years there is an ingrowing interest in developing interval-based methods, which allow studying the dynamics of PDEs. There are two main trends in this subject.

One approach is to transform a dynamical problem into a zero finding problem in a proper (usually infinite-dimensional) Banach space. This approach originates in works by Nakao, Watanabe and Plum, and later on has been successfully developed by other researchers; just to mention few names: Gianni Arioli, Ferenc Bartha, Jordi-Lluís Figueras, Marcio Gameiro, Hans Koch, Jean-Philippe Lessard, Rafael de la Llave, Michael Plum, Warwick Tucker, Thomas Wanner. Although this approach has proved to be efficient in computation of bounded solutions (like periodic orbits or connecting orbits), it has one important weakness: it is rather difficult to apply it to study global

dynamics of the system because the method focuses on computation of a single, isolated and bounded solutions.

The second category of interval-based methods for PDEs is to work directly in the (infinite-dimensional) phase space of the system. To the best of our knowledge there are three existing methods for rigorous integration of PDEs.

The first algorithm has been proposed by Zgliczyński in a series o papers [3-5]. This approach reduces a problem of rigorous integration of a dissipative PDE into rigorous integration of a well chosen differential inclusion.

Arioli and Koch [6] proposed a method for rigorous integration of PDEs which bases on enclosing certain integral operator. This is the only existing algorithm which provides tools for validation of hyperbolicity of a periodic orbit.

The last method by Cyranka [7] is a modification of the algorithm by Zgliczyński. It uses the FFT algorithm to speed up computation of some convolutions. The core of the method, however, remains the same as in [3-5] and a PDE is solved by integration of a differential inclusion.

New algorithm for integration of dPDEs

In the present work we propose a new algorithm for validated integration of dissipative PDEs. The core of the method is an algorithm for automatic differentiation in infinite-dimensional space. The phase space is split into a direct sum of a finite-dimensional space, in which the projected trajectories are bounded using methods known from ODEs, and an infinite-dimensional subspace in which analytic estimates are used. The existence of solutions over a time-step interval is validated by means of two combined algorithms: high-order enclosure for ODEs [8] and dissipative enclosure [4].

Since the algorithm allows to compute continuous-time enclosures for the trajectories, we are able to enclose rigorously their intersection with a Poincaré section. In consequence, a validated bounds for associated Poincaré map can be computed.

Conclusion

We have developed a new algorithm for validated computation of trajectories in dissipative PDEs and associated Poincaré maps. This allows us to check geometric conditions which guarantee the presence of chaos.

The above approach has been successfully applied to the well known Kuramoto-Sivashinsky equation [9]. To the best of our knowledge, this is the first computer-assisted proof of chaos in a PDE.

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Computing Attracting Ellipsoids for Nonlinear Systems using an Interval Lyapunov Equation

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Keywords: asymptotic stability, interval arithmetic, attraction basin

Introduction

The asymptotic stability of an equilibrium point x^* for a nonlinear system $\dot{x} = f(x)$ can be proven by constructing a Lyapunov function $V(x)$. Such a quadratic Lyapunov function can be obtained by solving the Linear Lyapunov Equation (LLE) for a linear system that approximates the non-linear one around x^*. However, nothing can be said about the size of the bassin of attraction, which can be arbitrarily small for arbitrarily strongly attracting equilibria.

It is thus valuable to additionally build a basin of attraction for a given equilibrium x^*. One way of doing this is to consider a neighborhood N of x^* and to prove that the *Lie derivative* $\dot{V}(x) := \nabla V(x)^T f(x)$ is negative, that is:

$$\forall x \in N \setminus \{x^*\}, \quad \dot{V}(x) < 0. \tag{1}$$

It is indeed well-known that the level-sets of V included in N are then attracting. Interval analysis seems promising for proving such an inequality, but faces two key difficulties. First, proving (1) while $\dot{V}(x^*) = 0$ cannot be addressed directly by interval analysis, which can only prove generic properties. Second, the equilibrium x^* cannot be computed exactly and is ususally enclosed inside a box, leading to uncertain Lyapunov functions and level-sets.

Contribution

We first propose a new sufficient condition for an ellipsoid to be inside the basin of attraction. Our approach considers the solution V of the LLE and checks if this solution also satisfies an interval variant of the LLE, which involves an interval evaluation of the derivatives of f over an input box $[x]$. If it does, then all the level sets of V included in $[x]$ are attracting. We then take into account the uncertainty of the equilibrium to find the largest level set which certaintly encloses x^*.

Noticeably, our method just requires interval evaluations of the first-order derivatives of f. It is thus simpler and less expensive than the one proposed in [1], which requires interval evaluations of second-order derivatives of f. Also, it does not require an a priori intial subset of the basin, as in the method proposed in [2].

These approaches (including ours) have the common pitfall that they require an initial box, which can either be too large for succeeding or too small for providing a significantly large ellipsoid. Therefore, it has to be embedded in some heuristic search.

To cope with this issue, a second approach is finally proposed. It requires this time second-order derivatives but gives directly an attracting ellipsoid for arbitrary large input box $[x]$.

Finally, our two approaches generate an initial ellipsoid that can be enlarged by handling (1) as a nonlinear minimization problem. The attracting ellipsoids computed with our two approaches and with that of the papers cited above are compared on several systems.

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Contraction, propagation and bisection on a validated simulation of ODE[5]

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Keywords: contractor programming, ordinary differential equations, guaranteed numerical integration.

Introduction

Many tools used for the validated simulation of initial value problem of ordinary differential equations (ODE) provide abstraction of the solution under the form of a list of boxes obtained by time discretization and local polynomial interpolation. Majority of them are based on Taylor methods (Vnode-LP or CAPD) or on Runge-Kutta methods (DynIbex). If some information on the system state at a given time are given (*e.g.*, from measurements, or guard intersection [1]), it is complex to take this information into account with a low computation cost. We propose a contraction/propagation algorithm to use this information in an elegant manner. This approach will allow one to avoid some costly steps which would appear in running a new simulation (time-step computation and Picard operator).

Main idea

We consider an interval initial value problem of ODE of the form:

$$\dot{y} = f(y, p) \text{ with } y(0) \in [y_0] \text{ and } p \in [p] \ . \tag{11}$$

A validated simulation of (11) is then given in form of two lists of boxes: i) the *a priori* enclosures: $\{[\tilde{y}_0], \ldots, [\tilde{y}_N]\}$, with $y(t) \in [\tilde{y}_i]\ \forall t \in [t_i, t_{i+1}]$ and ii) the tight enclosures $\{[y_0], \ldots, [y_N]\}$, with $y(t_i) \in [y_i]$.
If an information provides $y(t^*) \in [y^*]$, then a contractor is used at $t = t^*$ following the two steps

- add a k^{th} integration step to the time discretization:
  $\{[y_0], \ldots, [y_i], [y_k], [y_{i+1}], \ldots, [y_N]\}$ s.t. $y(t^*) \in [y_k]$ and $t_k = t^*$
- apply the basic contractor $[y_k] := [y_k] \cap [y^*]$

Then a Picard contractor [2] on $[\tilde{y}_i]$ and a validated Runge-Kutta contractor [2] on $[y_i]$ can be apply on each integration step i, in order to propagate (in forward for $t > t^*$ and backward for $t < t^*$) this information on the whole simulation, *i.e.*, on all the boxes in the lists.

Remarks The bisection can be seen as a copy followed by two contractions, then the bisection of a simulation w.r.t. a given time is available with our approach. A propagation of a contraction on one state is a contraction on a simulation [3]. It is easy to generalize to a contraction on parameters. We can also generalize to an interval of time during one we obtain information.

Applications Bisection on initial state to avoid an obstacle; attainability of an objective at a given time; parameter synthesis w.r.t. some constraints; etc.

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Set-based methods in programs and systems verification

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This will be a survey talk on set-based methods we developed and used in program and (hybrid) systems analysis.

We will first explain how abstract-interpretation based program validation is related to set-based methods ; we will exemplify this in particular with the affine arithmetic methods we developed over the last 12 years or so in the tool FLUCTUAT.

Abstract interpretation computes program invariants, we need for that to enhance the set-based methods with least-fixed point computation methods. We will show how we dealt with it on affine arithmetic, and will mention some related topics, among which set-based constraint solving and characterization of invariants for continuous systems (to make a quick link to Daniel Wilczak's invited tutorial).

One step beyond proving safety properties, using program invariants, is to consider temporal properties (in the sense of temporal logics).

In order to prove these more general properties, we need not only outer-approximations of sets of reachable states, but also inner-approximations, parameterized by time. In our current work, in progress, we are deriving these outer and inner approximations on general hybrid systems, including a continuous part modeled by ODEs, from affine arithmetic, modal intervals and Taylor models.

We will briefly show, if time permits, how to exploit these abstractions to verify or falsify general temporal properties.

Control of an Autonomous Underwater Vehicle under robustness constraints

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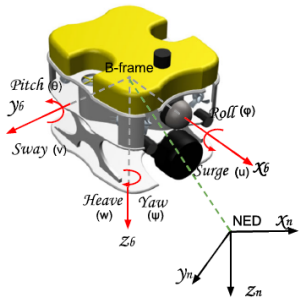
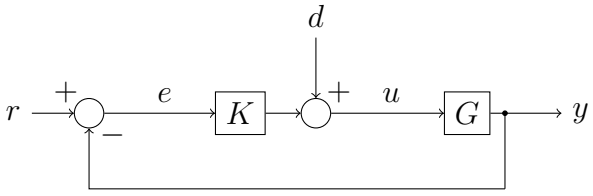
Keywords: robust control, Hinfinity control, AUV, global optimization.

AUV modeling and Control law synthesis

AUV are usually designed to operate in the ocean environment, that is why their hydrodynamic models suffer from uncertainties. Due to these identification and modeling problems, both modeling and control are challenging. The Ciscrea AUV is shown in Figure 9(a).



(a) B-frame and NED-frame of Underwater Vehicles



(b) Control scheme

Figure 9: Ciscrea

We aim to control the yaw of the Ciscrea, see Figure 9(a). We consider that there are no dependencies between the yaw dynamic and dynamics along other axis. Equation 12 has been proposed in [3] as a linear model of the yaw dynamic, where x denotes the yaw angle, I_{YRB} and I_{YA} are inertia coefficients, τ_i is the resultant torque generated by the horizontal propellers, D_{YLA} is a damping coefficient and δ represents a model uncertainty, $\delta \in [-0.25D_{YLA}, 0.25D_{YLA}]$.

$$(I_{YRB} + I_{YA})\ddot{x} + (D_{YLA} + \delta)\dot{x} = \tau_i, \tag{12}$$

We aim to control the yaw with respect to the following criteria:

- The error between the CISCREA yaw angle and the desired yaw angle must be small. This is a performance constraint.
- The CISCREA must not be sensitive to torque perturbations due to the environment. This is a robustness constraint.

Such constraints can be formulated as H_∞ constraints. Let us consider the control scheme in Figure 9(b) where K represents the controller we aim to synthesize, G the Ciscrea dynamic model, r the yaw angle target signal, e the tracking error signal, u the control signal, y the measure signal and d the torque perturbation input. In our case, $u = \tau_i$ is the torque control signal and y is the yaw angle measured by the compass embedded in the Ciscrea. The first constraint can be formulated by Equation 13.

$$||T_{r\to e}(s)W_e(s)||_\infty \leq 1 \iff \forall\omega \geq 0,\ |T_{r\to e}(i\omega)| \leq |W_e^{-1}(i\omega)| \iff \sup_{\omega>0}|T_{r\to e}(i\omega)W_e(i\omega)| \leq 1 \quad (13)$$

Where $s = i\omega, \omega \geq 0$ denotes the laplacian variable, and $T_{r\to e}(s)$ the transfer function from r to e. The weighting filter $W_e(s)$ is chosen according to the first constraint. The second constraint can be formulated by equation 14.

$$||T_{d\to y}(s)W_y(s)||_\infty \leq 1 \iff \forall\omega \geq 0,\ |T_{d\to y}(i\omega)| \leq |W_y^{-1}(i\omega)| \iff \sup_{\omega>0}|T_{d\to y}(i\omega)W_y(i\omega)| \leq 1 \quad (14)$$

Where $T_{d\to y}(s)$ denotes the transfer function from d to y. The weighting filter $W_y(s)$ is chosen according to the second constraint.
We propose to synthesize a filtered proportional integral derivative (PID) controller to ensure the respect of Constraints 13 and 14. Thus, $K(k,s) = k_p + \frac{k_i}{s} + \frac{k_d s}{1+Ts}$, $k = (k_p, k_i, k_d, T)$ and both $T_{r\to e}(k,s)$ and $T_{d\to y}(k,s)$ depends on k. Moreover, the stability of the closed loop system must be ensure. Such a constraint can be dealt with interval analysis [2].
The problem we aim to solve can be defined as q minimax problem under constraint:

$$\begin{gathered}\min_k \sup_{\omega>0}\{\max(|T_{r\to e}(k,i\omega)W_e(i\omega)|, |T_{d\to y}(i\omega,s)W_y(i\omega)|)\} \leq 1 \\ \text{subject to the Routh Hurwitz criterion is respected}\end{gathered} \quad (15)$$

We propose to solve Problem 15 considering the nominal linear model, that means we set $\delta = 0$, with an interval branch and bound algorithm [1]. Proceeding this way, we can provide an enclosure of the solution, and thus state whether there exist a controller $K(k,s)$ such as H_∞ constraints are satisfied and the closed loop system is stable for the nominal system. Then, we analyze the robustness of the solution with respect to model uncertainties. The control law is robust if both stability and H_∞ constraints are respected $\forall\delta \in [-0.25D_{YLA}, 0.25D_{YLA}]$. This can be proved as true or false in a guaranteed way using interval arithmetic.

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Consensus Control: Fundamentals and Some Recent Developments

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There have been some significant progresses in control of network-connected systems, also often referred as multi-agent systems. The subsystems often have similar or identical dynamics, and are connected by a communication network to perform a common task. Two common formations are leaderless consensus, and leader-follower control, and these two formulations have applications in formation control of robots and UAV etc. The key feature in the control design is to use the information obtained from the neighbouring subsystems, instead of using a centralized command systems. The control design and stability analysis benefit from the application of classical results in graph theory such as the properties of Laplacian matrices. In this talk, we will briefly review the fundamentals of consensus control by showing the typical results for single integrators, the simplest subsystem dynamics, and then the results for general linear systems, with some details on disturbance rejection of linear systems. We also will highlight typical results for consensus control of nonlinear systems, systems with input delay, adaptive consensus control and output regulation of network connected systems. Then, using specific tools from interval methods, we will address the consensus control in presence of uncertainties. The purpose of the talk is to encourage applications of consensus control with interval methods.

Robust Hybrid State Estimation using Interval Methods

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Keywords: Hybrid systems, Differential equations, Zonotope enclosures, Estimation.

Introduction

State estimation is a key engineering problem when addressing control or diagnosis issues for complex dynamical systems. The issue is still challenging when the latter systems must be modelled as hybrid discrete-continuous dynamics, which is true for many complex and safety-critical systems.

Main Results

We consider a hybrid automaton given by

$$H = (\mathcal{Q}, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \text{Inv}, \mathcal{F}). \tag{16}$$

Here $\mathcal{Q}$ is a set of locations. Given a location $q \in \mathcal{Q}$, the continuous dynamics, and hence flow transitions, are described by non-autonomous differential equations $f_q \in \mathcal{F}$ of the form

$$\text{flow}(q): \quad \dot{x}(t) = f_q(x, p, t), \tag{17}$$

where $f_q : \mathcal{D} \times \mathcal{P} \times \mathbb{R}^+ \mapsto \mathcal{D}$ is nonlinear and assumed sufficiently smooth over $\mathcal{D} \subseteq \mathbb{R}^n$, with dimension n that may depend on q, and $p \in \mathcal{P}$, where $\mathcal{P}$ is an uncertainty domain for the parameter vector p. Inv is an invariant, which assigns a domain to the continuous state space of each location. It is defined by the following system of inequalities:

$$\text{Inv}(q): \quad \nu_q(x(t), p, t) \leq 0,$$

where $\nu_q : \mathcal{D} \times \mathcal{P} \times \mathbb{R}^+ \mapsto \mathbb{R}^m$ is a vector-valued nonlinear function, the negativity constraint applies componentwise, and the number m of inequalities may also depend on q. The set $\mathcal{A}$ is the set of discrete transitions $\{e = (q \to q')\}$ each given by the 5-uple $(q, \text{guard}, \sigma, \rho, q')$, where q and q' represent upstream and downstream locations respectively; guard is a condition given as the system of equations

$$\text{guard}(e): \quad \gamma_e(x(t), p, t) = 0, \tag{18}$$

σ is an event, and ρ is a reset function. A transition $q \to q'$ may occur when the continuous flow resides within the guard set, i.e. when the continuous state satisfies condition (18). Let us also consider the following measurement equation

$$y(t) = g_q(x(t), p, t), \tag{19}$$

where function $g_q : \mathcal{D} \times \mathcal{P} \times \mathbb{R}^+ \mapsto \mathbb{R}^m$ may be nonlinear.

We assume that one can have access to measurements of the output vector $y(t)$. The latter is usually measured at some sampling time instants t_j taken over a time-grid $t_0 < t_1 < t_2 < \ldots < t_{n_T}$. Measurement noise is taken as a discrete time signal assumed additive and bounded with known bounds. Denote by $\boldsymbol{y}_i$ the feasible domain for output vector y_i at time t_i. Recall that the parameter vector p is uncertain $p \in \mathcal{P}$. We assume that the initial hybrid state vector is only partially known, i.e. $x(t_0) \in \boldsymbol{x}_0$. The goal of the bounded error estimation is to compute conservative outer enclosures for feasible sets for both the discrete modes and associated continuous variables that are consistent with the feasible domains for measurements, the chosen hybrid model, and the assumptions about the uncertainties and perturbations acting on the system. That is, given two measurements $\boldsymbol{y}_i$ and $\boldsymbol{y}_{i+1}$ gathered at the two time instants t_i and t_{i+1}, the estimation problem boils down to simultaneously:

(i) Detect and identify all possible discrete transitions $(t^\star, e)$ that may occur at time $t^\star \in [t_i,\, t_{i+1}]$. Because of the uncertainties, there might be a continuum of time instants, where events may occur. There may also exist several such time intervals in $[t_i,\, t_{i+1}]$.

(ii) Reconstruct the sets of hybrid states $(\boldsymbol{q}(t_i), \boldsymbol{x}(t_i))$ and $(\boldsymbol{q}(t_{i+1}), \boldsymbol{x}(t_{i+1}))$ that are consistent with the switching sequence reconstructed in (i), the modelling (16)-(19), and the inclusions $y(t_i) \in \boldsymbol{y}_i$ and $y(t_{i+1}) \in \boldsymbol{y}_{i+1}$.

These two steps must be done for all i, $0 \leq i \leq n_T - 1$.

We review some recent results related to hybrid state estimation in the bounded-error framework using reliable and robust methods [1-2], then introduce our new method [3]. We essentially discuss how the use of our recent method for nonlinear hybrid reachability based on zonotopic and interval enclosures [4], can be used to extend the classical prediction-correction approach to truly hybrid systems. From discrete-time measurements of system outputs, our new method simultaneously reconstructs, at each time instant, the set of consistant switching sequence and the corresponding set of consistant continuous state vectors. The method is illustrated with simple well tuned examples.

Acknowledgments

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An Interval Branch and Bound Algorithm for Parameter Estimation

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Keywords: parameter estimation, shape recognition, interval branch and bound

Parameter Estimation

Parameter estimation is a difficult problem widely studied by engineering sciences. Calibration or geolocation can be viewed as specific parameter estimation problems. The problem consists in determining the n numerical parameters of a model based on m observations. A parameterized model is defined by an implicit equation $f(\mathbf{x}, \mathbf{p}) = 0$, $\mathbf{p} = (p_1, \ldots, p_n)$ being the vector of parameters to be determined. An observation $\mathbf{o}_i$ is a d-dimensional vector of observed data. o_i is compatible with the parameters vector $\mathbf{p}$, using a tolerance value τ, when it satisfies an *observation constraint* $-\tau \leq f(\mathbf{o}_i, \mathbf{p}) \leq +\tau$. The parameter estimation problem becomes challenging when the function f used to define the parametric model is not linear and/or in presence of *outliers*.

In this paper, given a finite set of observations $\{\mathbf{o_1}, \ldots, \mathbf{o_i}, \ldots, \mathbf{o_m}\}$, we search for the parameterized models that are compatible with at least q of the m corresponding observation constraints. We improve a deterministic method handling this problem and apply it to shape detection problems in a point cloud. The optimization version of this problem simply consists in finding *one* parameterized model maximizing the number q of observation constraints satisfied.

RANSAC: parameter estimation heuristic coping with outliers

The random sample consensus algorithm (RANSAC) is a state-of-the-art tool used by the computer vision and image processing communities for achieving parameter estimation robust to outliers. This stochastic algorithm proceeds by randomly sampling observations for determining a model (n observations for determining n parameters), before checking the number of other observations compatible with this model.

Deterministic interval constraint programming approach

A deterministic parameter estimation method based on interval constraint programming and robust to outliers was described in [2, 1]. It performs a tree search to exhaustively explore the parameter space $[\mathbf{p}]$. (A Cartesian product of intervals $[\mathbf{p}] = [p_1] \times ... \times [p_n]$ is called a (parallel-to-axes) *box*.)

- $[\mathbf{p}]$ is recursively subdivided: one variable p_i in $\mathbf{p}$ is selected, its domain $[p_i]$ is bisected into two sub-intervals and the two corresponding sub-boxes are explored recursively. The combinatorial process stops when a precision is reached, i.e. when the current box reaches a precision size.
- At each node of the tree, a box $[\mathbf{p}]$ is handled:

1. *Contractions* of [**p**] are achieved using each of the m observation constraints by a well-known forward-backward procedure, which produces an m-set S of sub-boxes of [**p**].
2. The q-intersection box of S is returned. The q-intersection is the box of smallest perimeter that encloses the set of points of $\mathbb{R}^n$ belonging to at least q boxes of S.

The q-intersection of boxes is a difficult (DP-complete) problem and we have resorted to a time $O(nm\log(m))$ non optimal projection heuristic that reasons on each dimension independently.

Improvements to the interval constraint programming approach

We have proposed several generic improvements to the deterministic parameter estimation code, and several improvements specific to shape recognition problems.

In the search tree, two data structures are maintained. First, the set of *possible* observations: if an observation constraint leads to an empty box using a (forward-backward or q-intersection) contraction at a given node, this observation will be removed from the possible observations in the subtrees. Second, the number of *valid* observations is maintained by testing every possible observation at given punctual parameters vectors inside the box studied. The valid observations form a subset of the possible observations. A stopping condition in the current branch of the search is reached when the two sets are the same. We also perform a q-projection on an additional direction where we hope to obtain small intervals, thus favoring a failure of the q-intersection. To this end, we linearize and relax every observation constraint, and project the parallelograms obtained on the direction corresponding to the mean normal vector of the "parallelogram" gradients.

Instead of running a general forward-backward contraction algorithm using a library for interval arithmetic computations and backward projections (e.g., as implemented in `Ibex`), we can rewrite interval computations dedicated to the analytical form of observation constraints. Finally, we have proposed bisection choice strategies dedicated to both shape recognition problems studied.

Experiments and discussion

The algorithms are implemented in the Interval Based EXplorer (`Ibex`), a free C++ library devoted to interval computing. The combination of the improvements brings a significant speedup of two orders of magnitude on each tested instance of 3D plane and 2D circle detection problems. These experiments suggest that our interval branch and contract algorithm can guarantee the models computed while ensuring a good performance.

A question is whether the approach scales up in higher dimension, and we have studied stereo vision problems. To this end, we have designed an interval branch and bound algorithm for parameter estimation that computes a model maximizing the number of valid observations (inliers) of a parameterized model. First experiments seem to show that the current interval branch and bound algorithm cannot cope with the fundamental matrix estimation problem (dimension 7) while promising results have been obtained on the essential matrix estimation (dimension 5).

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OMNE is a Maximum Likelihood Estimator

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Keywords: interval analysis, relaxed intersection, maximum likelihood.

Introduction

OMNE (Outlier Minimal Outlier Estimator) introduced in [**1**] is a well known bounded-error estimator which is robust with respect to outliers. It returns the set of parameter vectors that are consistent with the maximal number of data bar. OMNE proposes to minimize the number of outliers or equivalently to maximize the number of inliers. The relaxed intersection made it possible to implement OMNE in a reliable way for nonlinear models [**2**]. OMNE and has been used in several applications such as the localization of robots [**3**] and has received some probabilistic interpretations [**4**]. Here, we show that OMNE corresponds to a maximum likelihood estimator.

OMNE

Consider the static parameter estimation model

$$\mathbf{y} = \boldsymbol{\psi}(\mathbf{p}) + \mathbf{e}$$

where $\boldsymbol{\psi} : \mathbb{R}^n \to \mathbb{R}^m$ is the model, $\mathbf{p} \in \mathbb{R}^n$ is the parameter vector to be estimated, $\mathbf{y} \in \mathbb{R}^m$ is the measurement vector and $\mathbf{e}$ the noise. Define the function $\mathbf{f}(\mathbf{p}) = \mathbf{e} = \mathbf{y} - \boldsymbol{\psi}(\mathbf{e})$. Given an erreur interval $[e] \subset \mathbb{R}$ that is supposed to contain the error e_i if the corresponding data y_i is an inlier. In practice, $[e]$ is a small interval which contains 0. The OMNE estimator returns the set of all $\mathbf{p}$ such as the property $f_i(\mathbf{p}) \in [e]$ is satisfied for a maximal number of data. More precisely, OMNE returns the set

$$\hat{\mathbb{P}} = \arg\max_{\mathbf{p}\in\mathbb{R}^n} \sum_i \mathbf{1}_{[e]}(f_i(\mathbf{p}))$$

where $\mathbf{1}_{\mathbb{X}}$ denotes the characteristic function the set $\mathbb{X}$ or equivalently, $\mathbf{1}_{[e]}(f_i(\mathbf{p})) = 1$ iff $f_i(\mathbf{p}) \in [e]$ and $\mathbf{1}_{[e]}(f_i(\mathbf{p})) = 0$ otherwise, as illustrated on Figure 10.

Main result

Theorem. Assume that the error vector $\mathbf{e} = (e_1, \ldots, e_n)$ is white, *i.e.*, all e_i are independant and identically distributed with the probability density function π_e which is half uniform. More precisely, $\pi_e(e_i) = a$ if $e_i \in [e]$ and $\pi_e(e_i) = b < a$ otherwise. Then the maximum likelihood estimator corresponds to OMNE.

Proof. The likelihood is defined by

$$\begin{array}{rcl} \pi(\mathbf{y} \mid \mathbf{p}) & = & \prod_i \pi_e(y_i - \psi_i(\mathbf{p})) = \prod_i \pi_e(f_i(\mathbf{p})) \\ & = & \prod \left(a \cdot \mathbf{1}_{[e]}(f_i(\mathbf{p})) + b \cdot \mathbf{1}_{\mathbb{R}\setminus[e]}(f_i(\mathbf{p}))\right). \end{array}$$

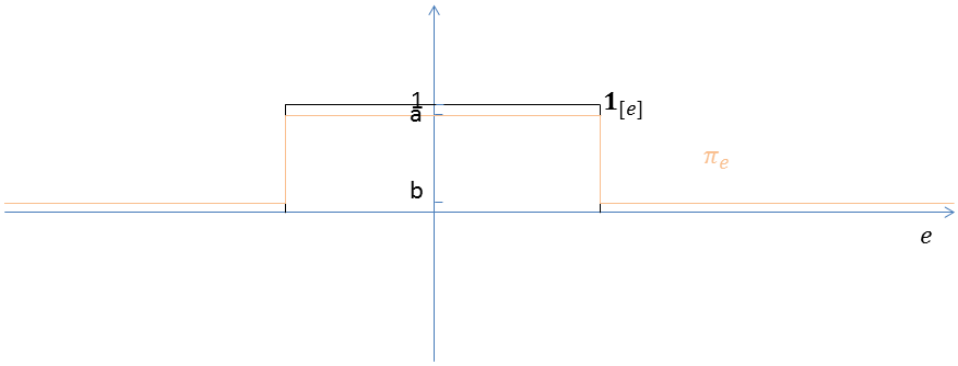

Figure 10: Illustration of $\pi(e)$ and $\mathbf{1}[e]$

It is maximal if the log-likelihood

$$\begin{array}{rcl} \log \pi(\mathbf{y} \mid \mathbf{p}) & = & \sum_i \log\left(a \cdot \mathbf{1}_{[e]}(f_i(\mathbf{p})) + b \cdot \mathbf{1}_{\mathbb{R}\setminus[e]}(f_i(\mathbf{p}))\right) \\ & = & \sum_i \log\left((a-b) \cdot \mathbf{1}_{[e]}(f_i(\mathbf{p})) + b\right) \end{array}$$

is also maximal. Now, since the function $\log((a-b)\cdot x + b)$ is increasing with respect to x, we conclude thatlog $\pi(\mathbf{y} \mid \mathbf{p})$ is maximal if $\sum_i \mathbf{1}_{[e]}(f_i(\mathbf{p}))$ is maximal, which corresponds to what OMNE computes.

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Interval Analysis for the Representation of Phoneme Databases in Speech Recognition Systems: Fundamentals of a Computer-Based Assistance System in Speech Therapy

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Keywords: Speech recognition; Interval-based phoneme representation; Frequency estimation

Abstract

Language disorders can be classified into the three major linguistic levels of lexicon, grammar, and pronunciation [2]. Due to the fact that most patient-oriented sessions at therapists' offices involve an enormous amount of work that is related to the analysis of spoken language, it is desired to develop a software-based assistance system allowing a therapist to focus his/ her valuable time on the actual therapy work instead of a (partially tedious) analysis of recorded speech sequences.

For that reason, a research project bringing together the fields of signal processing and speech therapy has been started recently. It consists of the following aims: (i) automatic transcription and preprocessing of spoken text involving erroneous pronunciation, (ii) automatic classification of pronunciation disorders, (iii) grammatical analysis of freely spoken language. This contribution is focused on the first and second project aim by providing an observer and filter-based substitute for the offline frequency analysis that is currently used in many state-of-the-art language recognition systems. Moreover, a new interval-based representation of a phoneme database is developed that can be used during a transcription of spoken language into a computer-processible format. In addition, this database also serves as a basis for the fundamental stages of classifying pronunciation disorders.

In general, phonemes can be classified into voiced and unvoiced sounds [2, 7, 5]. Voiced sounds (e.g. normal vowels) are characterized by several relatively sharp formant frequencies produced by vibrations of the vocal folds representing a fluidic resistance against the outflow of air expelled from the lungs. In contrast, unvoiced sounds (e.g. whispered vowels and fricatives such as ch, ss, sh, f) are caused by a turbulent, partially irregular, air flow with negligible vibrations of the vocal folds. To some extent, they are produced by fizzing sounds originating between teeth and lips as well as between tongue and hard or soft palate. Here, sharp formant frequencies are characteristic for voiced phonemes, whereas wide frequency bands are typical for unvoiced ones.

From that point of view, automatic speech recognition systems [4, 1] have to be capable of simultaneously dealing with signals containing sharp characteristic frequencies as well as signals with a broad-band behavior. In the state-of-the-art offline frequency analysis of speech signals, this is done by first cutting the sound sequence into short temporal slices of typically $10 - 50\,\text{ms}$ length, second performing a short-time Fourier analysis for each of these time slices (partly with overlapping time windows), and third determining a measure of similarity with phoneme-dependent frequency spectra (usually by the application of cross-correlation functions in the frequency domain.)

For both voiced and unvoiced sounds, the authors have proposed a stochastic filtering approach that can be employed to estimate expected values of the formant frequencies and their associated amplitudes and covariances. Here, the broad-band nature of unvoiced speech is reflected by (co-)variance estimates that are significantly larger than for voiced sounds [5]. Transitions between subsequent phonemes are indicated by rapid changes in the above-mentioned estimation results [3]. Within the proposed filter, phoneme boundaries are detected automatically by threshold operations on both the absolute values and the variation rates of the estimated quantities [6].

In order to extract speech features from the estimated formant frequencies and signal amplitudes on the level of individual phonemes, it is necessary to develop a compact representation of a suitable phoneme database. This database has to contain representative information concerning formant frequencies (after a speaker-dependent normalization), information about the associated signal amplitudes and the related covariances of the estimated frequencies. In this contribution, such kind of representation is developed in terms of a feature classification that relies on interval analysis.

Estimation results for selected speech samples are presented with a focus on the classification of correctly pronounced phonemes and the detection of potential pronunciation disorders, related to results that are not yet included in the database. To make the classification of speech sequences applicable in practice (involving e.g. children's speech with a large number of mispronunciations) the expert classification by a speech therapist is used to validate the estimation accuracy and to include features that are not yet contained in the database. This contribution is concluded with an outlook on future work that will aim also at the classification of stuttering disorders, which will be assumed to be characterized by repetitions of individual and/ or multiple phonemes and/ or syllables.

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Inner approximation of a capture basin of a dynamical system

Thomas Le Mézo, Luc Jaulin and Benoît Zerr

Keywords: interval analysis, dynamical system, capture basin

Introduction

Consider a dynamic system $\mathcal{S}$ defined by the following state equation:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t)) \tag{20}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector and $\mathbf{f} : \mathbb{R}^n \mapsto \mathbb{R}^n$ is the evolution function of $\mathcal{S}$. Interval analysis has been used for many years to deal with dynamical systems. It is for instance possible to perform guaranteed integration as shown in [1,3,5,6] which is used in control theory [4] or robotics [2]. More and more tools for interval arithmetic exist as it is the case for the new package for Octave [3]. Our objective is to compute capture basin that is now defined. Let $\boldsymbol{\varphi}$ be the flow map of $\mathcal{S}$, *i.e.*, with the initial condition $\mathbf{x}_0 = \mathbf{x}(0)$, the system $\mathcal{S}$ reaches the state $\boldsymbol{\varphi}(t, \mathbf{x}_0)$ at time t. The *capture basin* of the *target* $\mathbb{T} \subset \mathbb{R}^n$ is the set $Capt(\mathbb{T})$ of initial states $\mathbf{x}$ from which at least one evolution of $\mathcal{S}$ reaches the target $\mathbb{T}$ in finite time:

$$Capt(\mathbb{T}) = \{\mathbf{x}_0 \mid \exists t \geq 0, \boldsymbol{\varphi}(t, \mathbf{x}_0) \in \mathbb{T}\}\,. \tag{21}$$

Note that $\mathbb{T} \subset Capt(\mathbb{T})$. We propose here a new method to compute an inner approximation of $Capt(\mathbb{T})$.

Dead path

A *trajectory* is a smooth function $\mathbf{x}(\cdot)$ from $\mathbb{R}^+$ to $\mathbb{R}^n$. The *path* associated with a trajectory $\mathbf{x}(\cdot)$ is the set of all $\mathbf{x}(t) \in \mathbb{R}^n$ and an orientation with respect to t. A path which satisfies (20) is said to be *feasible*. A path is *elected* if at least one of its points is inside $\mathbb{T}$. Otherwise it is a *dead path*. A state $\mathbf{x}$ which belongs to a dead path is outside $Capt(\mathbb{T})$. Figure provides five paths. All of them satisfy (20) except (v) which makes a loops and this cannot satisfy the state equation. The path (ii) enters in $\mathbb{T}$ and converges to an equilibrium point. The path (iii) is elected since it enters in $\mathbb{T}$, but since it leave it later, it contains some subpaths that are dead. The path (iv) corresponds to a limit cycle which is dead since it does not enter in $\mathbb{T}$.

Methods

To compute an inner approximation of $Capt(\mathbb{T})$ we search for a dead path using a contractor-based approach. Note that the fact that the set of dead paths has a dimension equal to infinity is not a problem for our method. Then, we derive a finite dimensional polygonal contractor for $Capt(\mathbb{T})$. Using a paver in $\mathbb{R}^n$, we will show that we are able to obtain an inner approximation of $Capt(\mathbb{T})$ without any interval integration.

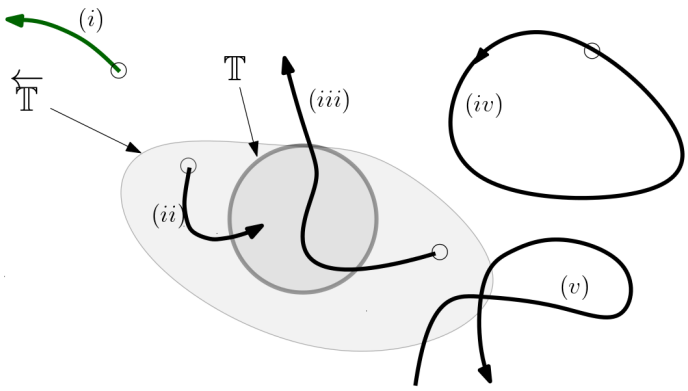

Figure 11: The paths (i),(iv) are dead, (v) does not satisfy (20) since it loops, and paths (ii), (iii) are elected

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Interval trajectory tracking with flatness[6]

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Keywords: set-membership computation, flatness, nonlinear predictive control, discrete-time systems.

Introduction

We consider the computation of admissible controls for trajectory tracking of the class of nonlinear discrete-time systems described by:

$$(S_d)\begin{cases} x_{k+1} = f(x_k, u_k) \\ y_k = h(x_k) \end{cases} \tag{22}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^m$ and $y_k \in \mathbb{R}^p$ are resp. the state, the input and the output of the system at the current time k, x_0 is the initial condition. The reference trajectory to be tracked belongs to a set $\mathbb{Y}^{\text{ref}} = \{\mathbb{Y}^{\text{ref}}_{k+1}, \mathbb{Y}^{\text{ref}}_{k+2}, \dots\}$ and the objective is to determine the set of admissible controls $\mathcal{U}^*_i$, at time i, such that:

$$\mathcal{U}^*_i = \{u_i \in \mathbb{R}^m \mid \forall j = i, \dots, i + n_p - 1, h(f(x_j, u_j)) \in \mathbb{Y}^{\text{ref}}_{j+1}\}.$$

In [3], the computation of all the guaranteed sequences of control was considered but only for a short prediction horizon because of the time complexity of the SIVIA algorithm in $O(\exp(m \times n_p))$.

Main Result

We propose a direct method to compute an inner approximation of the admissible control set for flat systems [1].

Definition 2 (Flatness)**.** *The system (S_d) described in* (22) *is flat if there exists an output $F_k \in \mathbb{R}^m$ such that for all k:*

$$x_k = \psi(F_k, F_{k+1}, \dots, F_{k+r-1}) \tag{23}$$
$$y_k = h(\psi(F_k, F_{k+1}, \dots, F_{k+r-1})) \tag{24}$$
$$u_k = \varphi(F_k, F_{k+1}, \dots, F_{k+r}) \tag{25}$$

where r is the relative degree of the system [5].

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We denote $z_k \in \mathbb{R}^{r.m}$ the vector composed of the flat output F_k and its $r-1$ advances. The control input can be expressed using (25) to evaluate the set of controls. The reference trajectory is an admissible trajectory for the flat output. The set of admissible controls $\mathcal{U}_k^*$ at each time k is then

$$\mathcal{U}_k^* = \{\varphi(z_k, v_k) \in \mathbb{R}^m \mid z_k \in \mathbb{Y}^{\text{ref}}_{k..k+r-1}, v_k \in \mathbb{Y}^{\text{ref}}_{k+r}\}. \tag{26}$$

Characterizing this set in Eq. (26) is equivalent to compute the image S of a set $\mathbb{X}$ by a function $\mathcal{F}$: $S = \{\mathcal{F}(x) \mid x \in \mathbb{X}\}$ where S, $\mathcal{F}$ and $\mathbb{X}$ correspond to $\mathcal{U}_k^*$, φ and $\mathbb{Y}^{\text{ref}}$ respectively. The computation of such a set S has already been addressed in [2, 4]. In [2], an inner approximation of the set S of dimension 1 is produced using generalized affine forms. In [4], a branch & prune algorithm is considered in the case of a dimension greater than 1. Two methods are then possible for the computation of an inner approximation of the set according to the dimension of the control input. In the latter method, time complexity is then in $O(n_p \times \exp(m \times r))$ allowing larger prediction horizon n_p than in [3].

Conclusion

The computation of admissible control set for an uncertain trajectory tracking purpose can be viewed as the computation of the image of a set by a function. The two methods proposed in this paper address both precision and computational time requirements.

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Homotopy perturbation method for solving nonlinear interval differential equations

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Keywords: interval analysis, nonlinear ODEs, two parametric form, Vanderpol oscillator, Rayleigh oscillator

Introduction

In dynamics, many engineering problems are governed by second order Nonlinear Ordinary Differential Equations (NODEs). In this regard, few authors have proposed different methods to solve NODEs. A new analytic technique known as Homotopy Perturbation Method (HPM) was proposed by He [1] for solving nonlinear differential equations. Using such HPM and Parameterized Perturbation Method (PPM), Samaee and Ganji [2] solved various nonlinear oscillator differential equations. Akbari et al. [3] discussed solving NODEs of vibration viz. Vanderpol, Rayleigh and Duffing equations using Algebraic Method (AGM).

In general, the parameters in such governing differential equations are considered as crisp values. But due to maintenance induced errors etc., one may obtain uncertain parameters (in terms of intervals). In this regard, interval arithmetic and computations have been discussed by Alefeld and Herzberger [4] and Moore et al. [5]. Tapaswini and Chakraverty [6] discussed single and double parametric methods for conversion of intervals to crisp forms.

As such, this paper discusses solving second order NODEs by considering the initial conditions as intervals. Initially, the interval initial conditions have been converted in terms of parameters β_1 and β_2 using two parametric form. Then, the NODE having parametric initial conditions has been solved using HPM. As such, the NODE solution may contain two parameters viz. $\beta_1, \beta_2 \in [0, 1]$. Finally, the lower and upper solution bounds may be computed by varying β_1 and β_2 over $[0, 1]$. So, the present section gives the introduction and the next section discusses HPM along with parametric form for solving second order Nonlinear Interval Ordinary Differential Equations (NIODEs).

Proposed procedure

Parametric forms

Single parametric form: (Tapaswini and Chakraverty [6]) The parametric form in terms of β of each interval $a_i^I = [\underline{a}_i, \overline{a}_i]$ reduces a_i^I to crisp form as $\beta(\overline{a}_i - \underline{a}_i) + \underline{a}_i$ where $\beta \in [0, 1]$ for $i = 1, 2, \cdots$. Accordingly, the lower and upper bounds of an interval a^I from $\beta(\overline{a} - \underline{a}) + \underline{a}$ may be obtained for $\beta = 0$ and $\beta = 1$ respectively.

Two parametric form: The two parametric form is proposed as an extension of single parametric form. As per the targetted second order NIODEs, there are two initial conditions. Accordingly, uncertainity for each interval a_i^I corresponding to each initial condition is expressed in terms of parameters β_i as $\beta_i(\overline{a}_i - \underline{a}_i) + \underline{a}_i$ where $\beta_i \in [0, 1]$ for $i = 1, 2$.

Two parametric HPM

Let us consider a nonlinear differential equation

$$L(u) + N(u) + f(t) = 0 \tag{27}$$

having initial conditions $u(0) = \beta_1(\overline{a} - \underline{a}) + \underline{a}$ and $u'(0) = \beta_2(\overline{b} - \underline{b}) + \underline{b}$ where $\beta_i \in [0, 1]$ for $i = 1, 2$. Here, $L(u)$ and $N(u)$ represent the respective linear and nonlinear differential operators whereas $f(t)$ is the forcing function. Then the standard HPM may be used to solve Eq. (27) subject to the two parametric initial conditions.

Numerical examples

In this section, two second order NIODEs viz. Vanderpol (Samaee and Ganji [2]) and Rayleigh (Akbari et al. [3]) oscillator equations have been considered and solved using the proposed procedure.

Conclusion

This investigation presents solving second order NIODEs (NODEs having interval initial conditions). Accordingly, the lower and upper solution bounds have been obtained for $\beta_1, \beta_2 \in [0, 1]$. The proposed procedure may also be applied to other practical problems where the governing equation may be in the form of second order NODEs involving interval initial conditions.

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Exact solution to a parametric linear programming problem

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Introduction

Let $A(p)x = b(p), p \in \boldsymbol{p}$ be a square linear interval parametric (LIP) system of size n whose elements, $a_{ij}(p)$ and $b_i(p)$, are affine linear functions of an m-dimensional vector of parameters p. A new type of solution $\boldsymbol{x}(p)$ (called parameterized or p-solution) to the LIP system has been recently introduced in [1]. It is of the following linear parametric (LI) form $\boldsymbol{x}(p) = Lp + \boldsymbol{a}, p \in \boldsymbol{p}$, where L is a real $n \times m$ matrix and $\boldsymbol{a}$ is an n-dimensional interval vector. An iterative method for determining $\boldsymbol{x}(p)$ has been suggested in [1]. The new solution $\boldsymbol{x}(p), p \in \boldsymbol{p}$ has a number of useful properties such as: direct determination of an outer interval solution $\boldsymbol{x}$ or an inner estimation $\boldsymbol{\xi}$. Combined with a constraint satisfaction technique, it permits to determine each component $\boldsymbol{x}_i^*$ of the hull solution $\boldsymbol{x}^*$ as the global solution of two equality-constrained optimization problems.

The objective of the present paper is to generalize the approach from [1] to address the following parametric linear programming (PLP) problem: given a linear parametric objective function

$$l(x, p) = c^T(p)x(p), \tag{28}$$

where $c_i(p)$ are, in general, nonlinear functions of p, and constraint

$$A(p)x = b(p), p \in \boldsymbol{p}, \tag{29}$$

determine the range

$$\boldsymbol{l}^* \left(A(p), b(p), c(p), \boldsymbol{p}\right) = \left\{l(x, p) : \ A(p)\, x \ = b(p), p \in \boldsymbol{p}\right\}. \tag{30}$$

The PLP (28), (29) is a parametric generalization of the known interval linear programming problem where interval matrix $\boldsymbol{A}$ and interval vectors $\boldsymbol{b}$, $\boldsymbol{c}$ are involved.

Obviously, the endpoints $\underline{l}^*$ and $\overline{l}^*$ of the range (30) can be determined as the global solutions of the following two optimization problems

$$\underline{l}^* = \min \left\{l(x, p) : A(p)x = b(p), p \in \boldsymbol{p}\right\}, \tag{31}$$

$$\underline{l}^* = \max \left\{l(x, p) : A(p)x = b(p), p \in \boldsymbol{p}\right\}. \tag{32}$$

Iterative method

In the present paper, an iterative method for determining the range (30) by globally solving (31) and (32) is suggested.

Determination of $\underline{l}^*$

A simple iterative method for solving (31) is proposed. Its computational scheme is as follows. Starting with an initial domain $\boldsymbol{p}^{(0)} = \boldsymbol{p}$, in v-th iteration ($v \geqslant 0$) find in $\boldsymbol{p}^{(v)}$ an upper bound l^u on $\underline{l}^*$. Using l^u and a related constraint equation, an attempt is made to reduce the initial domain $\boldsymbol{p}^{(v)}$ to a narrower domain $\boldsymbol{p}^{(v+1)}$ by applying a constraint satisfaction technique. The progress in the domain reduction is measured by the distance $q(\boldsymbol{p}^{(v)}, \boldsymbol{p}^{(v+1))})$. If $q(\boldsymbol{p}^{(v)}, \boldsymbol{p}^{(v+1))})$ is larger than a given threshold ε_q, the iterations are resumed. The iterative process continues until the width of the current domain becomes smaller than a given threshold ε_p.

The constraint equation is determined at each current v-th iteration corresponding to the v-th domain $\boldsymbol{p}^{(v)}$ and the parametrized solution is computed $\boldsymbol{x}(p) = Lp + \boldsymbol{a}$, $p \in \boldsymbol{p}^{(v)}$. Now $l(x, p)$ is put in the form

$$l(x, p) = \sum_i c_i(p)\boldsymbol{x}_i(p), p \in \boldsymbol{p}^{(v)} \tag{33}$$

An upper bound l^u on $\underline{l}^*$ is found as well (using an inner estimation $\boldsymbol{\xi}$ of the hull solution $\boldsymbol{x}^*$ in the current domain or applying a local optimization method). Thus, the constraint equation

$$\sum_i c_i(p)\boldsymbol{x}_i(p) = l^u, \;\; p \in \boldsymbol{p}^{(v)} \tag{34}$$

is obtained. A simple constraint satisfaction technique is now applied, trying to reduce the current domain $\boldsymbol{p}^{(v)}$ to a narrower domain $\boldsymbol{p}^{(v+1)}$. If no progress in the reduction of $\boldsymbol{p}^{(v)}$ has been achieved (either $\boldsymbol{p}^{(v)} = \boldsymbol{p}^{(v+1)}$ or the reduction is negligible), another possibility is to use the monotonicity conditions

$$\partial l(x, p)/\partial p_i = (\partial c_i(p)/\partial p_i)x_i(p) + c_i(p)(\partial x_i(p)/\partial p_i) \tag{35}$$

The upper bound $\overline{l}^*$ on the range $\boldsymbol{l}^*$ is determined in, essentially, the same manner. The only difference is that at each iteration use is made of a lower bound l^l on $\underline{l}^*$.

To illustrate the performance of the proposed method, some numerical examples will be provided.

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An implementation of a posteriori interval analysis technique and its application to linear algebra problems

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Keywords: interval computations, a posteriori, determinant, linear systems

Introduction

In traditional interval computations often as a result we have a very wide interval. This is mainly due to the dependence of the variables in the calculation. To solve this problem special interval radius reduction methods have been developed. One of the first such methods is the *Generalized Interval Arithmetic* proposed by Hansen in [1]. Later in [2] Yu.Matiyasevich proposed new technique called *aposteriori interval analysis*. The work presents the implementation of aposteriori interval computations library for arbitary programs in C++ and its application to the computation of the determinant and solving linear systems.

Proposed implementation

We denote the interval as a pair $x = (val, err) = [val-err, val+err]$. Suppose we have a program with input values $x_1, ..., x_n$ and output value $y = (val_y, err_y)$. Then the idea of Generalized and Aposteriori methods is to replace err_y by $\sum_{i=1}^{n} \left|\frac{\partial y}{\partial x_i}\right| err_{x_i}$. Generalized Interval Arithmetic calculates the required derivatives during the computation of the main program and there are redundant computations that can be avoided. If the complexity of the program ($O(T(n))$ then the complexity of Generalized method will be $O(nT(n))$. On the other hand, Aposteriori method calculates derivatives after main program, so we have two steps:

- main program; y computation;
- calculation of derivatives and error estimate.

And using automatic differentiation techniques we can get the $O(T(n))$ complexity for program with one output variable. If we have $m = m(n)$ output variables then the complexity will be $O(mT(n))$.

For traditional interval computations with arbitrary precision used the Arb [3] library. For automatic generation of second step of Aposteriori method used simulation of Aposteriori machine proposed in [4]. Implementation has a user-friendly interface — it has two additional types for intermediate values and result. Then one set result value y to intermediate value x_m automatically called error estimation step and y will have a new error value. This approach will be called *dynamic*, because the second stage is generated during the execution of main program.

The second possible approach is to write the second step of Aposteriori method by hand. It can give the increase in speed and spatial complexity. This approach will be called *static*. Static versions of determinant computation and solving linear systems using Gauss elimination have been implemented. Static determinant computation algorithm has $O(n^3)$ time complexity and $O(n^2)$ spatial complexity, static linear solver has $O(n^4)$ time and $O(n^2)$ spatial complexity. Dynamic implementations have same time, but $O(n^3)$ spatial complexity due to the storage of all intermediate results.

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A Concoction of Zonotope Abstraction and Constraint Programming for finding an Invariant

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Synthesizing an invariant is a key concept in formal verification ensuring correctness of programs and finding bugs [1]. The standard method for finding an invariant is to look for *inductive invariants*, which is a stronger form of invariant. This work deals with the challenges associated while combining abstract interpretation (zonotopes) and constraint programming for infering inductive invariants.

The well-known approach for finding inductive invariants is abstract interpretation which works by computing the least fixpoint of a functional $\mathbb{P}$ over a (sufficiently structured) partially-ordered domain of program states, defining the program semantics. As $\mathbf{lfp}\mathbb{P} \subseteq I$ implies $\mathbb{P}(I) \subseteq I$, any such I is an inductive invariant, $\mathbf{lfp}\mathbb{P}$ being the smallest one.

A classical way to determine $\mathbf{lfp}\mathbb{P}$ is to use Kleene's theorem (e.g. when $\mathbb{P}$ is continuous and the semantic domain is a complete partial order), which amounts to iterating $\mathbb{P}$ until the least fixed point is reached.

In order to ensure convergence of Kleene iterations within finite number of steps, extrapolation method like widening is used. However, such method overshoots the fixpoint resulting in loss of precision (weak invariants) [2]. In order to circumvent this issue, a descending iteration towards a fixpoint known as narrowing is used. Nevertheless, this technique does not guarantee to find the strongest inductive invariant (for instance, whenever the transition relation of a loop is reflexive, narrowing fails to refine the inductive invariant achieved by widening [2]).

In the light of all these issues, a recent work combines abstract interpretation with constraint programming by interactively splitting and tightening (analogous to the concept of consistency in contractor programming [3]) a collection of abstract elements (boxes and octagons) until the following properties are met [4], [5]. The properties are 1) set of abstract elements contain the entry states, also known as *necessary* abstract elements, 2) they entail the invariant (this property makes it possible to apply concept of contractors), and 3) they are inductive (most indespensible property for proving invariance), also known as *benign* abstract elements.

Owing to the fully relational property of zonotopes, we are interested to explore the refinement in the inference of inductive invariant if any, when zonotope abstract domain is used in conjunction with constraint programming. However, the research issue to circumvent is, zonotopes are not closed under intersection or under splitting.

A first method that we will present is to split zonotopes by overlapping zonotopes : because of these potential overlaps, the method of [5] has to be revisited. Another way to handle this issue would be the use of constrained zonotopes trying to keep a parametrization of the zonotope by the input noise symbols [6]. The main idea behind constrained zonotopes is to tranfer the constraint from the variables' domain to noise symbols' domain by replacing each variable with its corresponding affine form. Then, the constraint is used for all non-linear computations. We also contemplate that the work in connection with zonotope bundles ($\mathfrak{Z}^{\cap} = \{\bigcap_{i=1}^{s} \mathcal{Z}_i | \mathcal{Z}_i \in \mathfrak{Z}\}$, i.e. the intersection of zonotopes $\mathcal{Z}_i$ where $\mathcal{Z} = (c, g^{(1)}, \ldots, g^{(p)})$) [6] can be a suitable choice. A zonotope or a zonotope bundle can

be split by over-approximating it by parallelotope. The parallelotope can be obtained as a matrix whose columns are the directions of span of the parallelotope. The matrix can be calculated using various methods like box, principal component analysis (PCA), generator filtering and flow method [7]. The algorithm presented in [4] has to be modified in the context of zonotopes (at least while taking care of the *necessary* abstract elements after partitioning).

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