# Guaranteed confidence region characterization for source localization using LSCR

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# Localization in Wireless Sensor Network

- Essential and fundamental
  - Positions are required to process collected information
- Challenging
  - Estimation problem with a model non-linear in its parameters
  - Very noisy measurements

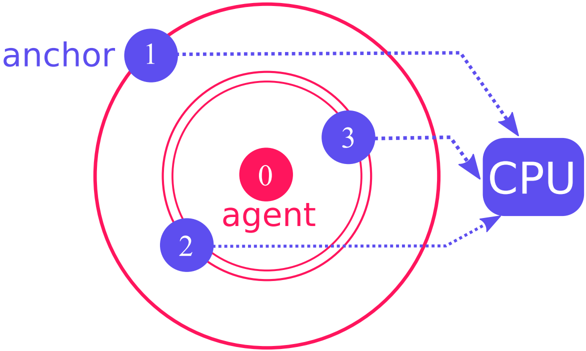
# Estimation of Localization

Received Signal Strength (RSS) (Sklar, 2001)

- Evaluate distance using received signal power
- No need for time synchronization

# Problem Formulation

- Agent with unknown location $\theta_0 = [\theta_0^1, \theta_0^2]^T$
- Anchors $a, a = 1, \ldots, N_a$ located at $\theta_a = [\theta_a^1, \theta_a^2]^T$




Anchors measure RSS from agent and send it to CPU

# Problem Formulation

The RSS measurement from anchor $a$:

$$y_{\mathrm{m}}(a,\mathbf{p}) = P_0 - 10 n_P \log_{10} \frac{\|\theta_0 - \theta_a\|}{d_0},$$
$$a = 1, \ldots, N_a$$

where

- $\mathbf{p} = [P_0, n_P, \theta_0^1, \theta_0^2]$ : unknown parameters
- $\mathbf{p}^*$ : true values of unknown parameters
- $P_0$ : reference power received at a reference distance $d_0$
- $n_P$ : path-loss exponent

# LS, ML, or MAP estimation I

- Estimate the location, e.g., via ML estimation (Wymeersch et al., 2009, Vaghefi et al., 2013)

  Given model $\boldsymbol{y}^{\mathrm{m}}(\mathbf{p})$ and measurements $\boldsymbol{y}$

$$\boldsymbol{y}_{\mathrm{m}}(\mathbf{p}) = \begin{bmatrix} y_{\mathrm{m}}(1, \mathbf{p}) \\ \vdots \\ y_{\mathrm{m}}(N_a, \mathbf{p}) \end{bmatrix}, \boldsymbol{y} = \begin{bmatrix} y(1) \\ \vdots \\ y(N_a) \end{bmatrix}$$

  minimize the cost function $J = \|\boldsymbol{y} - \boldsymbol{y}_{\mathrm{m}}(\mathbf{p})\|^2$

  Cons: solution may get trapped at local minimizer

# LS, ML, or MAP estimation II

- Get estimator confidence region, e.g., via Cramér Rao bound (Qi, 2003, Hossain and Soh, 2010)

  Noise is assumed to be normal or log-normal
  Hypothesis is difficult to verify asymptotic result

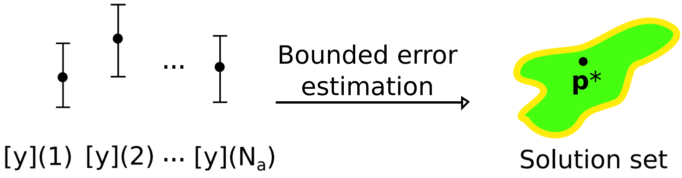
# Bounded Error Methods

- Bounded-error localization techniques (Léger and Kieffer, 2010, Mourad et al., 2011)

  Noise is assumed to be within a bound

  bounds too small may be violated : empty solution set

  bounds are too large lead to huge solution sets



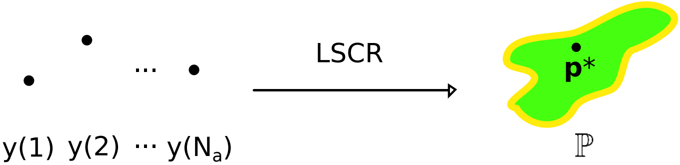
# Our approach with LSCR

Leave-out sign-dominant correlated regions (LSCR) (Campi and Weyer, 2005)

Requires very mild assumptions on noise samples

- Independent
- Symmetrically distributed

Provides a set $\mathbb{P}$ to which $\mathbf{p}^*$ belongs with specified probability



# Main Idea of LSCR

Consider prediction errors

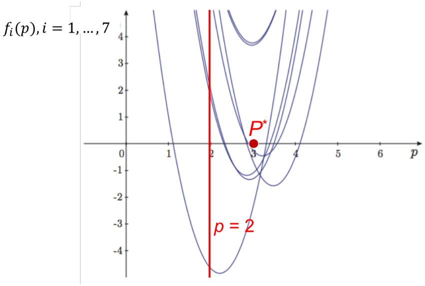
$$\varepsilon(a, \mathbf{p}) = y(a) - y_{\mathrm{m}}(a, \mathbf{p})$$

LSCR: estimates of empirical correlations should have random signs

Leave out subset of parameter space where sign is not random
(i.e. sign dominant)

# Example

- Consider model $y_{\mathrm{m}}(p) = p$
- $N_m = 8$ noisy data generated with $p^* = 3$
- $N_f = 7$ empirical correlation functions $f_i(p)$




We want to find $p$ such that $f_i(p)$ have random signs (which will be around $p^*$)

# Description I

1. Consider the prediction error

$$\varepsilon(a,\mathbf{p}) = y(a) - y_{\mathrm{m}}(a,\mathbf{p})$$

   such that $\varepsilon(a,\mathbf{p}^*)$ is realization of noise of anchor a.

2. Compute a vector $\boldsymbol{w}$

$$\boldsymbol{w}(\mathbf{p}) = \begin{bmatrix} \varepsilon(1,\mathbf{p})\varepsilon(2,\mathbf{p}) \\ \vdots \\ \varepsilon(N_a-1,\mathbf{p})\varepsilon(N_a,\mathbf{p}) \end{bmatrix}$$

# Description II

3. Compute empirical correlation functions

$$\begin{bmatrix} f_1(\mathbf{p}) \\ \vdots \\ f_{N_f}(\mathbf{p}) \end{bmatrix} = \boldsymbol{G}\boldsymbol{w}(\mathbf{p})$$

$\boldsymbol{G}$ : a binary square matrix with size $N_a - 1 = N_f$

4. Select an integer $q \geq 1$ and find $\mathbf{\Theta}$ such that at least $q$ of functions $f_i(\boldsymbol{p})$ are larger than 0 and at least $q$ are smaller than 0 :

$$\mathbf{\Theta} = \{\mathbf{p} \in \mathbb{P}_q \text{ such that } \mathrm{pos}(\boldsymbol{f}(\mathbf{p})) \geq q \text{ and } \mathrm{neg}(\boldsymbol{f}(\mathbf{p})) \geq q\}$$

where $\mathbb{P}_q$ is prior domain for $\mathbf{p}$

# LSCR
## Properties

The set $\boldsymbol{\Theta}$ is such that (Campi and Weyer, 2005)

$$\Pr(\mathbf{p}^* \in \boldsymbol{\Theta}) = 1 - 2q/N_f.$$

Shape and size of $\boldsymbol{\Theta}$ depend on

- values given to $q$
- size of $\boldsymbol{G}$

A procedure for generating $\boldsymbol{G}$ suggested in (Gordon et al., 1974).

# Example

- Consider $y_{\mathrm{m}}(p) = p$ and 8 noisy data generated by $p^* = 3$
- Matrix $\boldsymbol{G}$ with size $7 \times 7$

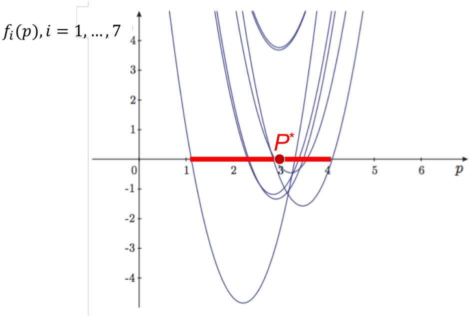
$$\boldsymbol{G} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Example

$$\begin{aligned} f_1(p) = & (y_m(1,p) - y_1(1))(y_m(2,p) - y(2)) \\ & + (y_m(2,p) - y_1(2))(y_m(3,p) - y(3)) \\ & + (y_m(4,p) - y_1(4))(y_m(5,p) - y(5)) \\ & + (y_m(5,p) - y_1(5))(y_m(6,p) - y(6)) \end{aligned}$$

# Example

- $N_f = 7$ empirical correlation functions is generated




- With threshold $q = 1$, $\Pr(p^* \in \Theta) = 1 - 2 * 1/7 = 71\%$

# Confidence region characterization using SIVIA I

- Inclusion function
  - $\boldsymbol{f}(\mathbf{p})$ –> $[\boldsymbol{f}]([\mathbf{p}])$
  - pos($\boldsymbol{f}$) –> [pos]([$\boldsymbol{f}$])
  - neg($\boldsymbol{f}$) –> [neg]([$\boldsymbol{f}$])

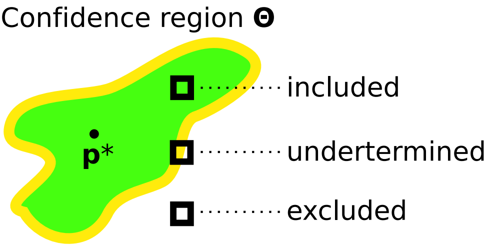
## Example

Calculate $[\text{pos}]([[1,2],[-1,1],[-2,-1]]^T)$

$$[\text{pos}]([[1,2],[-1,1],[-2,-1]]^T) = [1,1]+[0,1]+[0,0]$$
$$= [1,2]$$

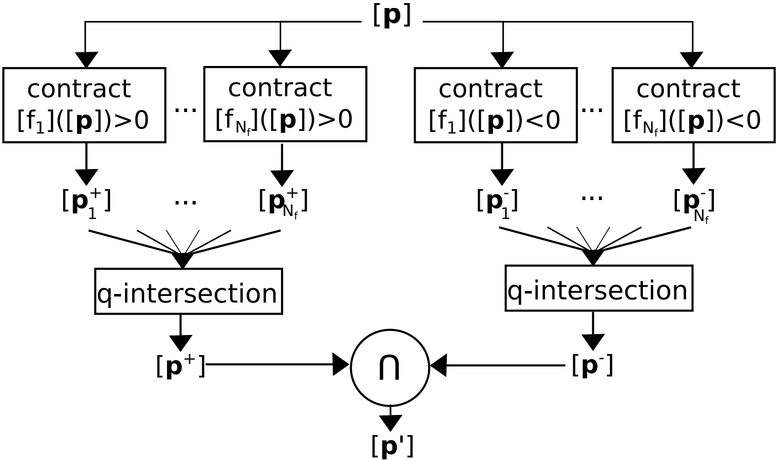
# Confidence region characterization using SIVIA II

- Included: $\underline{[\text{pos}]([\boldsymbol{f}]([\boldsymbol{p}]))} \geq q$ and $\underline{[\text{neg}]([\boldsymbol{f}]([\boldsymbol{p}]))} \geq q$
- Excluded: $q > \overline{[\text{pos}]([\boldsymbol{f}]([\boldsymbol{p}]))}$ or $q > \overline{[\text{neg}]([\boldsymbol{f}]([\boldsymbol{p}]))}$



# Confidence region characterization using Contractors

Contract [**p**] by $\underline{[\text{pos}]([\boldsymbol{f}]([\mathbf{p}]))} \geq q$ and $\underline{[\text{neg}]([\boldsymbol{f}]([\mathbf{p}]))} \geq q$



# Contract $f_i(\mathbf{p}) > 0$

- Centred form contractor (CF)
  - Represent $f_i$ in centred form

$$[f_i]([\mathbf{p}]) = f_i(\mathbf{m}) + ([\mathbf{p}] - \mathbf{m})^T [\boldsymbol{g}_i]([\mathbf{p}])$$

  - Isolate each $[p_k], k = 1, ..., N_p$ in $[\mathbf{p}]$

$$[p_k^{'}] = [p_k] \cap (([f_i]([\mathbf{p}]) \cap [0, \infty] - f_i(\mathbf{m}) - \sum_{j=1, j \neq k}^{N_p} ([p_j] - m_j)[g_{i,j}])/[g_{i,k}]([\mathbf{p}]) + m_k)$$

- Forward backward contractor (FB)
- 3B-Cid Contractor (3B)

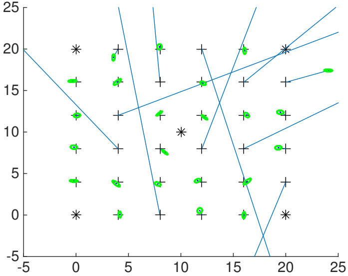
# Experimental Setup

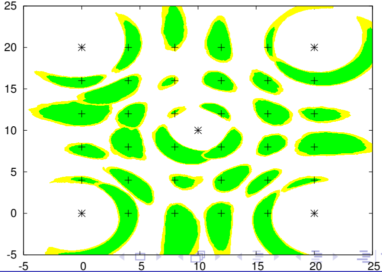
- $N_a = 5$ anchors $(*)$ and $N = 32$ agents $(+)$
- Anchor measures RSS of each agent $N_m = 10$ times
- Assume $P_0 = 30$ and $n_P = 4$ are same for the agents
- Noise: Gaussian-Bernoulli-Gaussian (GBG):

$$y(a) = y_{\mathrm{m}}(a, \mathbf{p}^*) + \varepsilon, \varepsilon = \begin{cases} N(0, \sigma^2 = 2) & \text{probability } 0.9 \\ N(0, \sigma^2 = 5) & \text{probability } 0.1 \end{cases}$$

# Compare to MLE method $\mathbf{p} = [P_0, \theta_0^1, \theta_0^2]$

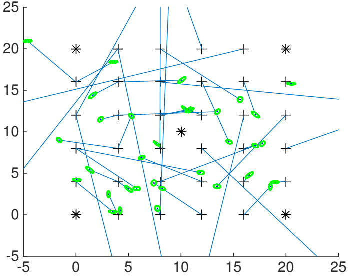
- Maximum likelihood + CRLB
  - blue lines: error between $\theta_0$ and $\theta_0^*$
  - green ellipses: CRB centered on the estimated location
- LSCR
  - Projections of 90% confidence regions using SIVIA

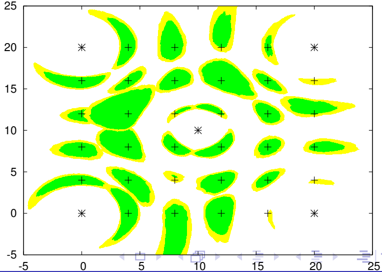


# Compare to MLE method $\mathbf{p} = [n_P, \theta_0^1, \theta_0^2]$

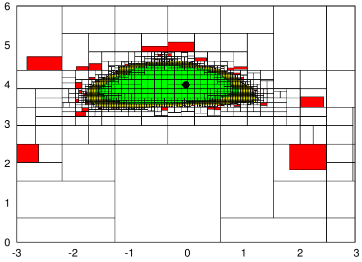
- Maximum likelihood + CRLB
  - blue lines: error between $\theta_0$ and $\theta_0^*$
  - green ellipses: CRB centered on the estimated location
- LSCR
  - Projections of 90% confidence regions using SIVIA



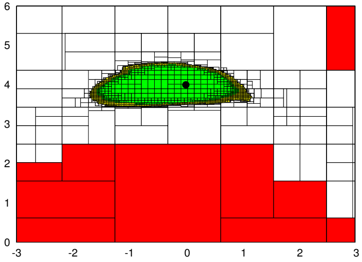



# Comparing CF and FB $\mathbf{p} = [\theta_0^1, \theta_0^2]$

- Forward-backward (FB) and Centred form (CF) Contractor
  - Red box : area eliminated by test SIVIA
  - White box : area eliminated by contractor



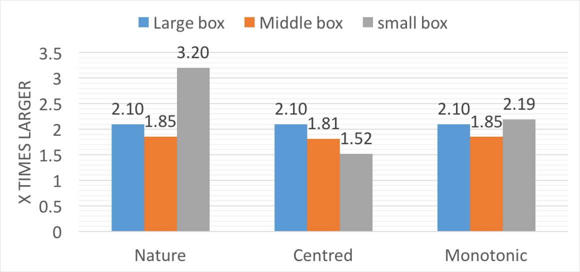

(e) FB contractor




(f) CF contractor

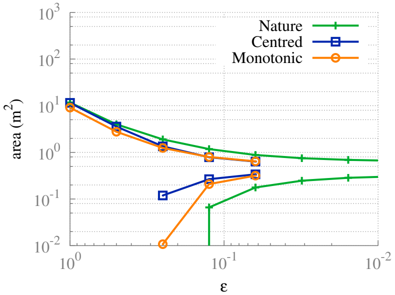
# Difficulties for Interval analysis $\mathbf{p} = [n_P, \theta_0^1, \theta_0^2]$

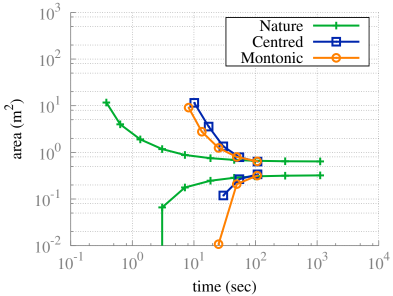
- Many occurrences of variables in $[f_i]([\mathbf{p}])$
  - Pessimism of inclusion function
  - Inefficient contractor
- During SIVIA, evaluation of overestimation of $[f_i]([\mathbf{p}])$
  - w(small box) $<$ 0.5m $\leq$ w(middle box) $<$ 5.0m $\leq$ w(large box)



# Different Inclusion Functions

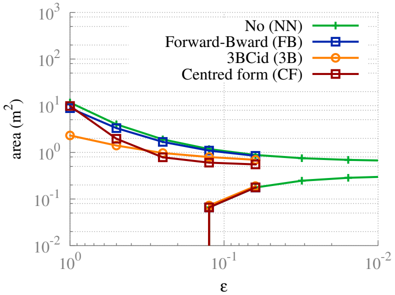
- Convergence respect to SIVIA $\varepsilon$
  - upper lines : volume for (included + undetermined) region
  - lower lines : volume for included region

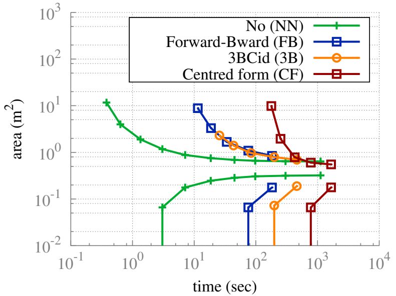


# Different Contractors

- Convergence respect to SIVIA $\varepsilon$
  - upper lines : volume for (included + undetermined) region
  - lower lines : volume for included region



# Conclusion

- Non-asymptotic confidence regions for localization using LSCR
- The results are better than MLE when considering $\mathbf{p} = (n_P, \theta_0^1, \theta_0^2)$ and $\mathbf{p} = (P_0, \theta_0^1, \theta_0^2)$
- Apply contractors and inclusion functions to improve the results
- Future works
  - Improve the contractor
  - Consider $\mathbf{p} = (P_0, n_P, \theta_0^1, \theta_0^2)$
  - Consider mobility model

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## Confidence region characterization using Contractors

1. Contract $[\mathbf{p}]$ by $\underline{[\text{pos}]([\boldsymbol{f}]([\mathbf{p}]))} \geq q$:
   1. Contract $f_i > 0$ , $i = 1, ..., N_f$ to obtain $[\mathbf{p}_i^+]$
   2. Perform $q$-intersection for every $[\mathbf{p}_i^+]$ to obtain $[\mathbf{p}^+]$
2. Contract $[\mathbf{p}]$ by $\underline{[\text{neg}]([\boldsymbol{f}]([\mathbf{p}]))} \geq q$
   1. Contract $f_i < 0$ , $i = 1, ..., N_f$ to obtain $[\mathbf{p}_i^-]$
   2. Perform $q$-intersection for every $[\mathbf{p}_i^-]$ to obtain $[\mathbf{p}^-]$
3. $[\mathbf{p}'] = [\mathbf{p}^+] \cap [\mathbf{p}^-]$
   - $[\mathbf{p}'] \subset [\mathbf{p}]$
   - $[\mathbf{p}'] \cap \mathbb{P} = [\mathbf{p}] \cap \mathbb{P}$