Interval technique to check the performance of control laws applied to wind turbines

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Outline

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Motivation

$\dot{x} \in \mathcal{F}(x, p, u)$, $x(0) \in [x_0], p \in [p]$

Non linear , uncertain

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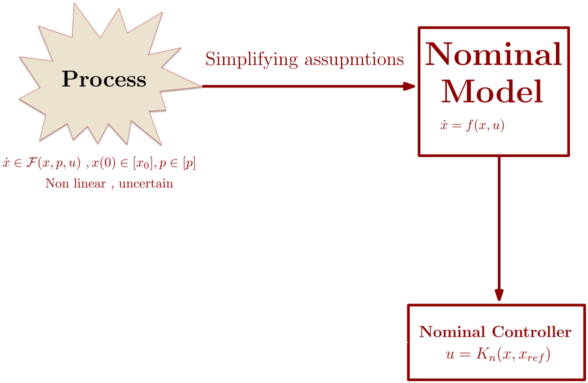
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Simplifying assupmtions
Nominal Model
$\dot{x} = f(x,u)$
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Non linear , uncertain
Nominal Controller
$u = K_n(x, x_{ref})$

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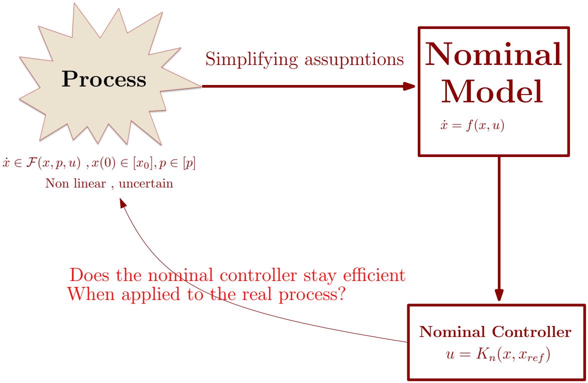
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When applied to the real process?
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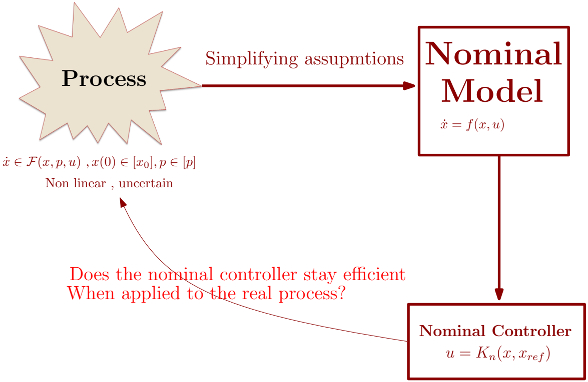

Objective: Propose a numerical technique able to evaluate a priori the expected performances of the nominal controller when applied to the real process .

Outline

1. Motivation
2. **Problem formulation**
3. Interval Control Verification Technique
   - Step 1: set-membership Formulation of the desired specifications
   - Step 2: Compute an outer-approximation of the reachable set of the closed-loop
   - Step 3: Set membership inclusion tests
4. Case study: Wind turbine
   - Simulation example
   - Experimental Example
5. Conclusion and perspectives

Problem formulation

Consider a complex system

$$\dot{\mathbf{x}} \in \mathcal{F}(\mathbf{x}, \mathbf{p}, \mathbf{u}), \quad \mathbf{x}(t_0) \in \mathcal{X}_0 \subset \mathbb{R}^n, \quad \mathbf{p} \in \mathcal{P} \subset \mathbb{R}^p$$

To design a feedback control

$$\mathbf{u} = \mathbf{k}(\mathbf{x}, \mathbf{x}_{\text{ref}})$$

A simple nominal model is needed

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$$

Is this nominal controller stay efficient when applied to the real system ?

$\Longrightarrow$ To check that ,we propose a technique based on **reachability analysis** in order to **evaluate** the performance of the nominal controller.

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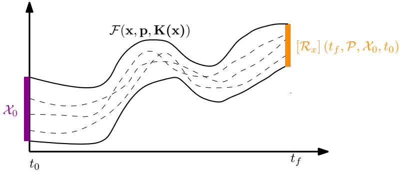
Reachability analysis

The controlled real system

$$\dot{\mathbf{x}} \in \mathcal{F}(\mathbf{x}, \mathbf{p}, \mathbf{K}(\mathbf{x})), \quad \mathbf{x}(t_0) \in \mathcal{X}_0, \quad \mathbf{p} \in \mathcal{P}$$

We can compute an over-approximation of the reachable set denote here by:

$$[\mathcal{R}_x]([t_0, t_f], \mathcal{P}, \mathcal{X}_0, t_0)$$



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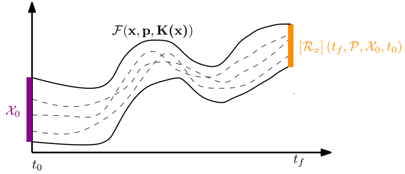

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To compute the over-approximation of the reachable set (for NL systems), we use:

- **Interval Taylor methods**, [*N.S.Nedialkov*],[*R.Rihm*],[*R.J.Lohner*]
- **Comparison theorems for differential inequalities**, [*N.Ramdani*, *N.Meslem*]

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Interval Taylor methods

Interval integration using Taylor expansion:

$$[x_{j+1}] = [x_{j+1}] + \sum_{i=1}^{k-1} h_j^i \mathcal{F}^{[i]}([x_j]) + h_j^k \mathcal{F}^{[k]}([\tilde{x}_j])$$

Comparison theorems for differential inequalities

Transform the uncertain system into two deterministic systems : $x(t) \in [\ \underline{x}(t), \overline{x}(t)\ ]$

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Principle of the technique

The three main steps of the technique are:

1. **Step 1** : Rewrite the desired control specifications as set- membership criteria
2. **Step 2** : Compute an outer-approximation of the reachable set of the closed-loop system.
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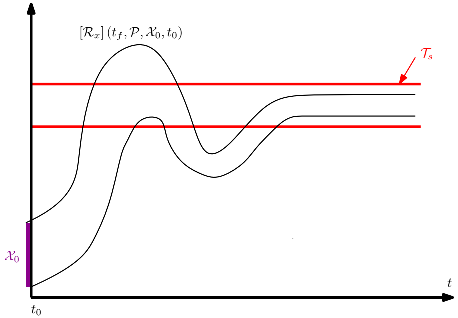
Step 1: set-membership Formulation of the desired specifications

First specification

- **Target set $\mathcal{T}_s$** : the desired behavior of the system at the steady state can be characterized by a set of state vector called target set. The ultimate bound of the closed-loop system must be enclosed in the target set



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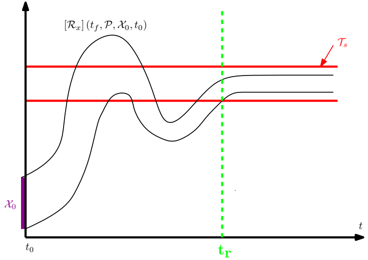
Second specification

- **Reaching time t_r** : in this context, the rapidity of the system is measured by its reaching-time t_r, which is equivalent to the classical settling time. More formally, t_r is the time instant for which:

$$[\mathcal{R}_x]\,(t_r, \mathcal{P}, \mathcal{X}_0, t_0) \subset \mathcal{T}_s$$

and $\forall t \geq t_r$ we get:

$$[\mathcal{R}_x]\,([t_r, t], \mathcal{P}, \mathcal{X}_0, t_r) \subset \mathcal{T}_s$$



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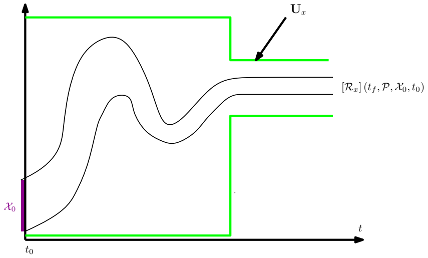
Third specification

- **Safety set $\mathcal{U}_x$** : denoted by $\mathcal{U}_x$, the safety set can be characterized by the state constraints and/or by authorized overshoot of the system outputs,. . . . So, the nominal controller must ensures the following inclusion:

$$[\mathcal{R}_x]\left([t_0, t_f], \mathcal{P}, \mathcal{X}_0, t_r\right) \subset \mathcal{U}_x$$

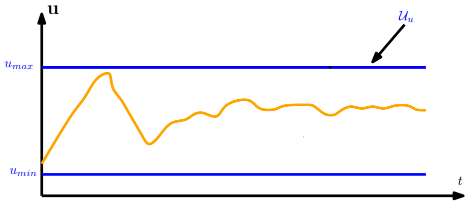


Step 1: set-membership Formulation of the desired specifications

Fourth specification

- **Feasible set $\mathcal{U}_u$** : denoted by $\mathcal{U}_u$, in practice, actuators can not generated a control vector with arbitrary values in $\mathbb{R}^m$. So, the set can be defined by the input constraints and the nominal controller must satisfy:

$$\mathbf{k}(\mathbf{x}, \mathbf{x_{ref}}) \in \mathcal{U}_u$$



Step 2: Compute an outer-approximation of the reachable set of the closed-loop

We compute an over-approximation of the reachable set of the closed-loop system that contains all possible solution $\mathbf{x}(t)$ generated from the set of initial conditions $\mathcal{X}_0$ and for all possible parameter vector $\mathbf{p} \in \mathcal{P}$.

Transitional regime

$$[\mathcal{R}_x]\left([t_0, t_r], \mathcal{P}, \mathcal{X}_0, t_0\right) \quad t \in [t_0, t_r]$$

At reaching time

$$[\mathcal{R}_x]\left(tr, \mathcal{P}, \mathcal{X}_0, t_0\right) \quad t = t_r$$

Steady regime

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Step 3: Set membership inclusion tests

In this context, a nominal controller is said **efficient** if all set-membership inclusion tests are true:

- $[\mathcal{R}_x]([t_r, \mathcal{P}, \mathcal{X}_0, t_0) \subset \mathcal{T}_s$
  (The ultimate bound of the closed-loop system is enclosed in the **target set**).
- $[\mathcal{R}_x]([t_r, t], \mathcal{P}, \mathcal{X}_0, t_r) \subset \mathcal{T}_s$
  (The target set is achieved at the **reaching time**).
- $[\mathcal{R}_x]([t_0, t_r], \mathcal{P}, \mathcal{X}_0, t_r) \subset \mathcal{U}_x$
  (**safety set**: state constraints are non violated).
- $\mathbf{k}(\mathbf{x}, \mathbf{x}_{\mathbf{ref}}) \in \mathcal{U}_u$
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Simulation example

Let's consider the Electric Wind Power Generation (EWPG) system defined by the following complete model:

$$\begin{cases} \dot{v} = -\alpha_v(v - \nu_0) + \eta \\ \dot{\omega} = \frac{1}{2J_{eq}} \rho N_x \pi r^2 C_p \frac{v^3}{w} - \frac{B_e}{J_e} w - \frac{K_e i_f}{J_{eq}} i_a \\ \dot{i_f} = \frac{U_f}{L_f} - \frac{R_f}{L_f} i_f \\ \dot{i_a} = \frac{K_e i_f}{L_a} w - \frac{R_a}{L_a} i_a - \frac{U_a}{L_a} \end{cases}$$

$[v, w, i_f, i_a]$ is the state vector, $u = [U_f, U_a]^T$ is the input, the measurements are uncertain

$v \in v_m + [-\epsilon_v, +\epsilon_v] \quad \epsilon_v = 0.1$
$w \in w_m + [-\epsilon_w, +\epsilon_w] \quad \epsilon_w = 0.1$
$i_f \in i_{f_m} + [-\epsilon_{i_f}, +\epsilon_{i_f}] \quad \epsilon_{i_f} = 0.1$
$i_g \in i_{g_m} + [-\epsilon_{i_g}, +\epsilon_{i_g}] \quad \epsilon_{i_g} = 0.1$
Uncertain parameters : $\rho \in [1.1875, 1.3125]$, $B_e \in [0.0142, 0.0158]$

Simulation example

Parameters	Value
Turbine Radio, R	$4.0\ m$
Turbine inertia, J_T	$1.5\ kgm^2$
Wind speed range	$2-12\ ms^{-1}$
Transmission ratio, N_x	7.5
Wind density, ρ	$1.25\ kgm^{-3}$
DC generator inertia, J_e	$0.3\ kgm^2$
Maximum DC generator output power, P_{gen}	$6.0\ kW$
Rated field voltage, V_f	$120\ V$
Rated armature voltage, V_a	$240\ V$
Rated field flow, ϕ_f	$0.12\ Wb$
Rated field current flow, I_F	$2\ A$
Field inductance, L_f	$60\ mH$
Field resistance, R_f	$60\ \Omega$
Armature inductance, L_a	$10\ mH$
Armature resistance, R_a	2.0Ω
Friction constant, B_e	0.015
Induced EFM constant, K_e	0.5
Wind speed time constant, α_v	0.2

Figure: Table of numerical values

Simulation example

The objective is to maximize the power generated by the wind turbine by ensuring the following performance criterion

$$Z = (\lambda_{opt} v - rw) = 0$$

based on the linearized model around the operating point $(v_0, w_0, i_{f0}, i_{g0}) = (5.3m/s, 82.48rad/s, 2A, 185.8A)$,$u_0 = (120V, 289.28V)$, an LQR controller $u = -k(x - x_0) + u_0$ is designed.

The objective of the controller is to minimize the criterion Z and make it close to 0. The LQR controller is obtained with the following weighting matrices :

$$Q = \begin{pmatrix} \lambda_{opt}^2 & -r\lambda_{opt} \\ -r\lambda_{opt} & r^2 \end{pmatrix} \quad R = 10^5$$

	Desired specification
Target set	$Z \in [-0.3, 0.3]$
Reaching time	$t_r \leq 8s$
Safety set	$(v, w, i_f, i_g) \in ([1, 15], [60, 160], [0, 7], [0, 215])$
Feasible set	$(Uf, Ua) \in ([0, 121], [0, 291])$

Table: Table of desired specifications

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Safety set	$(v, w, i_f, i_g) \in ([1, 15], [60, 160], [0, 7], [0, 215])$
Feasible set	$(Uf, Ua) \in ([0, 121], [0, 291])$

Table: Table of desired specifications

Simulation example

The objective is to maximize the power generated by the wind turbine by ensuring the following performance criterion

$$Z = (\lambda_{opt} v - rw) = 0$$

based on the linearized model around the operating point $(v_0, w_0, i_{f0}, i_{g0}) = (5.3m/s, 82.48rad/s, 2A, 185.8A)$,$u_0 = (120V, 289.28V)$, an LQR controller $u = -k(x - x_0) + u_0$ is designed.

The objective of the controller is to minimize the criterion Z and make it close to 0. The LQR controller is obtained with the following weighting matrices :

$$Q = \begin{pmatrix} \lambda_{opt}^2 & -r\lambda_{opt} \\ -r\lambda_{opt} & r^2 \end{pmatrix} \quad R = 10^5$$

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Table: Table of desired specifications

Simulation example

Compute an outer-approximation of the reachable set of the closed-loop system (upper and lower bounding systems).

$$
\left\{
\begin{array}{l}
\begin{cases}
\dot{\overline{v}} = -\alpha_v(\overline{v} - \nu_0) + \overline{\eta} \\
\dot{\overline{w}} = \frac{1}{2J_{eq}}\overline{\rho}N_x\pi r^2 C_p\frac{\overline{v}^3}{\overline{w}} - \frac{B_e}{J_e}\overline{w} - \frac{K_e\underline{i}_f}{J_{eq}}\underline{i}_a \\
\dot{\overline{i}}_f = \frac{-1}{L_f}(K_1(\overline{v} + \overline{\epsilon}_v - v_0) + K_2(\overline{w} + \overline{\epsilon}_w - w_0) + K_3(\overline{i}_f + \underline{\epsilon}_{if}) + K_4(\underline{i}_a + \overline{\epsilon}_{ia}) - u_{f_0}) - \frac{R_f}{L_f}\overline{i}_f \\
\dot{\overline{i}}_a = \frac{K_e\overline{i}_f}{L_a}\overline{w} - \frac{R_a}{L_a}\overline{i}_a - \frac{1}{L_a}(K_1(\underline{v} + \underline{\epsilon}_v - v_0) + K_2(\underline{w} + \underline{\epsilon}_w - w_0) + K_3(\overline{i}_f + \overline{\epsilon}_{if}) + K_4(\overline{i}_a + \overline{\epsilon}_{ia}) - u_{g_0})
\end{cases} \\
\begin{cases}
\dot{\underline{v}} = -\alpha_v(\underline{v} - v_0) + \underline{\eta} \\
\dot{\underline{w}} = \frac{1}{2J_{eq}}\underline{\rho}N_x\pi r^2 C_p\frac{\underline{v}^3}{\underline{w}} - \frac{B_e}{J_e}\underline{w} - \frac{K_e\overline{i}_f}{J_{eq}}\overline{i}_a \\
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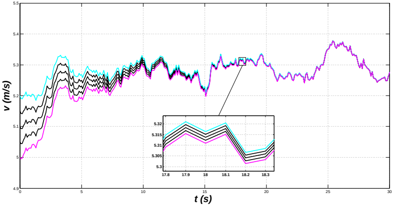
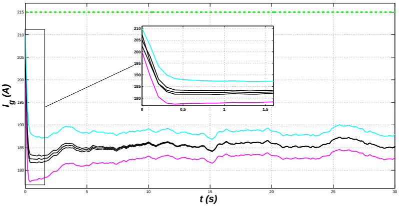
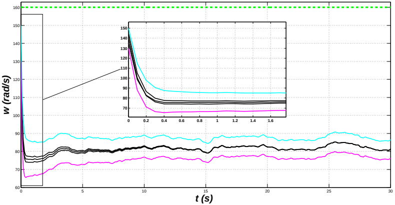
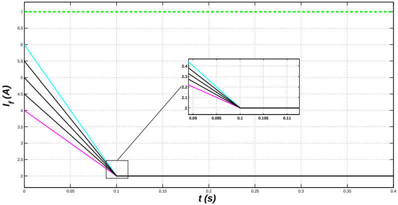
\dot{\underline{i}}_a = \frac{K_e\underline{i}_f}{L_a}\underline{w} - \frac{R_a}{L_a}\underline{i}_a - \frac{1}{L_a}(K_1(\overline{v} + \overline{\epsilon}_v - v_0) + K_2(\overline{w} + \overline{\epsilon}_w - w_0) + K_3(\underline{i}_f + \underline{\epsilon}_{if}) + K_4(\underline{i}_a + \underline{\epsilon}_{ia}) - u_{g_0})
\end{cases}
\end{array}
\right.
$$

Simulation results

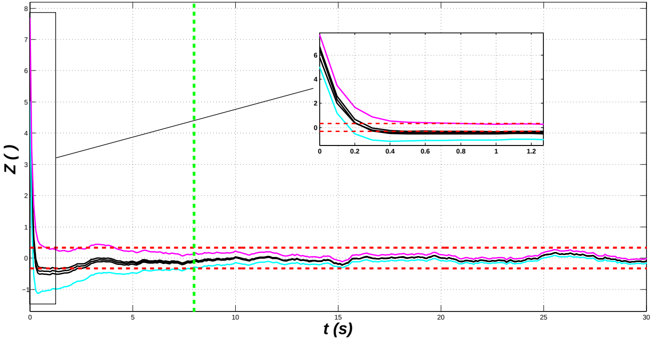




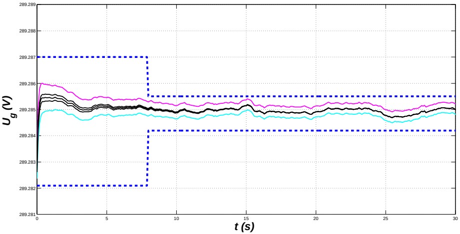


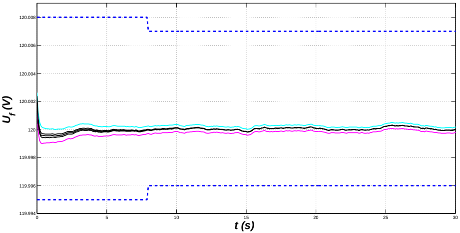


Simulation results

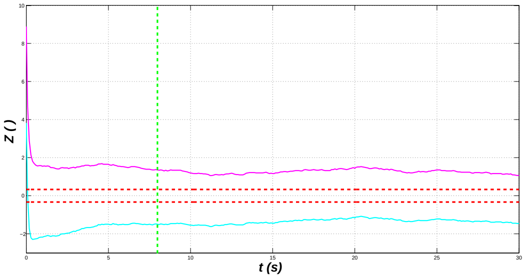


Simulation results



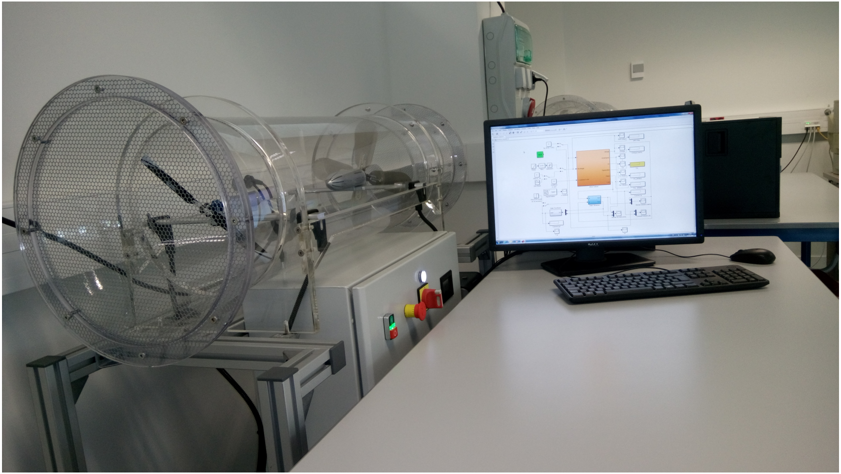


Case with big uncertainty of sensors: We will take a case of sensors that gives erroneous measurement, in this case with our technique, we can not conclude about the efficiency of the controller.



Outline

Test bench



Experimental model

The experimental wind turbine system is defined by the model:

$$\begin{cases} \dot{v} = -\alpha_v(v - v_0) + \eta \\ \dot{w} = \frac{1}{2J_e}\rho\pi r^2 \frac{v^3}{w} - \frac{[B_e]}{J_e}w - \frac{K_{eq}}{J_e}i_g \end{cases}$$

$[v, w]$ is the state vector, $u = i_g$ is the input, the measurements are uncertain
$v \in v_m + [-\epsilon_v, +\epsilon_v] \quad \epsilon_v = 0.1$
$w \in w_m + [-\epsilon_w, +\epsilon_w] \quad \epsilon_w = 0.1$
Uncertain parameters : $\rho \in [1.1875, 1.3125]$, $B_e \in [0.99, 1.21] \times 10^{-5}$

As before the LQR controller is designed by taking the linearized model around the operating point $x_0 = (2.1m/s, 156rad/s)$, $u_0 = 9.36 \times 10^{-4}$.

	Desired specifications
Target set	$Z \in [-0.3, 0.3]$
Reaching time	$t_r \leq 3s$
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Compute an outer-approximation of the reachable set of the closed-loop system.

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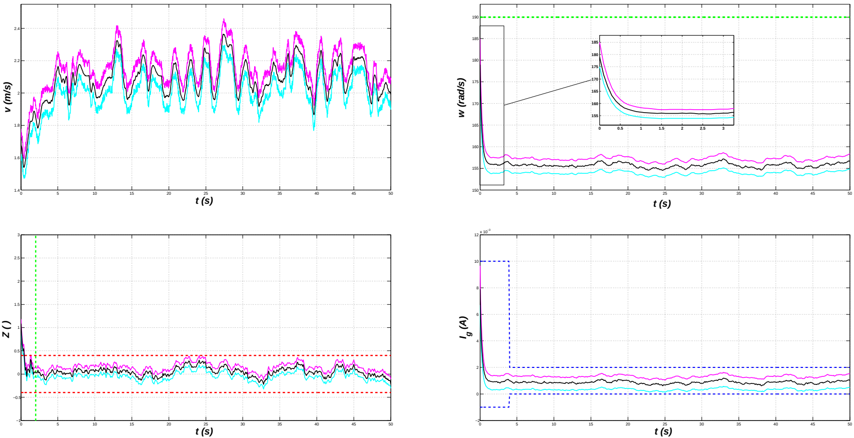
Simulation results



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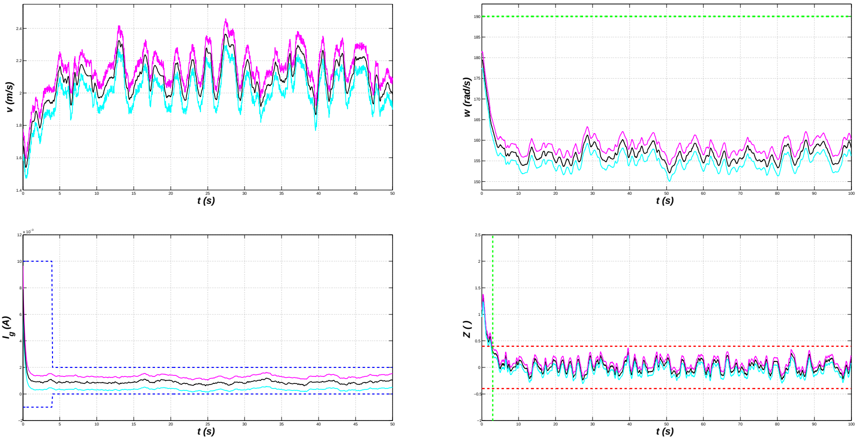

Outline

Conclusion and perspectives

- **Conclusion:** we proposed an interval technique based on reachability analysis to evaluate a priori the performance of nominal controller applied to real systems. To do that, a list of set-membership inclusion tests were used to verify the desired specifications.

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THANK YOU FOR YOUR ATTENTION