Robust Hybrid State Estimation using Interval Methods

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Outline

- Hybrid dynamical systems
- Set membership estimation
- Hybrid reachability based approach
- Example
- Research directions
Hybrid Cyber-Physical Systems

- Interaction discrete + continuous dynamics
- Safety-critical embedded systems
- Networked autonomous systems
Hybrid Cyber-Physical Systems

Modelling → hybrid automaton (Alur, et al. 1995)

- Non-linear continuous dynamics
- Nonlinear guards sets
- Nonlinear reset functions
- Bounded uncertainty

\[ H = (Q, \mathcal{D}, \mathcal{P}, \Sigma, \mathcal{A}, \text{Inv}, \mathcal{F}) \]

Continuous dynamics

\[
\begin{align*}
\text{flow}(q) : & \quad \dot{x}(t) = f_q(x, p, t), \\
\text{Inv}(q) : & \quad \nu_q(x(t), p, t) < 0,
\end{align*}
\]

Discrete dynamics

\[
\begin{align*}
\mathcal{A} \ni e : & \quad (q \rightarrow q') = (q, \text{guard}, \sigma, \rho, q'), \\
\text{guard}(e) : & \quad \gamma_e(x(t), p, t) = 0,
\end{align*}
\]

\[ t_0 \leq t \leq t_N, \quad x(t_0) \in X_0 \subseteq \mathbb{R}^n, \quad p \in \mathcal{P} \]
Example: the bouncing ball

Initial conditions:
\[ x = 10, v = 0 \]

Discrete transition:
\[ \dot{x} = v, \quad \dot{v} = -9.81 \]
\[ x \geq 0, \quad x = 0 \quad \text{and} \quad v' := -v \]
Example: the bouncing ball

- Initial conditions
- Free fall
- Continuous transitions
- Discrete transitions
Estimation of Hybrid State

- **Modelling → hybrid automaton**
  - Nonlinear …
  - Bounded uncertainty

- **Hybrid State Estimation → reconstruct system variables**
  - Switching sequence
  - Continuous variables
  - Hybrid solution trajectory tube

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**Fig. 3.** Time history of the $x_v$ component of the reachable set of $\text{lyvm}$ as obtained with Theorem $v_p$ with an initial domain for state vector of size $\text{utthr}$. The curve labelled 'no uncertainty' corresponds to no uncertainty in the parameter vector $\text{CPU time} = \text{w'rvz} \text{s PIV v GHzm}$ and the one labelled 'with uncertainty' corresponds to the presence of uncertainty in the parameter vector $\text{CPU time} = \text{w'ry' s PIV v GHzmr}$.

**Fig. 4.** Switching sequence for the hybrid automaton which drives the upper bounding system for the latter using functions $\uparrow(\cdot)$ defined in Rule $v_p$. The upper bounding systems are obtained by replacing parameter components in each algebraic expression of $f_i$ either by their upper or by lower bound for $m_q$ type modes or by using whole parameter uncertainty domain for $s_q$ type modes. Since there are $\text{ut}$ partial derivatives to monitor and $w$ possible values for the parameter components $l_{lower \text{ bound}} s$ upper bound $s$ whole uncertainty interval, the set $Q$ of discrete modes contains $w \times \text{ut}$ elements and we merely use a word of ternary digits of length $w \times \text{ut}$ to number the modes. Note however that not all of them may be activated.

**Fig. 5.** Shows the switching sequence for the hybrid automaton which derives the upper component-wise bounds of the reachable set of $\text{lywm}$ as generated by algorithm $\text{Hybrid UpperBounding}$. Some modes are active on very short time intervals. **Fig. 6.** Magnifies the switching sequence around $t = z_t$. In fact, such modes are $s_q$ type modes which are usually active only over one or two integration time intervals. The automaton which derives the lower component-wise bounds is obtained in a similar manner. The switching sequence for this automaton is shown in **Fig. 7.**

Note that both initial state vector and parameter vector are taken uncertain with large uncertainties.

**Fig. 8.** Shows the time history of the $x_{uv}$ component of the reachable set. Obviously, even for very large parameter boxes the hybrid bracketing method successfully computes the reachable set.
Outline

- Hybrid dynamical systems
- Set membership estimation
- Hybrid reachability based approach
- Example
- Research directions
Set Membership Estimation

Unknown but bounded-error framework

Hypothesis

Uncertainties and errors are bounded with known prior bounds

A set of feasible solutions

\[ S = \{ p \in \mathbb{P} \mid f(p) \in Y \} = f^{-1}(Y) \cap \mathbb{P} \]
State estimation with continuous systems

- Interval observers
  - (Moisan, et al. 2009), (Mazenc & Bernard, 2010),
  - (Meslem & Ramdani, 2011), (Raïssi, et al., 2012),
  - (Combastel, 2013), (El Thabet, et al. 2014), (Efimov, et al. 2015)

- Monotonicity
- Change of coordinates
- LMI ....
- Ensure practical stability
State estimation with continuous systems

- Prediction - Correction / Filtering approaches
  - (Raïssi et al., 2005), (Meslem, et al, 2010),
  - (Milanese & Novara, 2011), (Kieffer & Walter, 2011) …

- Reachability
  - + Set inversion
- Forward backward consistency
Set Membership Estimation

- Set inversion. Parameter estimation
  - Branch-&-bound, branch-&-prune, interval contractors ...
  - (Jaulin, et al. 93) (Raïssi et al., 2004)

\[ S = \{ z \in \mathcal{Z}, \ | \ f(z) \in \mathcal{Y} \} \quad \rightarrow \quad S \subseteq \bar{S} \]

- \( f([z]) \subseteq \mathcal{Y} \quad \rightarrow \quad [z] \subseteq S \) : inner approximation
- \( f([z]) \cap \mathcal{Y} = \emptyset \quad \rightarrow \quad [z] \notin \bar{S} \) : outer approximation
- otherwise \( \rightarrow [z] \subseteq \mathcal{Z} \setminus \bar{S} \) : partition ...

![Diagram](image-url)
Set Membership Estimation

- **State estimation with Continuous systems**
  - Interval observers
  - Prediction-correction / Filtering approaches

- **State estimation with Hybrid systems**
  - Piecewise affine systems (Bemporad, et al. 2005)
  - ODE + CSP (Goldsztejn, et al., 2010)
  - Nonlinear case (Benazera & Travé-Massuyès, 2009)
  - SAT mod ODE (Eggers, Ramdani, et al., 2012)
  - Reachability-based (Maïga, Ramdani, et al. 2015).
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Reachability based approach

- Predictor-Corrector approach for hybrid systems
Reachability based approach

- **Predictor-Corrector** approach for hybrid systems

\[
\begin{align*}
    e : g(x) & \geq 0 \\
x & \in \text{Init}(l) \\
x & \in \text{Inv}(l) \\
\dot{x} & \in \text{Flow}(l, x) \\
x' & = r(e, x) \\
x' & \in \text{Flow}(l', x') \\
x' & \in \text{Inv}(l') \\
\dot{x}' & \in \text{Flow}(l', x') \\
\end{align*}
\]
Reachability based approach
Reachability based approach

mode = 1

guard

reset mapping

mode = 2
Reachability based approach

mode = 1

X_1(t_0)
guard

reset mapping

mode = 2
Reachability based approach

\[ X_1(t_0) \quad \varphi(., t_0, X(t_0)) \]

\[ \text{mode} = 1 \quad \text{mode} = 2 \]

\[ \text{guard} \quad \text{reset mapping} \]
Reachability based approach

\[ \phi(., t_0, X(t_0)) \]

mode = 1

mode = 2

[Maïga, Ramdani, Travé Massuyes & Combastel, IEEE TAC 2016].
Reachability based approach

\[ X_{1}(t_0) \]

\[ \phi(., t_0, X(t_0)) \]

guard

reset mapping

mode = 1

[Maïga, Ramdani, Travé Massuyes & Combastel, IEEE TAC 2016].
Reachability based approach

\[ x(t_1) = x(t_0) \phi(., t_0, x(t_0)) \]

mode = 1

reset mapping

mode = 2

guard
Reachability based approach

\[ X_{1}(t_0) \]
\[ X_{1}^{+}(t_1) \]
\[ \text{guard} \]

mode = 1

\[ X_{2}(t_0) \]
\[ \text{reset mapping} \]

mode = 2
Reachability based approach

\[ X_1(t_0) \]

\[ X_1^+(t_1) \]

\[ \text{guard} \]

\[ g_1^{-1}(\cdot) \]

\[ g_2^{-1}(\cdot) \]

\[ \text{reset mapping} \]

\[ \text{mode} = 1 \]

\[ \text{mode} = 2 \]

\[ X_2^+(t_1) \]
Reachability based approach

\[ X_1(t_0) + (t_1) = X_1(t_1) \cap g_1^{-1}(Y(t_1)) \]

\[ X_1^+(t_1) \]

\[ \text{mode} = 1 \]

\[ g_1^{-1}(\cdot) \]

\[ \text{guard} \]

\[ \text{reset mapping} \]

\[ X_2(t_1) = X_2^+(t_1) \cap g_2^{-1}(Y(t_1)) \]

\[ X_2^+(t_1) \]

\[ \text{mode} = 2 \]
Reachability based approach

\[ X_1(t_1) = X_1(t_0) + (t_1) \]

\[ X_1^+(t_1) = X_1^+(t_1) \cap g_1^{-1}(Y(t_1)) \]

\[ X_2(t_1) = X_2(t_1) \cap g_2^{-1}(Y(t_1)) \]

\[ \text{mode} = 1 \]

\[ \text{mode} = 2 \]

Reconstructed Hybrid Solution

State Trajectory:

\[ t_1 \]

\{q=1, X_1(t_1)\} \cup \{q=2, X_2(t_1)\}
Reachability based approach

\[ X_1(t_0) \]

\[ X_1(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1)) \]

\[ X_1^*(t_1) \]

Mode = 1

\[ \text{guard} \]

\[ g_1^{-1}(\cdot) \]

\[ Y(t) \]

Mode = 2

\[ X_2^*(t_1) \]

\[ X_2(t_1) = X_2^*(t_1) \cap g_2^{-1}(Y(t_1)) \]

Reconstructed Hybrid Solution

State Trajectory:

\[ t_1 \]

\{ q=1, X_1(t_1) \} \cup \{ q=2, X_2(t_1) \} \]
Reachability based approach

**Reconstructed Hybrid Solution**

State Trajectory:

\[ X_2(t_1) = X_2^*(t_1) \cap g_2^{-1}(Y(t_1)) \]

\[ t_1 \]

\{q=1, X_1(t_1)\} \cup \{q=2, X_2(t_1)\}
Reachability based approach

\[ X_1(t_0) \]

\[ X_1(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1)) \]

\[ X_1^*(t_2) \]

\[ X_1^*(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1)) \]

mode = 1

\[ \text{guard} \]

\[ g_1^{-1}(.) \]

Reconstructed Hybrid Solution
State Trajectory:

\[ t_1 \]
\[ \{ q=1, X_1(t_1) \} \cup \{ q=2, X_2(t_1) \} \]
Reachability based approach

\[
X_{1}(t_0) + (t_1) = X_{1}(t_1) \cap g_{1}^{-1}(Y(t_1))
\]

\[
X_{1}(t_1) = X_{1}(t_1) \cap g_{1}^{-1}(Y(t_1))
\]

\[
X_{1}(t_2) = X_{1}(t_2) \cap g_{1}^{-1}(Y(t_2))
\]

\[
X_{1}(t_1) = X_{1}(t_1) \cap g_{1}^{-1}(Y(t_1))
\]

\[
X_{2}(t_1) = X_{2}(t_1) \cap g_{2}^{-1}(Y(t_1))
\]

\[
X_{2}(t_1) = X_{2}(t_1) \cap g_{2}^{-1}(Y(t_1))
\]

\[
g_{1}^{-1}(.)
\]

\[
g_{2}^{-1}(.)
\]

mode = 1

mode = 2

guard

reset mapping

Reconstructed Hybrid Solution

State Trajectory:

\[
t_{1}
\]

\[\{q=1, X_{1}(t_1)\} \cup \{q=2, X_{2}(t_1)\}\]
Reachability based approach

### Reconstructed Hybrid Solution

**State Trajectory:**

\[
t_1 \quad \{ q=1, X_1(t_1) \} \cup \{ q=2, X_2(t_1) \}
\]

\[
t_2 \quad \{ q=1, X_1(t_2) \} \cup \{ q=2, X_2(t_2) \}
\]
Reachability based approach

$X_1(t_0)$

$X_1^{*}(t_1)$

$X_1(t_1) = X_1^{*}(t_1) \cap g_1^{-1}(Y(t_1))$

$X_1(t_2) = X_1^{*}(t_2) \cap g_1^{-1}(Y(t_2))$

$X_1^{*}(t_2)$

$X_2^{*}(t_1)$

$X_2(t_1) = X_2^{*}(t_1) \cap g_2^{-1}(Y(t_1))$

$X_2(t_2) = X_2^{*}(t_2) \cap g_2^{-1}(Y(t_2))$

mode = 1

$g_1^{-1}(.)$

$g_1^{-1}(.)$

reset mapping

$g_2^{-1}(.)$ guard

Reconstructed Hybrid Solution State Trajectory:

$t_1$

$\{q=1, X_1(t_1)\} \cup \{q=2, X_2(t_1)\}$

$t_2$

$\{q=1, X_1(t_2)\} \cup \{q=2, X_2(t_2)\}$
Reachability based approach

\[
X_1(t_0) \cup \{q=1, X_1(t_1)\} \cup \{q=2, X_2(t_1)\}
\]

\[
t_1
\]

\[
X_1(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1))
\]

\[
X_1(t_2) = X_1^*(t_2) \cap g_1^{-1}(Y(t_2))
\]

\[
X_1(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1))
\]

\[
X_1(t_2) = X_1^*(t_2) \cap g_1^{-1}(Y(t_2))
\]

\[
X_1(t_1) = X_1^*(t_1) \cap g_1^{-1}(Y(t_1))
\]

\[
X_1(t_2) = X_1^*(t_2) \cap g_1^{-1}(Y(t_2))
\]

\[
X_2(t_1) = X_2^*(t_1) \cap g_2^{-1}(Y(t_1))
\]

\[
X_2(t_2) = X_2^*(t_2) \cap g_2^{-1}(Y(t_2))
\]

\[
X_2(t_1) = X_2^*(t_1) \cap g_2^{-1}(Y(t_1))
\]

\[
X_2(t_2) = X_2^*(t_2) \cap g_2^{-1}(Y(t_2))
\]

Reconstructed Hybrid Solution
State Trajectory:
Reachability based approach

**State Trajectory:**

\[ X_{1}(t_1) = X_{1}^*(t_1) \cap g_{1}^{-1}(Y(t_1)) \]
\[ X_{1}(t_2) = X_{1}^*(t_2) \cap g_{1}^{-1}(Y(t_2)) \]

**Mode = 1**

- \( X_{1}(t_0) \)
- \( X_{1}^*(t_1) \)
- \( g_{1}^{-1}(.) \)
- \( g_{1}^{-1}(.) \)

**Guard**

**Mode = 2**

- \( X_{2}(t_1) = X_{2}^*(t_1) \cap g_{2}^{-1}(Y(t_1)) \)
- \( X_{2}(t_2) = X_{2}^*(t_2) \cap g_{2}^{-1}(Y(t_2)) \)

**Reset Mapping**

**Reconstructed Hybrid Solution**

\( t_1 \)
\{ \text{q=1, } X_{1}(t_1) \} \cup \{ \text{q=2, } X_{2}(t_1) \} \)

\( t_2 \)
\{ \text{q=1, } X_{1}(t_2) \} \cup \{ \text{q=2, } X_{2}(t_2) \} \)
Reachability based approach

Reconstructed Hybrid Solution
State Trajectory:

\[ t_1 \]
\[ \{q=1, X_1(t_1)\} \cup \{q=2, X_2(t_1)\} \]

\[ t_2 \]
\[ \{q=1, X_1(t_2)\} \cup \{q=2, X_2(t_2)\} \]
Reachability based approach

mode = 1

\[
X_1(t_1) = \{ q=1, X_1(t_1) \} \cup \{ q=2, X_2(t_1) \}
\]

mode = 2

\[
X_2(t_1) = \{ q=2, X_2(t_1) \}
\]

\[
X_2(t_3) = \{ q=2, X_2(t_3) \}
\]

Reconstructed Hybrid Solution
State Trajectory:

Filter out mode q=1 at time t_3
Reachable set

\[ \mathbb{R}([t_0, t]; X_0) = \left\{ x(\tau), \; t_0 \leq \tau \leq t \mid \dot{x}(\tau) = f(x, p, \tau) \wedge x(t_0) \in X_0 \wedge p \in P \right\} \]

- **Set integration**
  - Interval Taylor methods
  - Bracketing enclosures
Reachable set

$$\mathbb{R}([t_0, t]; X_0) = \left\{ x(\tau), \ t_0 \leq \tau \leq t \mid \dot{x}(\tau) = f(x, p, \tau) \land x(t_0) \in X_0 \land p \in P \right\}$$

- Set integration
  - Interval Taylor methods
  - Bracketing enclosures
Continuous Reachability
Guaranteed set integration

- … with interval Taylor methods.
  - (Moore, 66) (Lohner, 88) (Rihm, 94) (Berz, 98) (Nedialkov, 99)

- … with interval Taylor models.
  - (Chen, 2012)

- also via interval Runge Kutta.
  - (Alexandre dit Sandretto & Chapoutot, 2015)
Guaranteed set integration

- ... with interval Taylor methods.
  - (Moore, 66) (Lohner, 88) (Rihm, 94) (Berz, 98) (Nedialkov, 99)
- ... with interval Taylor models.
  - (Chen, 2012)
- also via interval Runge Kutta.
  - (Alexandre dit Sandretto & Chapoutot, 2015)

Comparison theorems for differential inequalities

- Monotone systems
  - (Ramdani et al., 2010)
- Muller’s theorem
Guaranteed event detection & localization

- An interval constraint propagation approach
  
  (Ramdani & Nedialkov, Nonlinear Analysis Hybrid Systems 2011)
Hybrid Reachability Computation

- Guaranteed event detection & localization
  - An interval constraint propagation approach
    - (Ramdani & Nedialkov, Nonlinear Analysis Hybrid Systems 2011)

\[
\text{Time grid } \rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N
\]

Compute \([t^*, \tilde{t}^*] \times [X_j^*]\)
Guaranteed event detection & localization

- An interval constraint propagation approach

\( \dot{x}(t) = f(x, p, t), \quad t_0 \leq t \leq t_N, \quad x(t_0) \in [x_0], \quad p \in [p] \)

**Analytical solution** for \([x](t), \quad t \in [t_j, t_{j+1}]\)

\[
[x](t) = [x_j] + \sum_{i=1}^{k-1} (t - t_j)^i f[i]([x_j], [p]) + (t - t_j)^k f[k]([\dot{x}_j], [p])
\]
Guaranteed event detection & localization

- An interval constraint propagation approach

-(Ramdani & Nedialkov, Nonlinear Analysis Hybrid Systems 2011)

Time grid $\rightarrow$ $t_0 < t_1 < t_2 < \cdots < t_N$

Compute $[t^*, \bar{t}^*] \times [X_j^*]$
Guaranteed event detection & localization

- An interval constraint propagation approach
  -(Ramdani & Nedialkov, Nonlinear Analysis Hybrid Systems 2011)

\[
\text{Time grid } \rightarrow \quad t_0 < t_1 < t_2 < \cdots < t_N
\]

- \([x](t) = \text{Interval Taylor Series (ITS)}(t, [x_j], [\dot{x}_j])\)
- \(\gamma([x](t)) = 0\)

\[\Rightarrow \gamma \circ \text{ITS}(t, x_j, [\dot{x}_j]) \rightarrow \psi(t, x_j)\]

Solve CSP \(([t_j, t_{j+1}] \times [x_j], \psi(\ldots) \ni 0)\)
Detecting and localizing events

- Improved and enhanced version. A faster version.
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)
Detecting and localizing events

- Improved and enhanced version. A faster version.
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)
Detecting and localizing events

- Improved and enhanced version
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)
Detecting and localizing events

- Improved and enhanced version
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)
Detecting and localizing events

- Improved and enhanced version
  - (Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

Bouncing ball in 2D.
Detecting and localizing events

Improved and enhanced version

(Maïga, Ramdani, et al., IEEE CDC 2013, ECC 2014)

Bouncing ball in 2D.
Detecting and localizing events

- Improved and enhanced version
  - Impact of uncertainty on **sliding mode control**
    (Maïga, Ramdani, Travé-Massuyès, Combastel, IEEE TAC 2016)
Detecting and localizing events

- Improved and enhanced version
  - Impact of uncertainty on **sliding mode control**
    (Maïga, Ramdani, Travé-Massuyès, Combastel, IEEE TAC 2016)
Solution trajectory tube

\[
[x](t) = [x_j] + \sum_{i=1}^{k-1} (t - t_j)^i f^{[i]}([x_j], [p]) + (t - t_j)^k f^{[k]}([\bar{x}_j], [p])
\]

Mean value form + Lohner’s QR transformation method

\[
[x](t) = A(t) [r](t) \oplus [v](t) \rightarrow \text{MSBP}
\]

\[
[x](t) = c(t) \oplus R(t) B^{2n}
\]

is a particular zonotope

\[
c(t) = A(t) \text{mid}([r](t)) + \text{mid}([v](t)),
\]

\[
R(t) = (A(t) \text{diagrad}([r](t)) | \text{diagrad}([v](t))).
\]
Zonotope of minimum size enclosing the intersection of a zonotope and a strip

\[ Z(c, R) = c \oplus RB^p = \{ c + Rx : x \in B^p \} \]
and
\[ S = \{ x \in \mathbb{R}^n \mid \eta^\top x - d \leq \sigma \} \]

**Proposition 1** *(Vicino & Zappa, IEEE TAC 1996)*

*Given zonotope \( Z(c, R) \) and strip \( S \), the zonotope support strip is defined by*

\[ S_Z = \{ x \in \mathbb{R}^n \mid \rho_d \leq \eta^\top x \leq \rho_u \}, \]

\[ \rho_u = \eta^\top c + \| R^\top \eta \|_1 \]

\[ \rho_d = \eta^\top c - \| R^\top \eta \|_1 \]

**Theorem 1** *(Vicino & Zappa, IEEE TAC 1996)*

\[ Z = (c, R) \cap S = \emptyset \iff (\rho_d > d + \sigma) \lor (\rho_u < d - \sigma) \]

**Proposition 2** *(Alamo et al., Automatica 2005)*

*Given zonotope \( Z(c, R) \), strip \( S \), and \( \lambda \in \mathbb{R}^n \), define*

\[ \hat{c}(\lambda) = c + \lambda(d - \eta^\top c), \]

\[ \hat{R}(\lambda) = [(I - \lambda \eta^\top) R \mid \sigma \lambda], \]

\[ \Rightarrow Z \cap S = \hat{c}(\lambda) \oplus \hat{R}(\lambda) B^{(p+1)}. \]

\[ \lambda^* = \frac{RR^\top \eta \eta^\top R \eta + \sigma^2}{\eta^\top RR^\top \eta} \] minimizes the Frobenius norm of matrix \( \hat{R}(\lambda) \).
Hybrid Reachability based Predictor Corrector approach
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- Example
- Research directions
Hybrid Mass-Spring

* Velocity-dependent damping. Mode switching driven by velocity.

Parameter identification
Hybrid Mass-Spring

- Unknown initial mode.
- **CPU time approx. 1m20s**

```
start → q₁, c₁ = 0
x₂ < −v
x₂ < −v
q₁, c₁ ≠ 0
x₂ > −v
x₂ > v
q₃, c₁ = 0
```

State space

![Mode vs Time](image1)

![Velocity vs Time](image2)

![Position](image3)
Hybrid Mass-Spring

- Unknown initial mode.
- **CPU time approx. 1m20s**
Hybrid Mass-Spring

- Unknown initial mode.
- CPU time approx. 1m20s
Hybrid Mass-Spring

- Unknown initial mode.
- *CPU time approx. 1m20s*
Outline

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Research directions
Contractors for hybrid dynamical systems
- To build upon a hybrid reachability approach
- Push forward set membership estimation
  - SM hybrid state estimation of nonlinear hybrid systems
Research directions

- **Contractors** for hybrid dynamical systems
  - To build upon a *hybrid reachability* approach
- Push forward set membership *estimation*
  - SM *hybrid* state estimation of nonlinear hybrid systems
- Address SM estimation with *controlled* sampling
  - Event- & *Self-triggered* SM *hybrid* state estimation of nonlinear hybrid systems
Research directions

- **Contractors** for hybrid dynamical systems
  - To build upon a *hybrid reachability* approach

- Push forward set membership *estimation*
  - SM *hybrid* state estimation of nonlinear hybrid systems

- Address SM estimation with **controlled** sampling
  - Event- & **Self-triggered** SM *hybrid* state estimation of nonlinear hybrid systems

- Combine with decision making
  - Application to actual hybrid systems, in robotics, smart buildings, personalized medicine...

…


