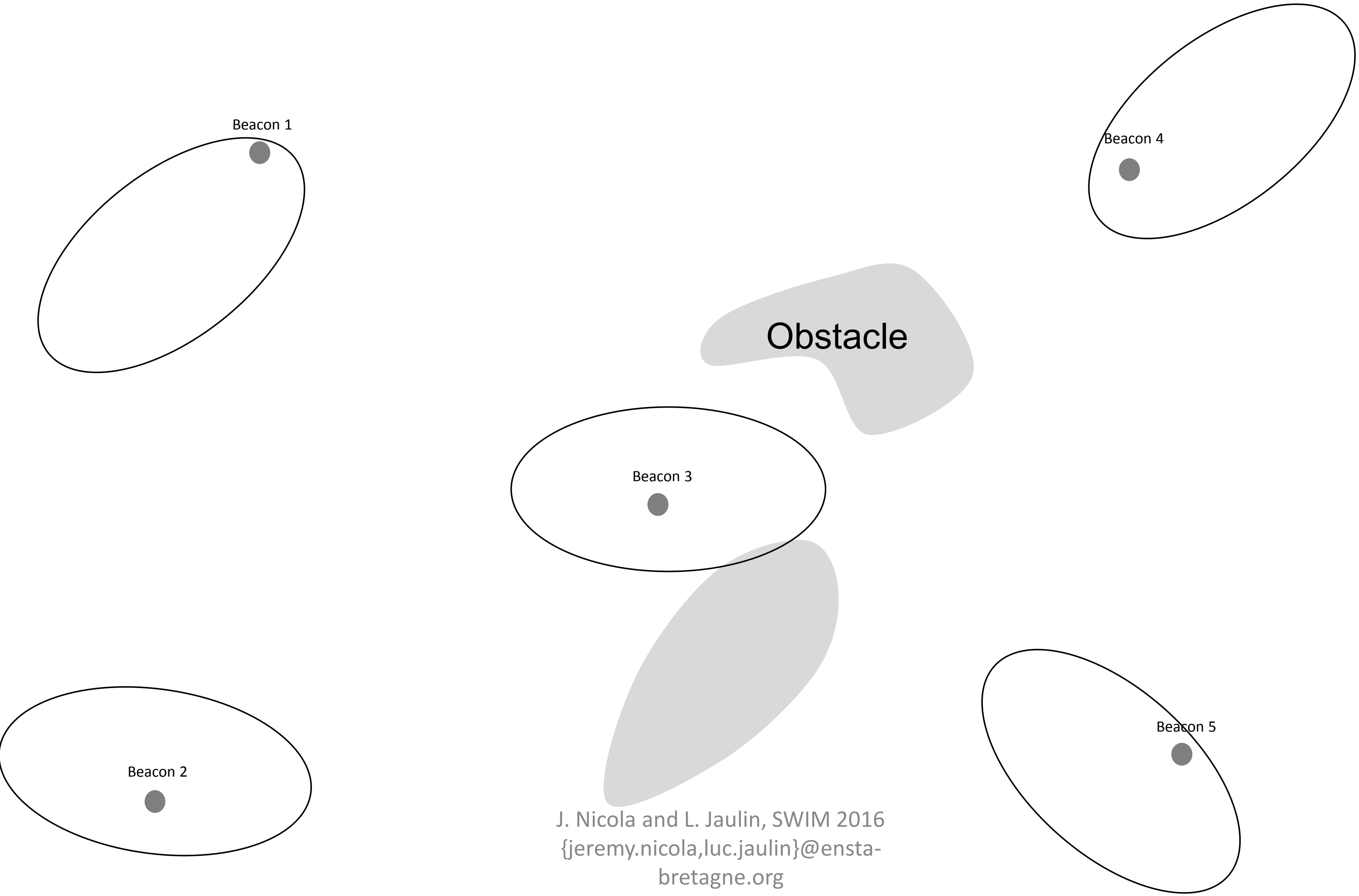
OMNE is a Maximum Likelihood Estimator

With application to robotics mobile mapping

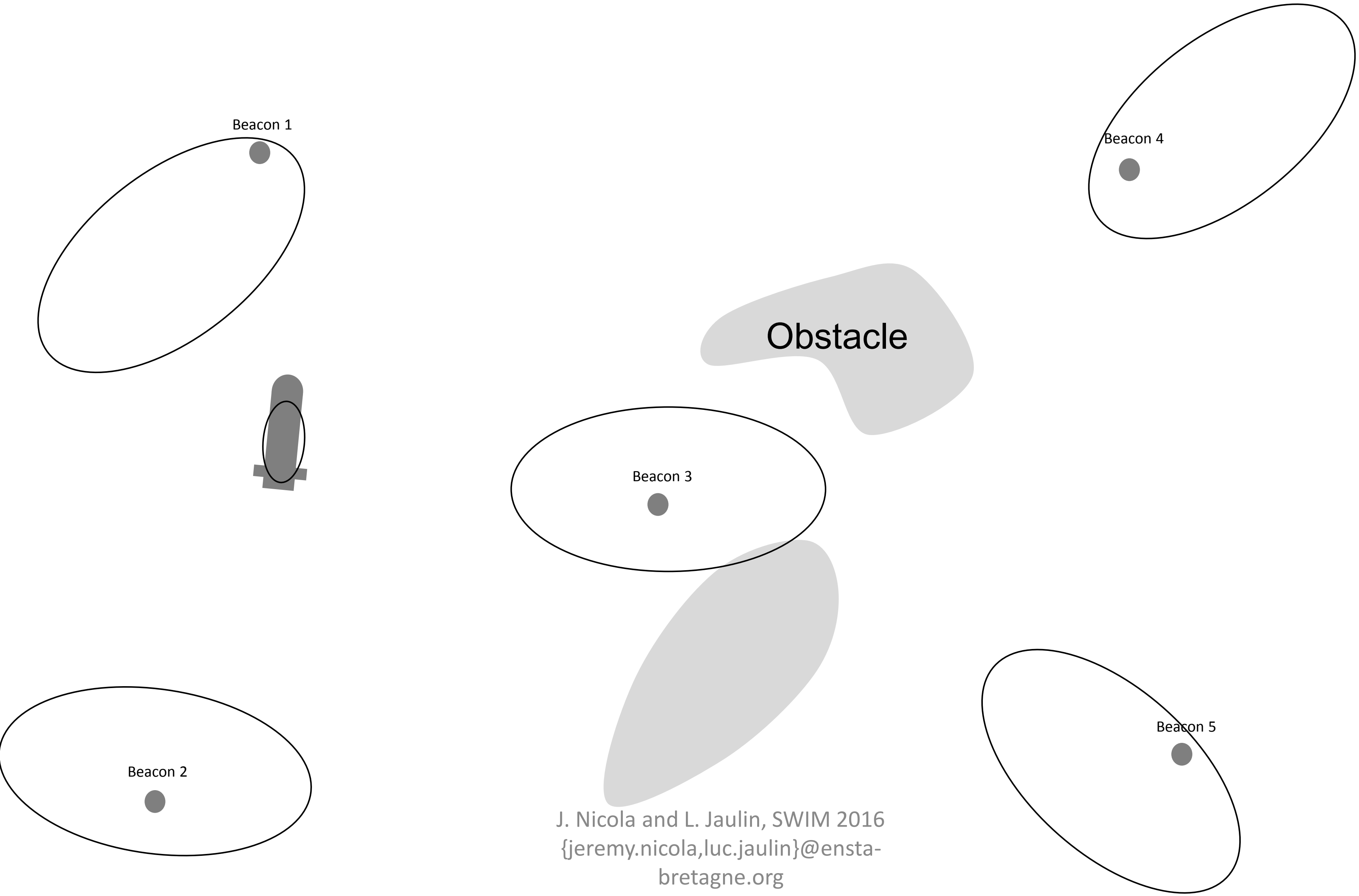
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

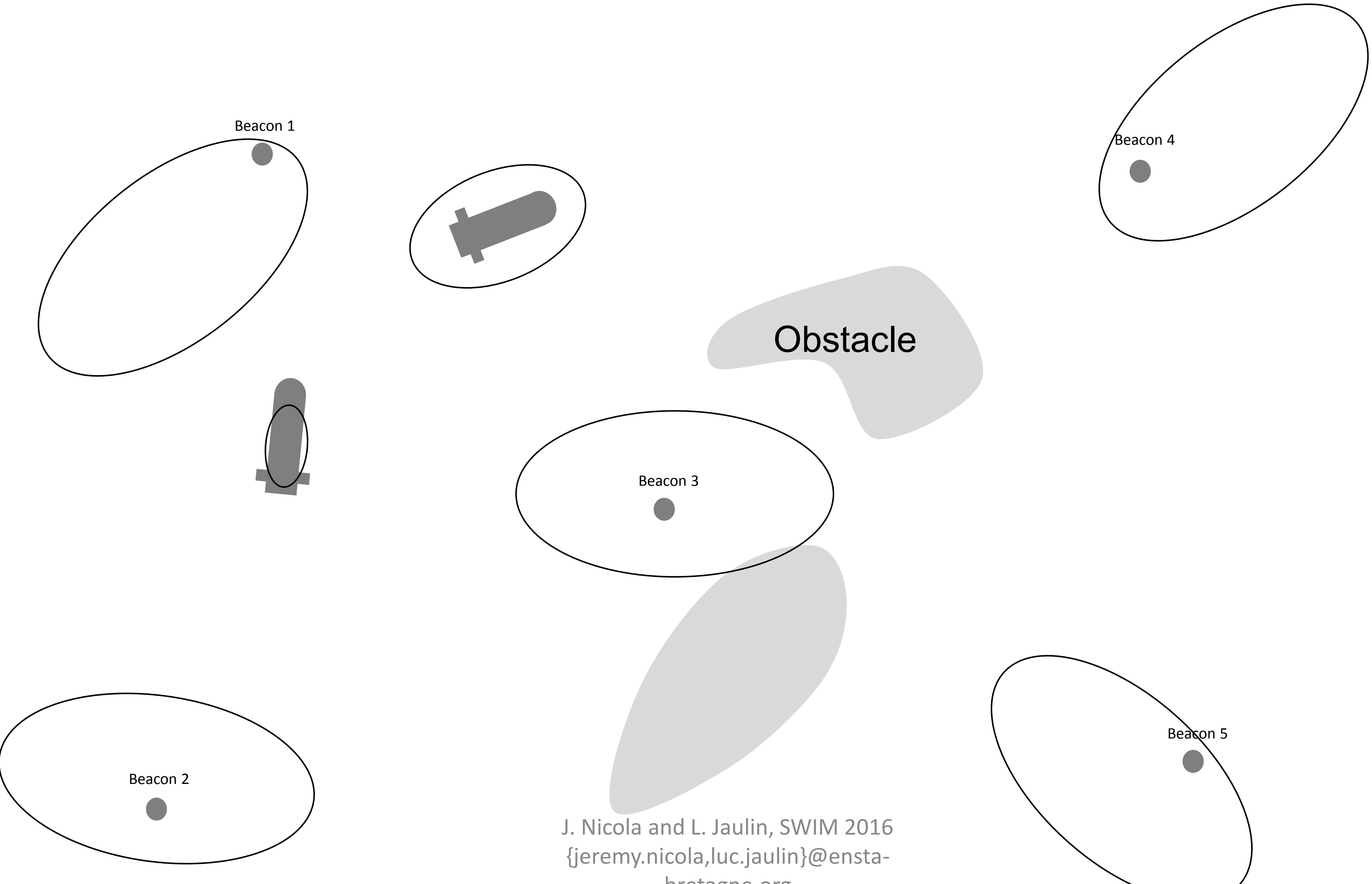
- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

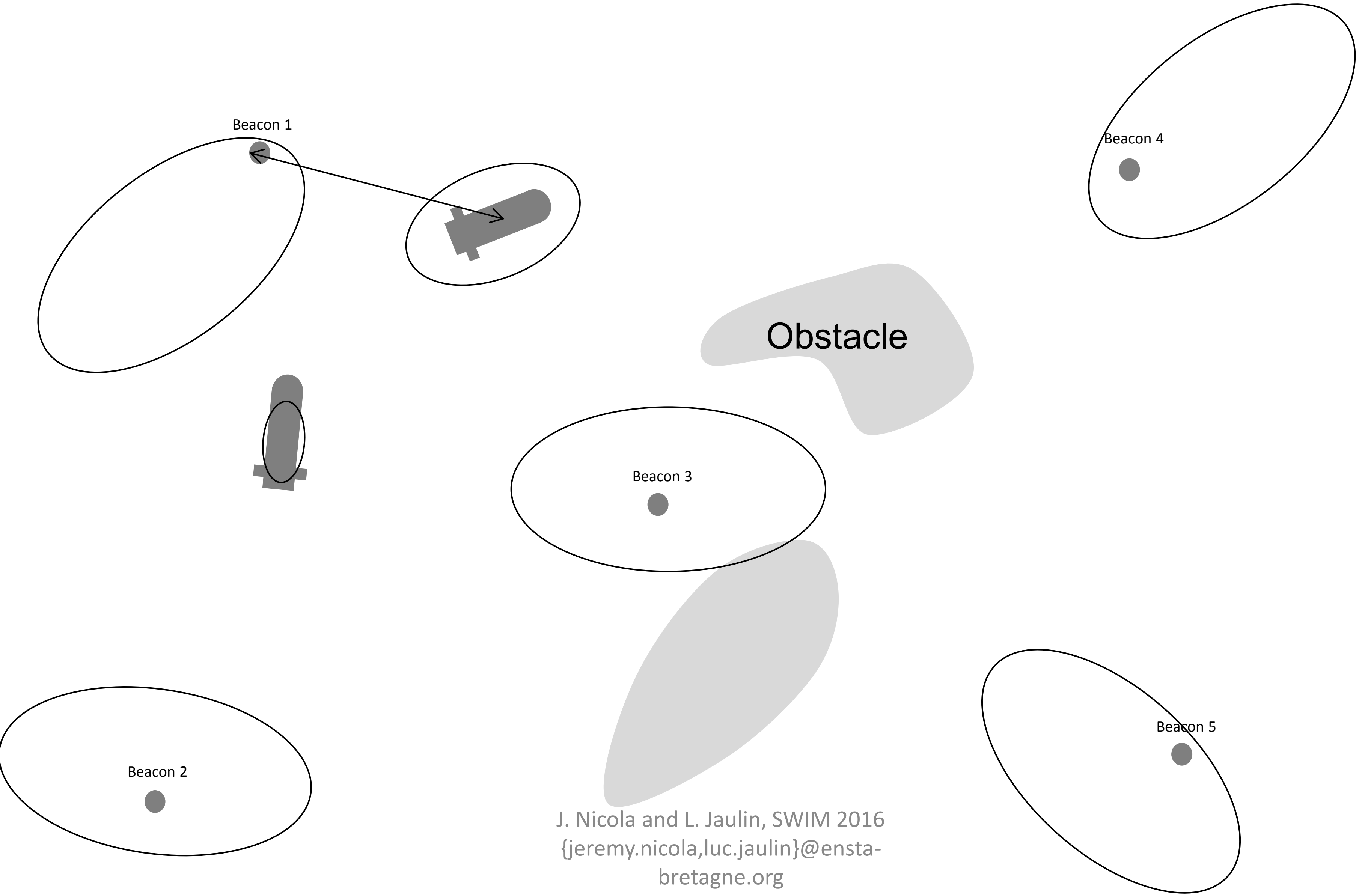


J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

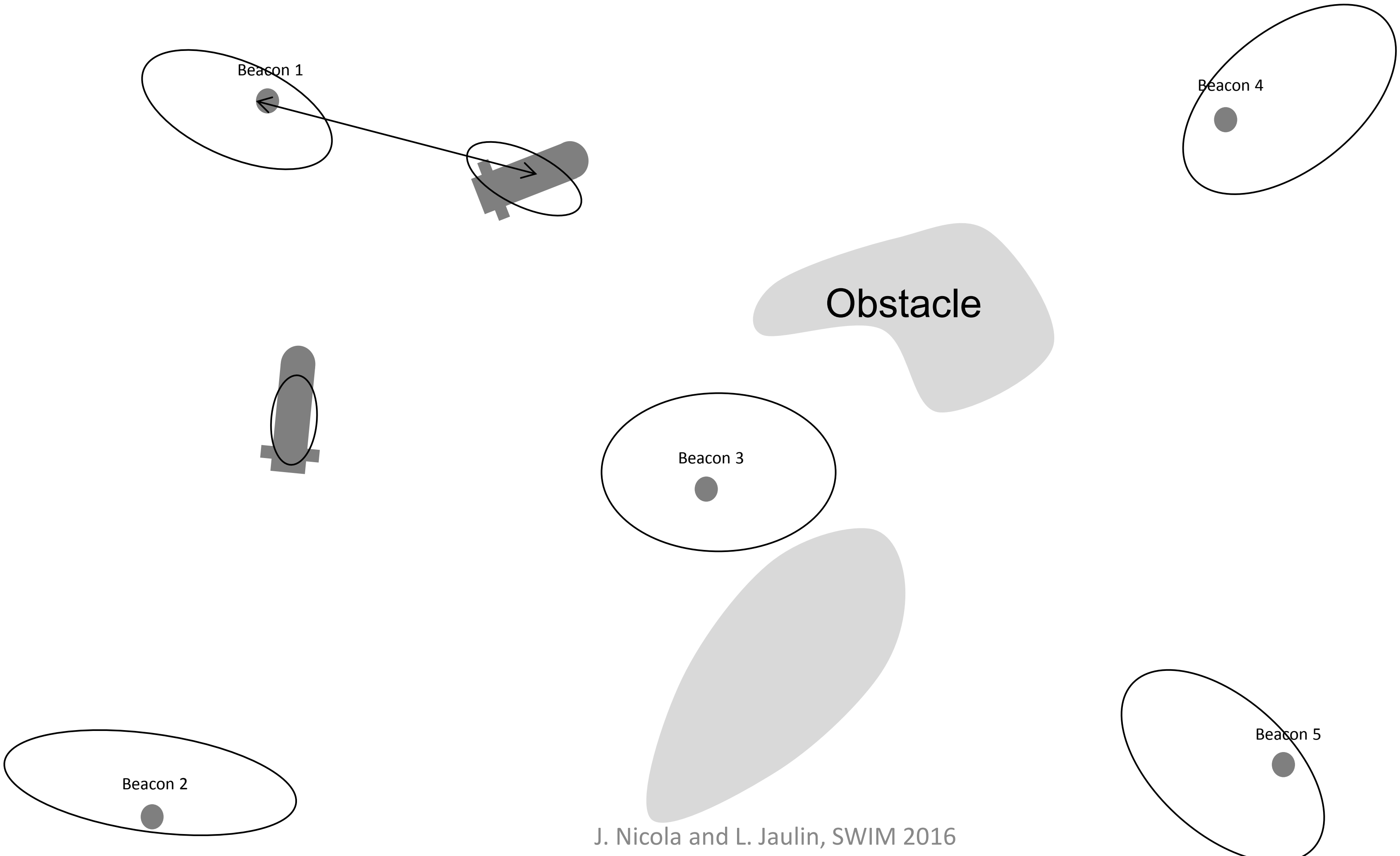


J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

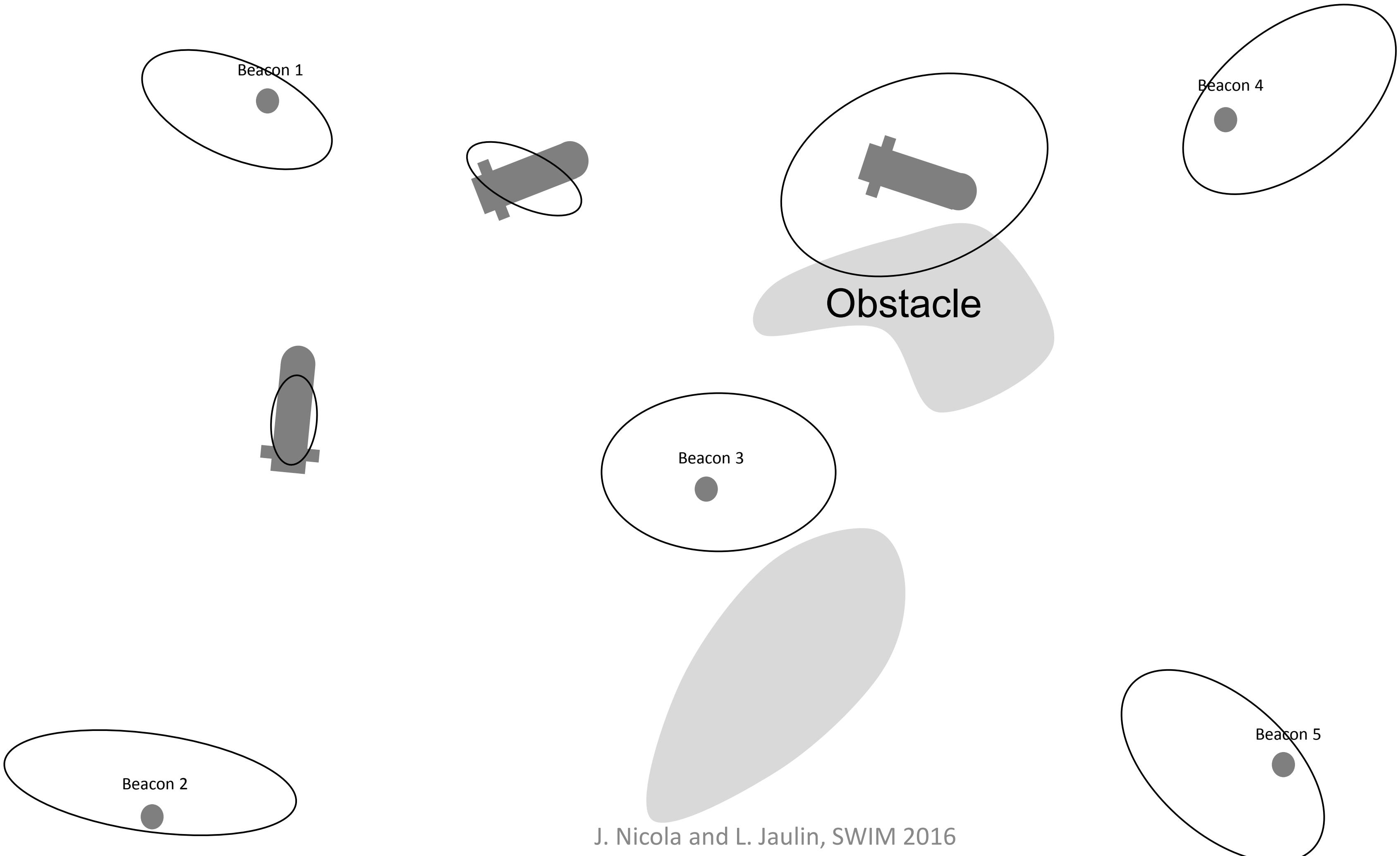

Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 2
Beacon 5



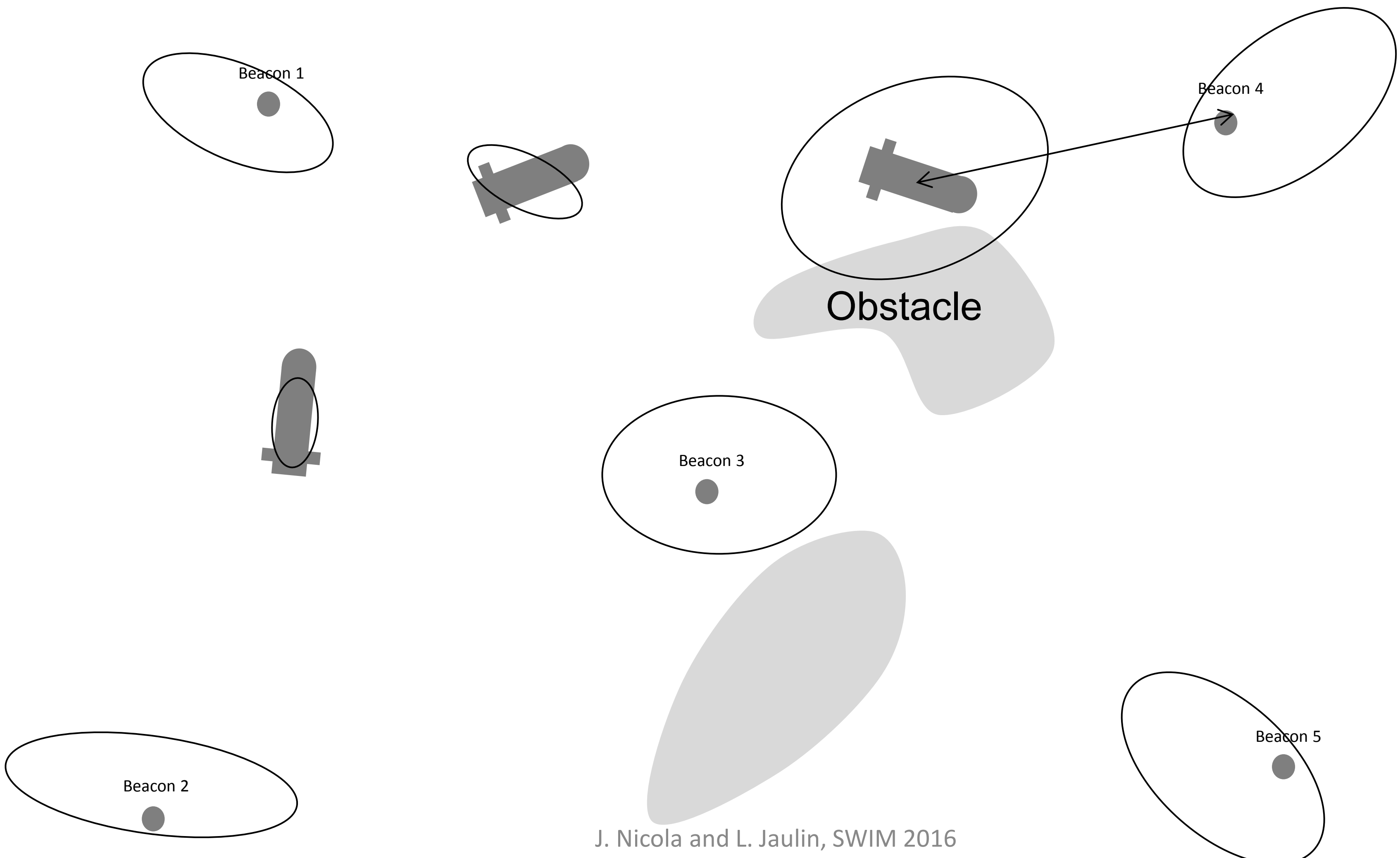
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

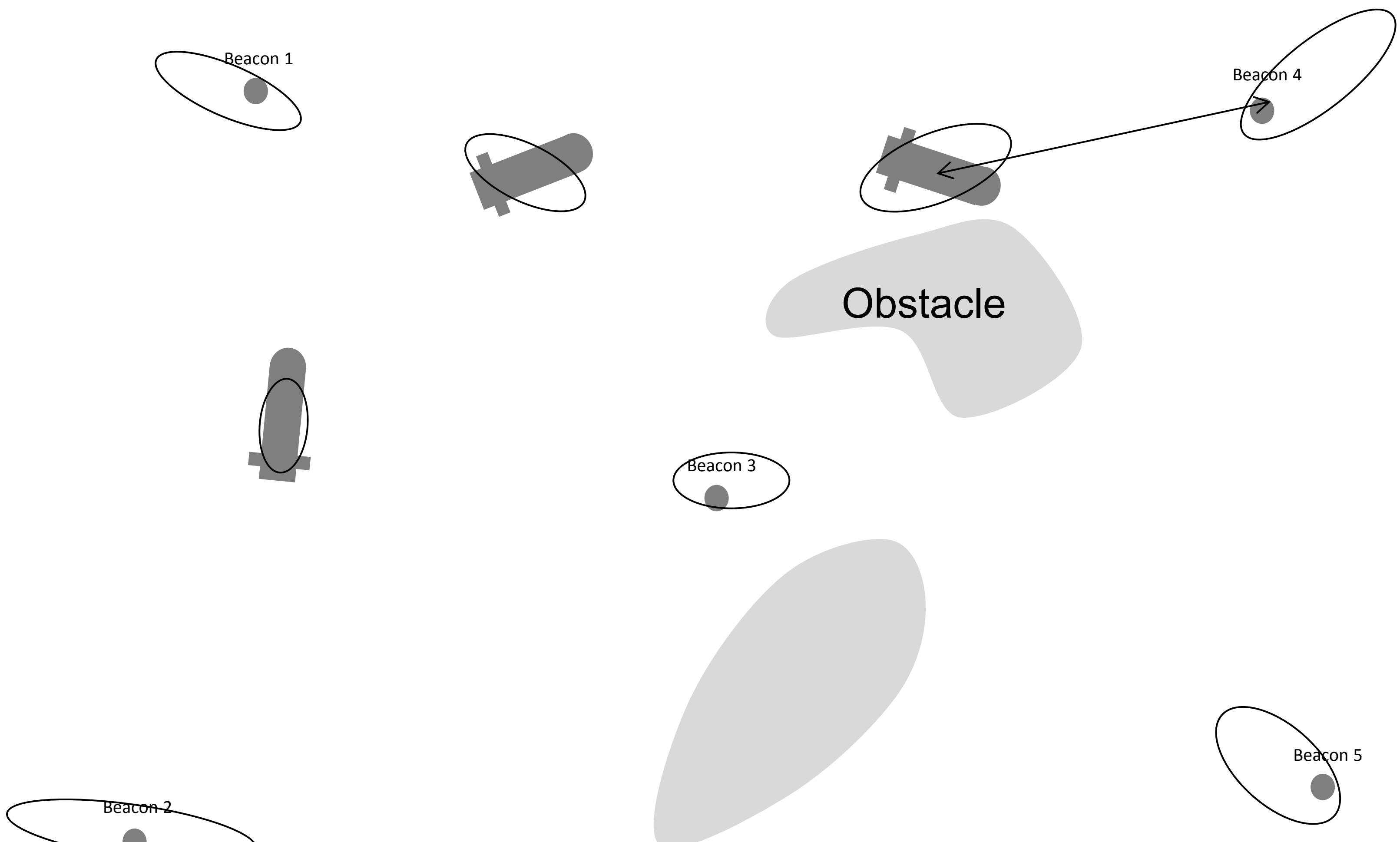


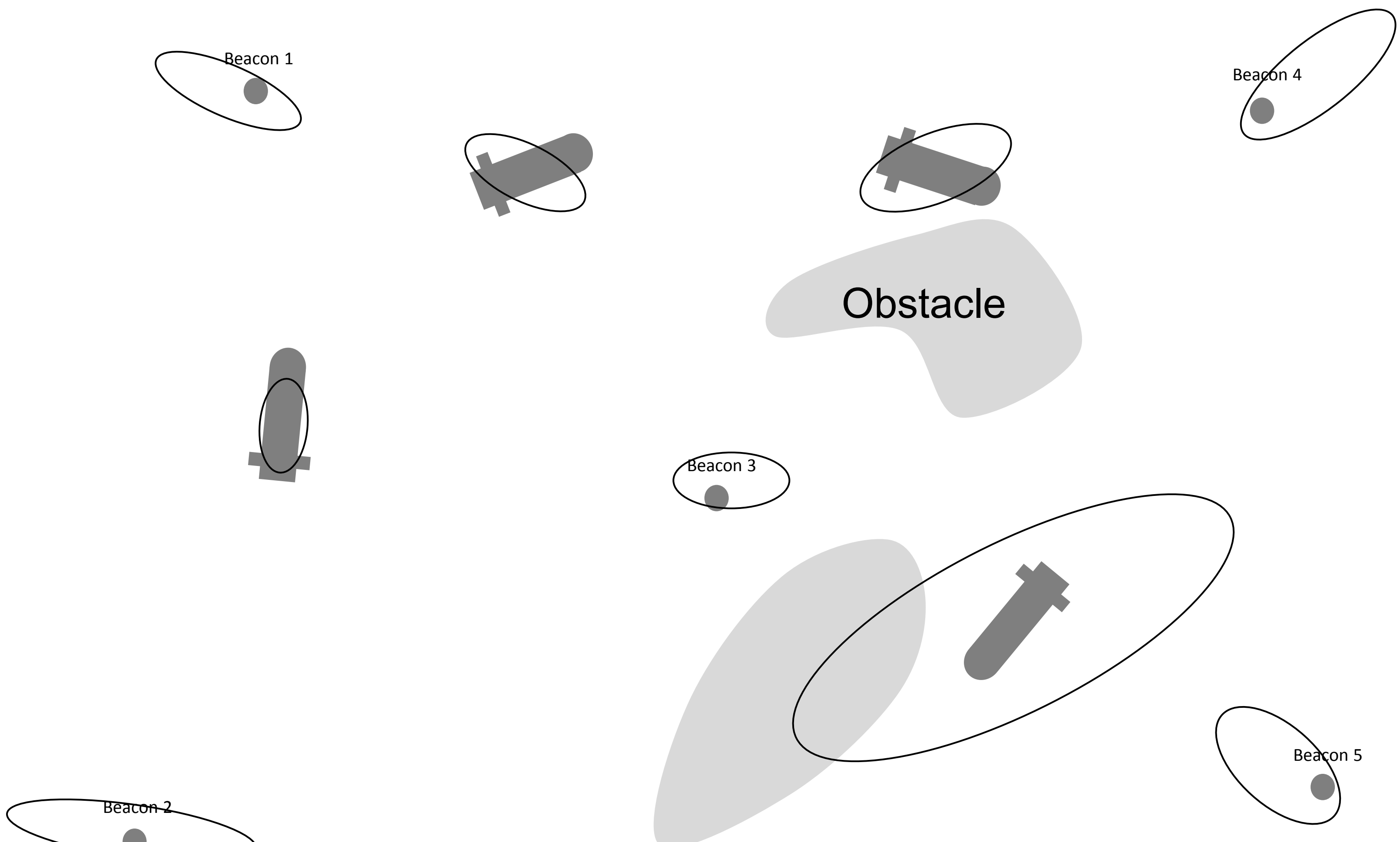
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

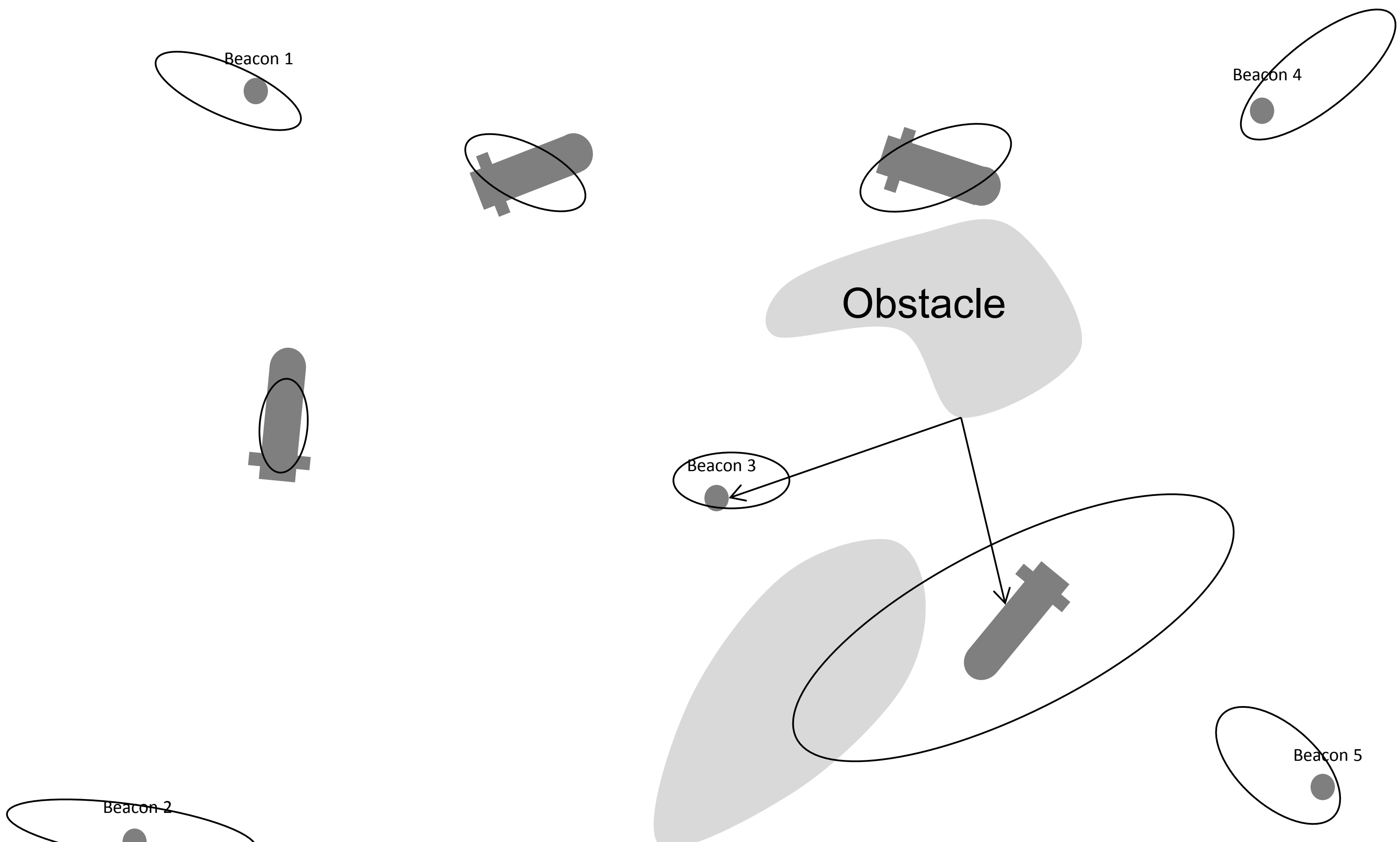


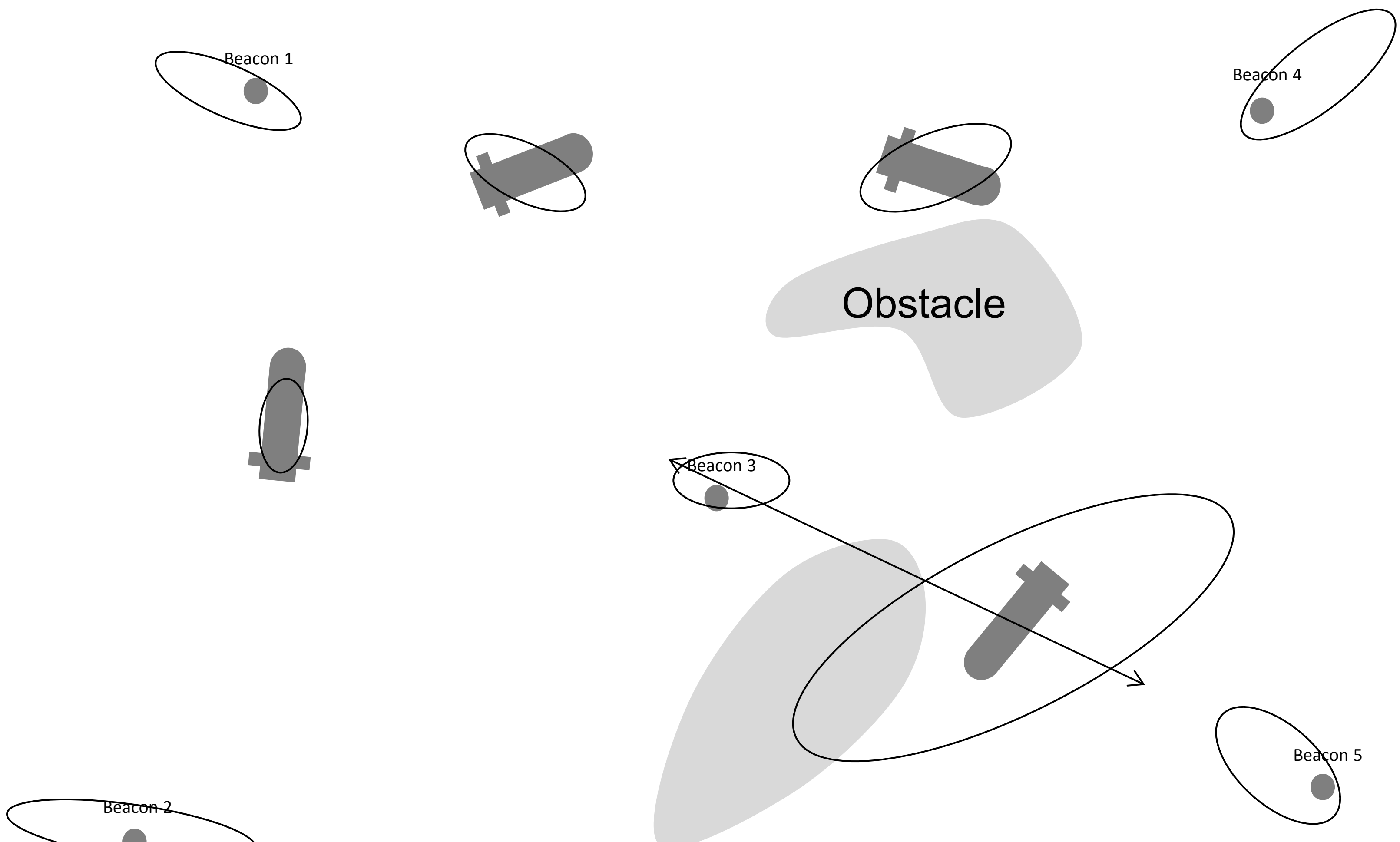
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

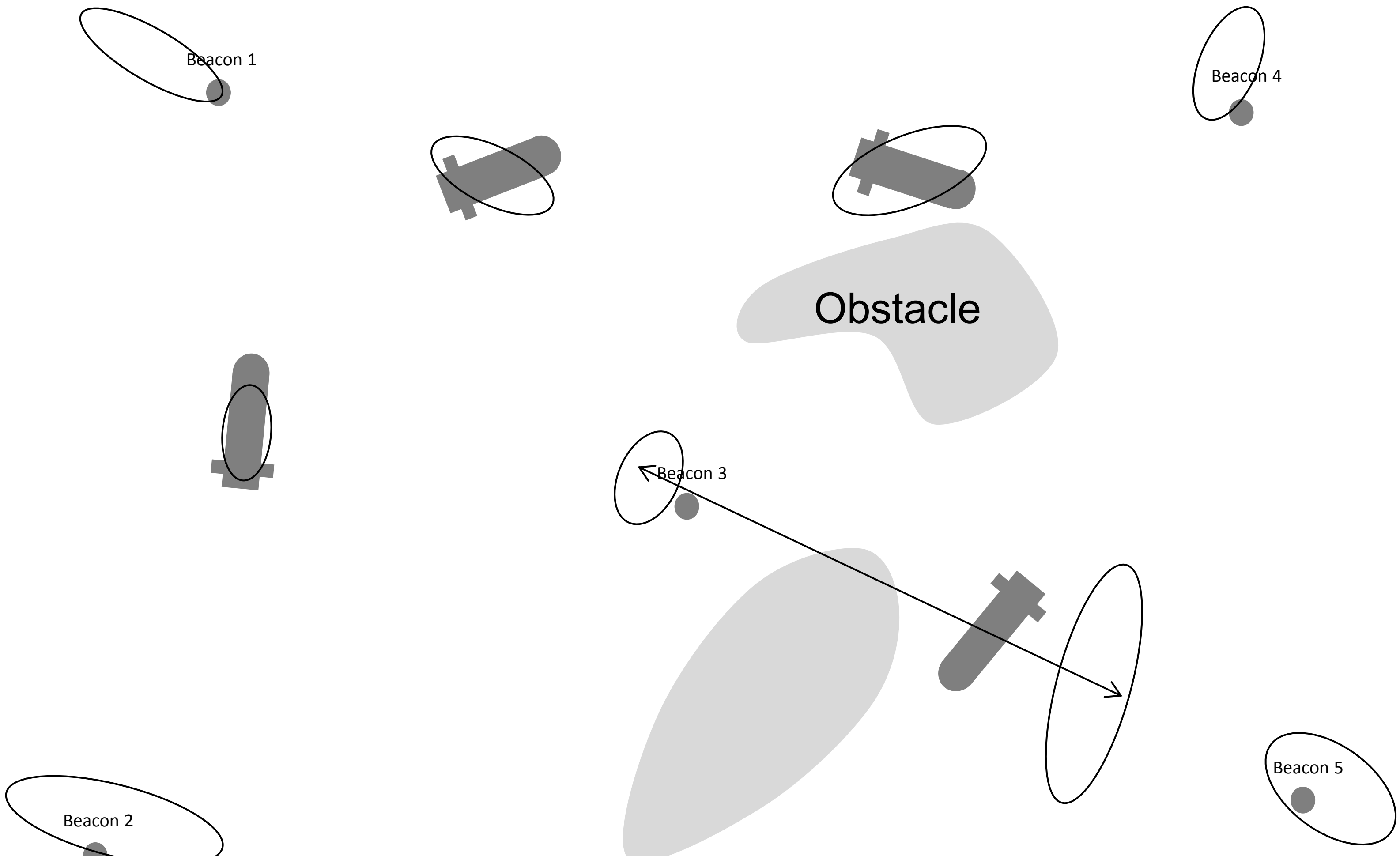

Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 5
Beacon 2


Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 5
Beacon 2


Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 5
Beacon 2


Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 5
Beacon 2


Beacon 1
Beacon 4
Obstacle
Beacon 3
Beacon 5
Beacon 2



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

- Simultaneous Localization and Mapping (SLAM) is a typical state estimation problem

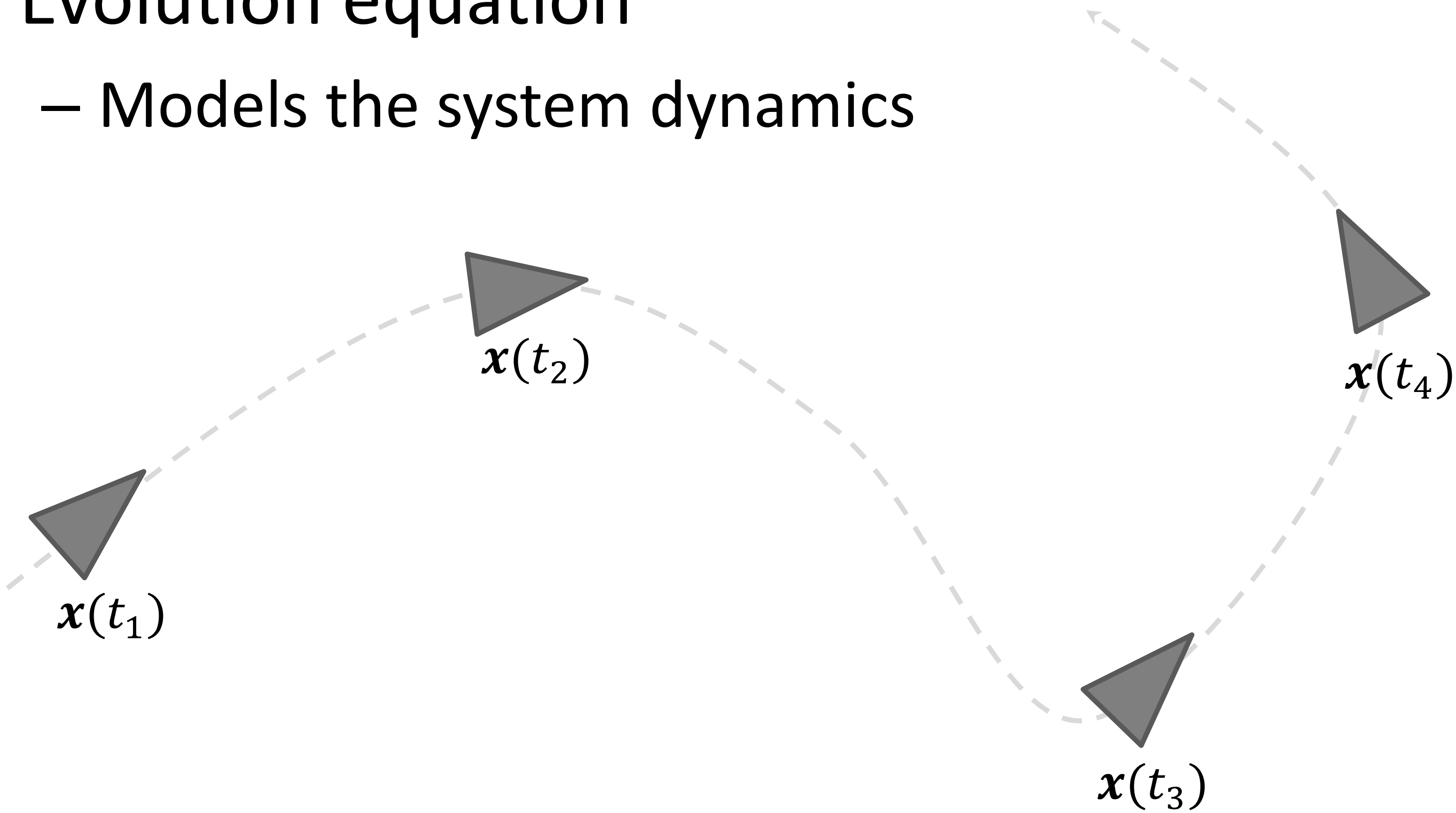
$$\begin{cases} \dot{x} = f(x,u) + \omega_\alpha \\ y = g(x) + \omega_\beta \end{cases}$$

Evolution equation

Observation equation

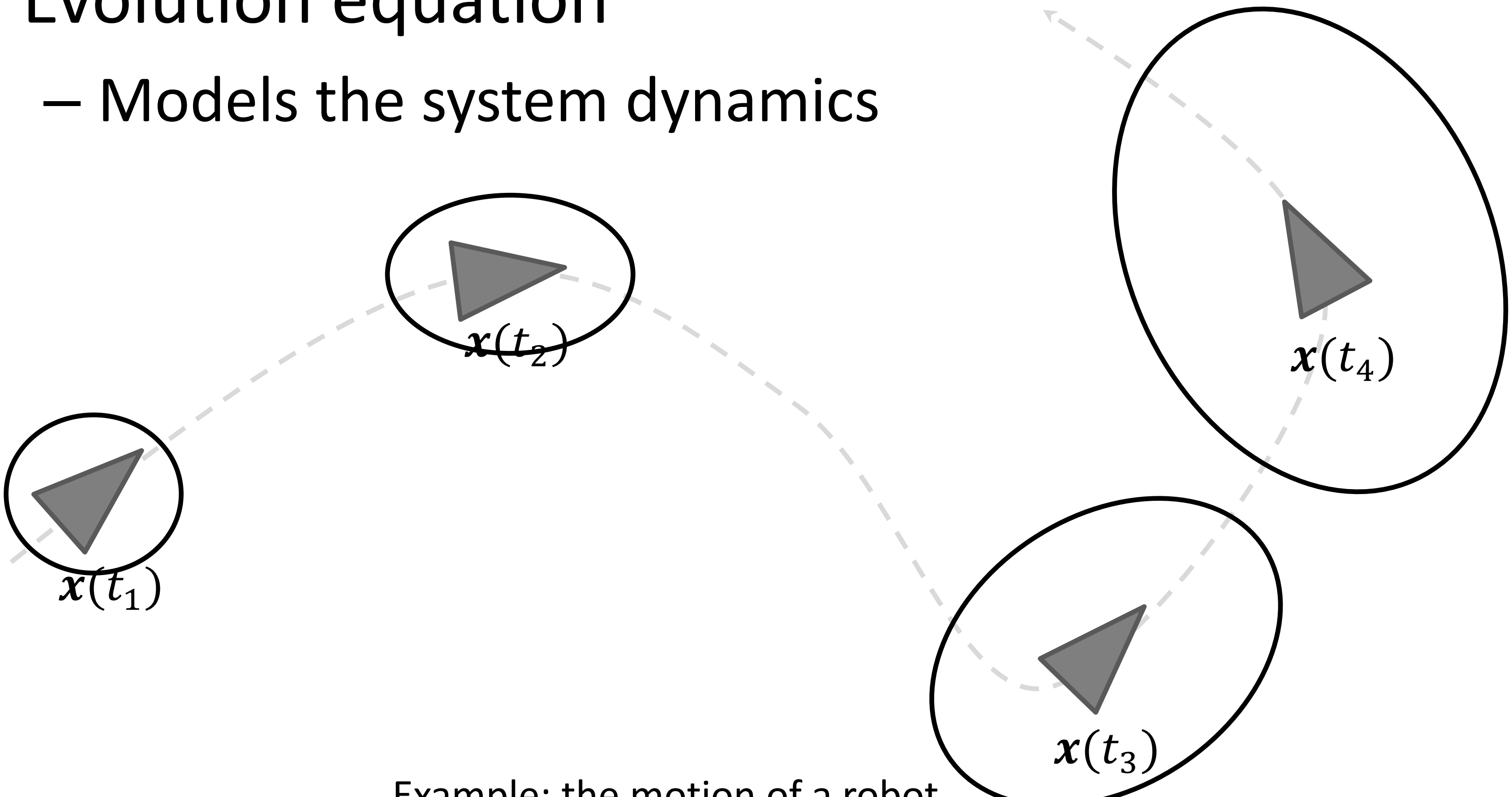
- Evolution equation
  - Models the system dynamics

- Evolution equation
  - Models the system dynamics



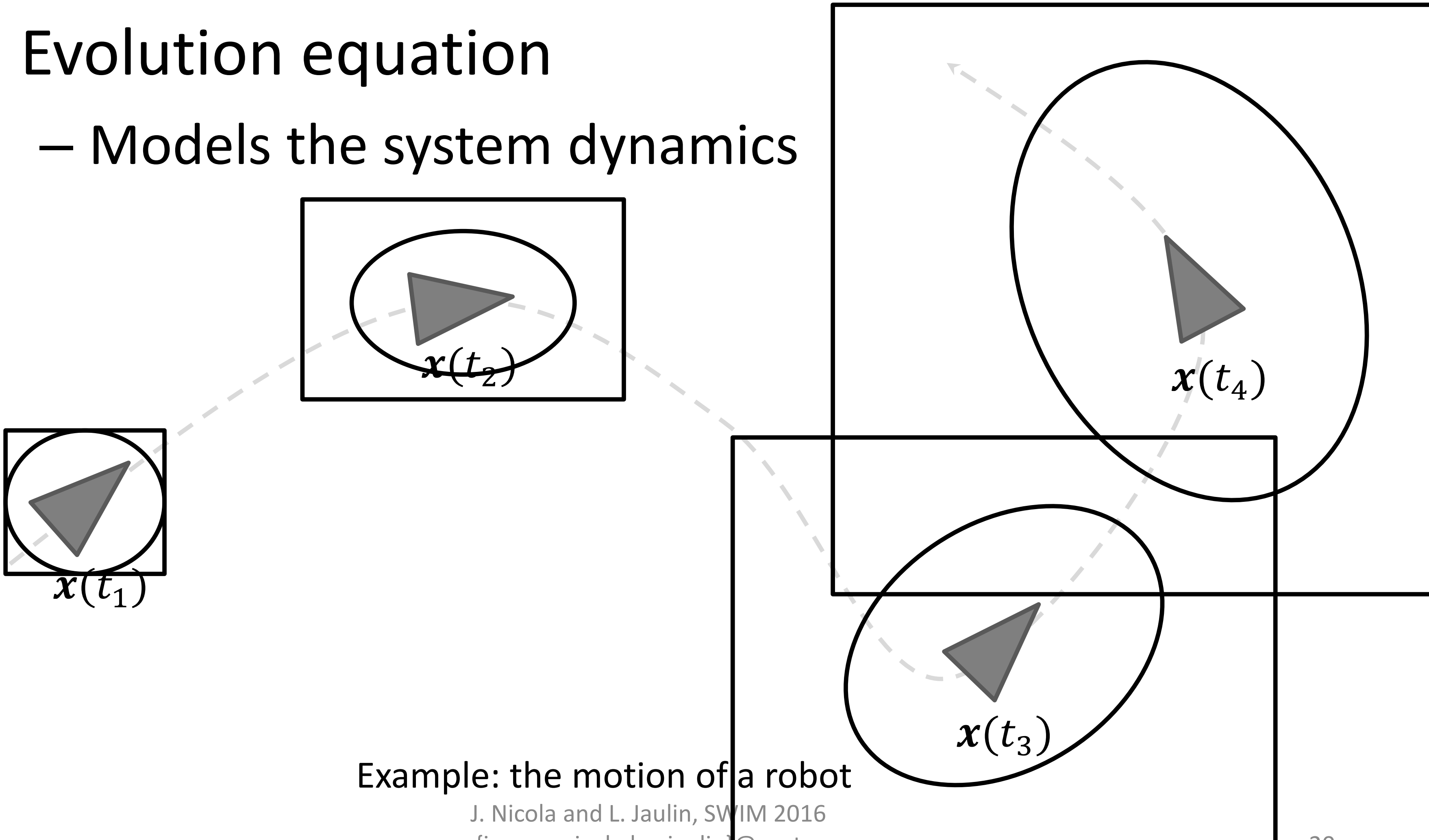
Example: the motion of a robot

- Evolution equation
  - Models the system dynamics



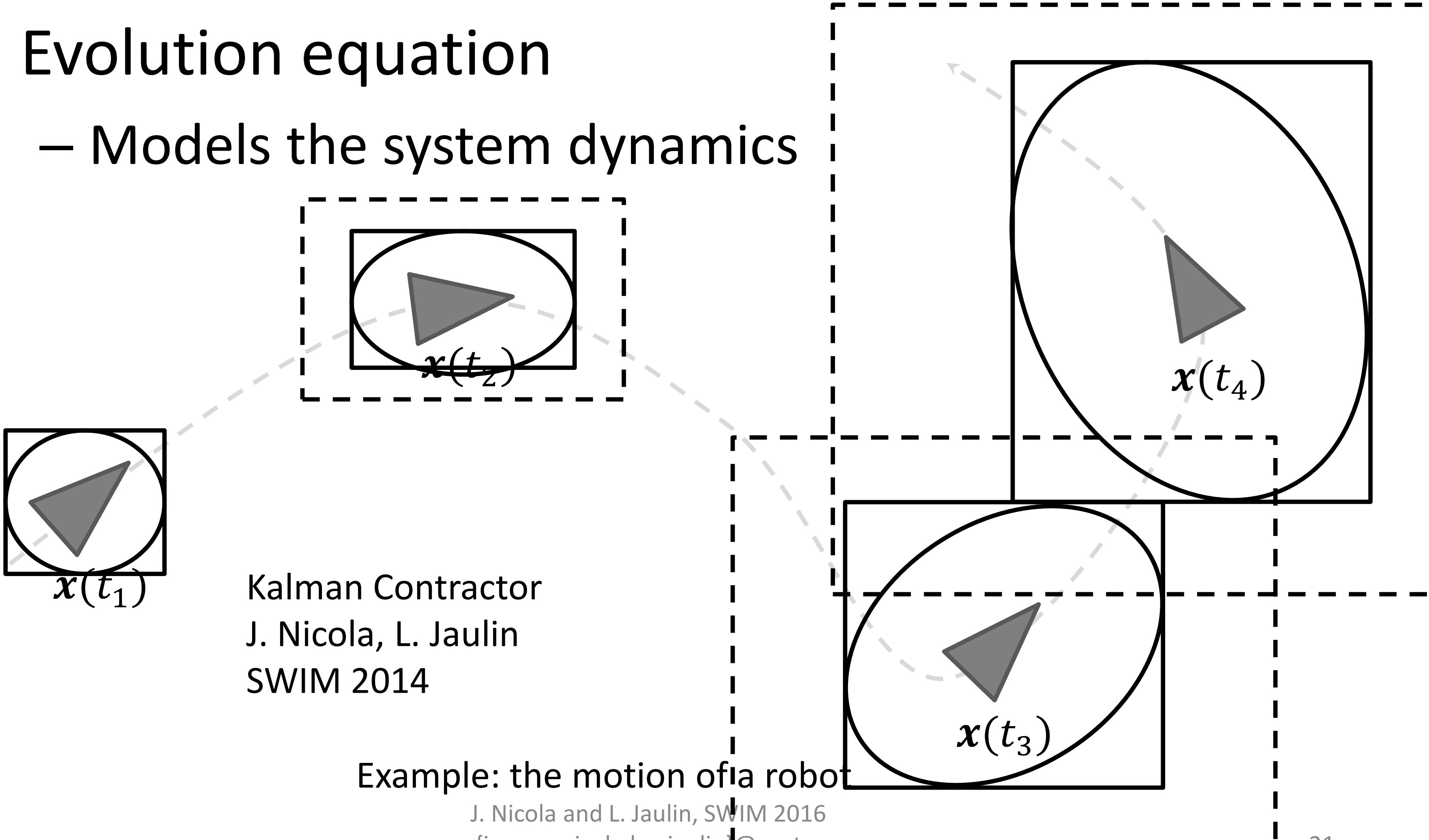
Example: the motion of a robot

- Evolution equation
  - Models the system dynamics



Example: the motion of a robot

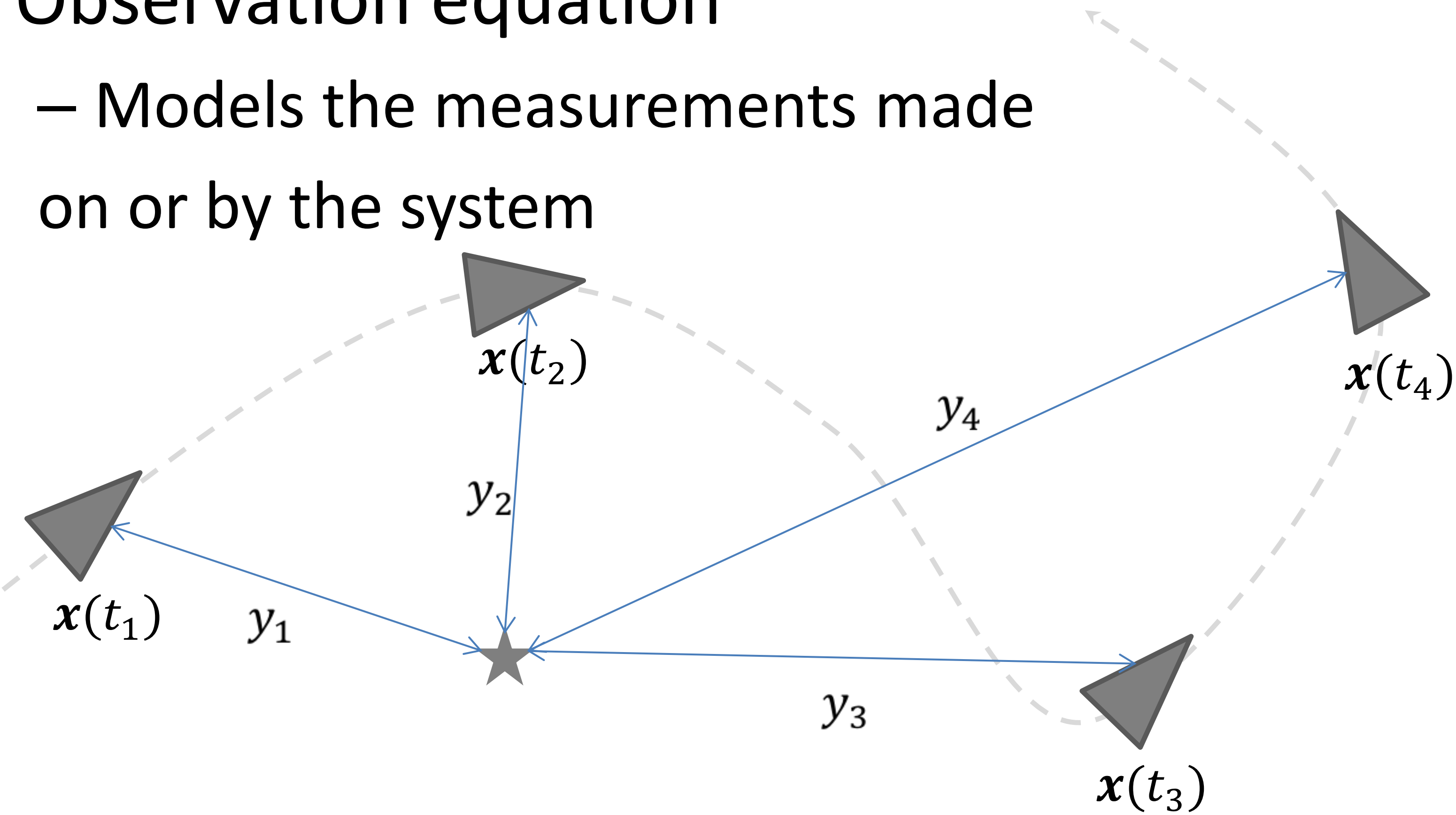
- Evolution equation
  - Models the system dynamics



Kalman Contractor
J. Nicola, L. Jaulin
SWIM 2014

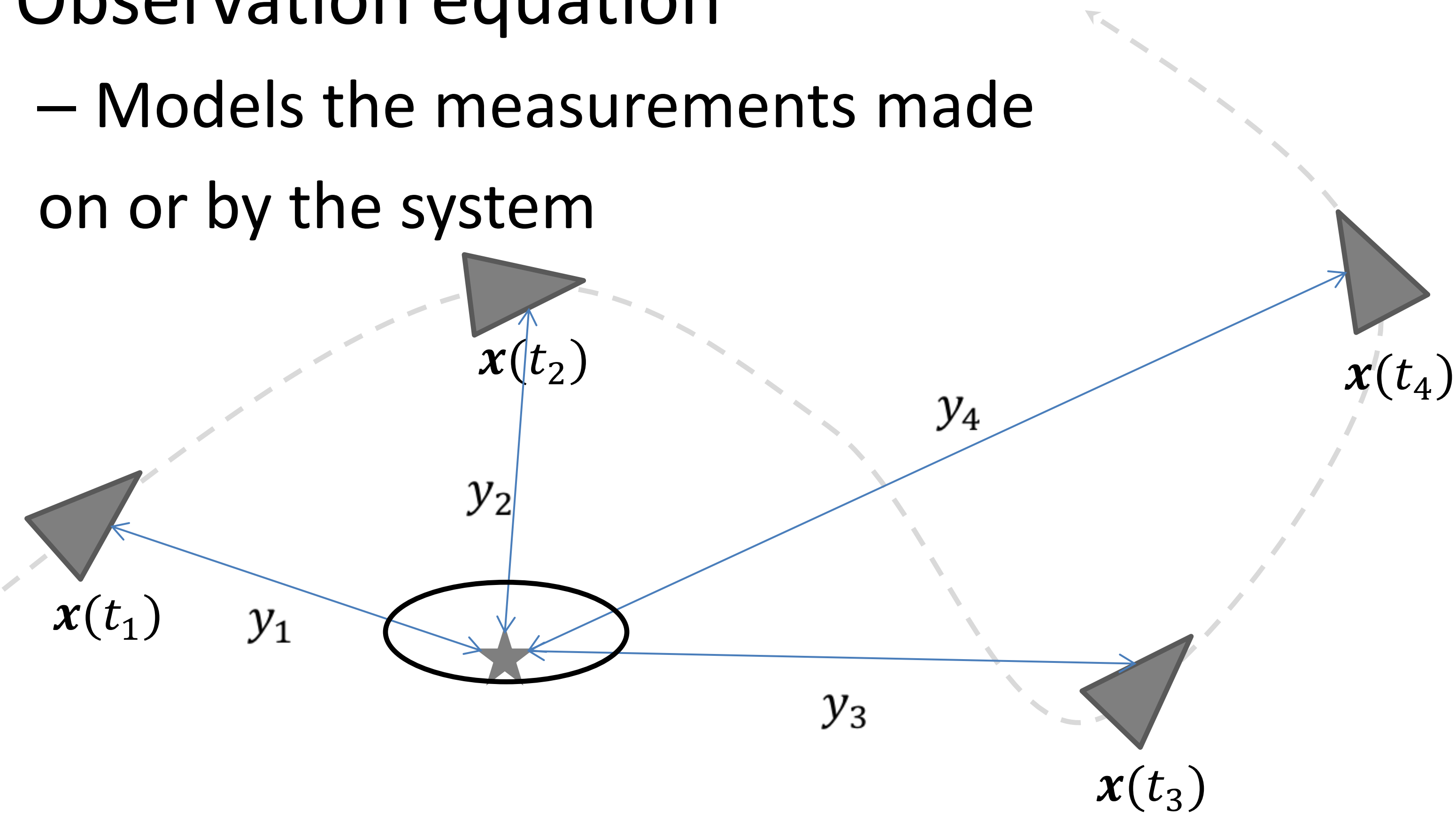
Example: the motion of a robot

- Observation equation
  - Models the measurements made on or by the system



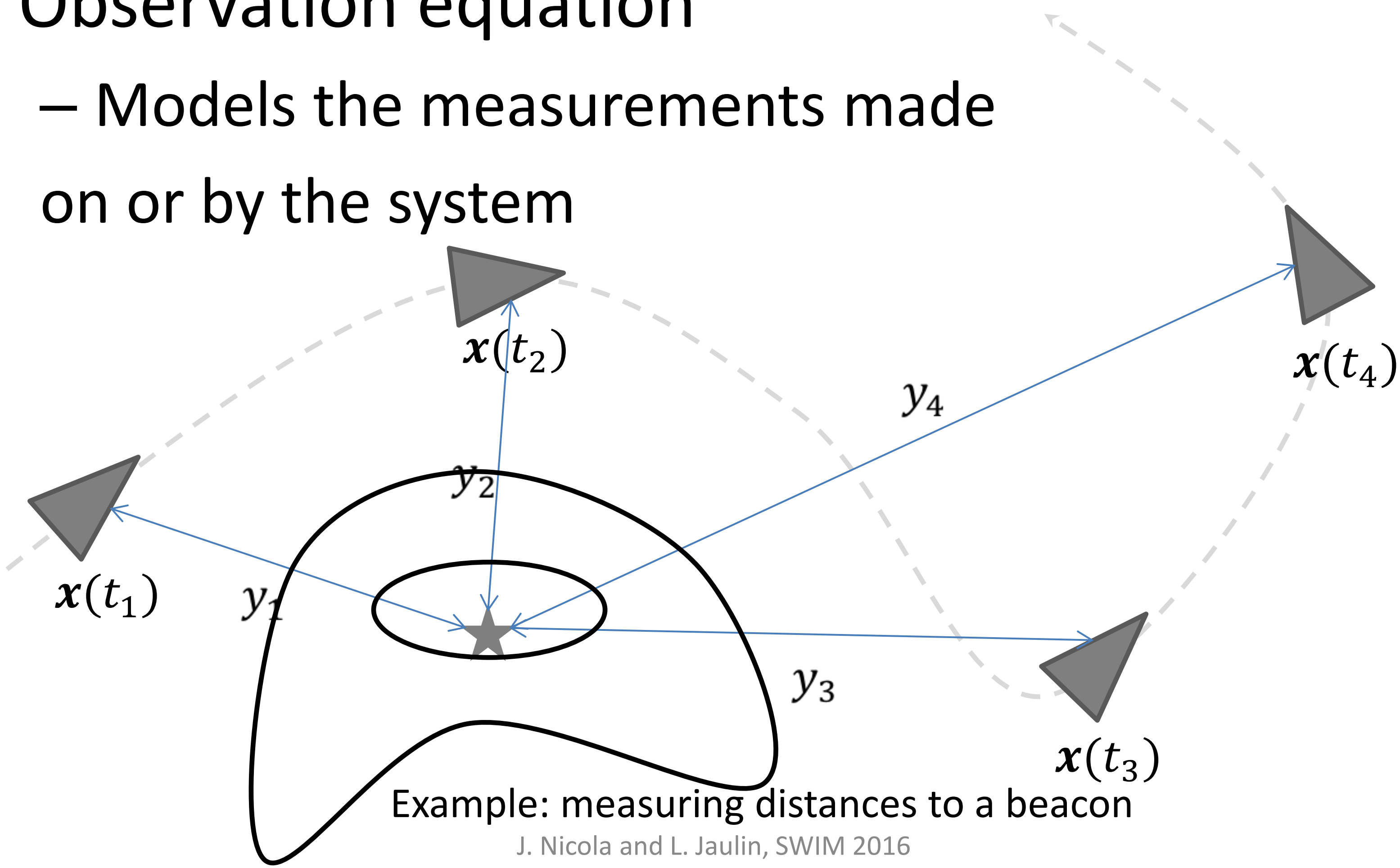
Example: measuring distances to a beacon

- Observation equation
  - Models the measurements made on or by the system



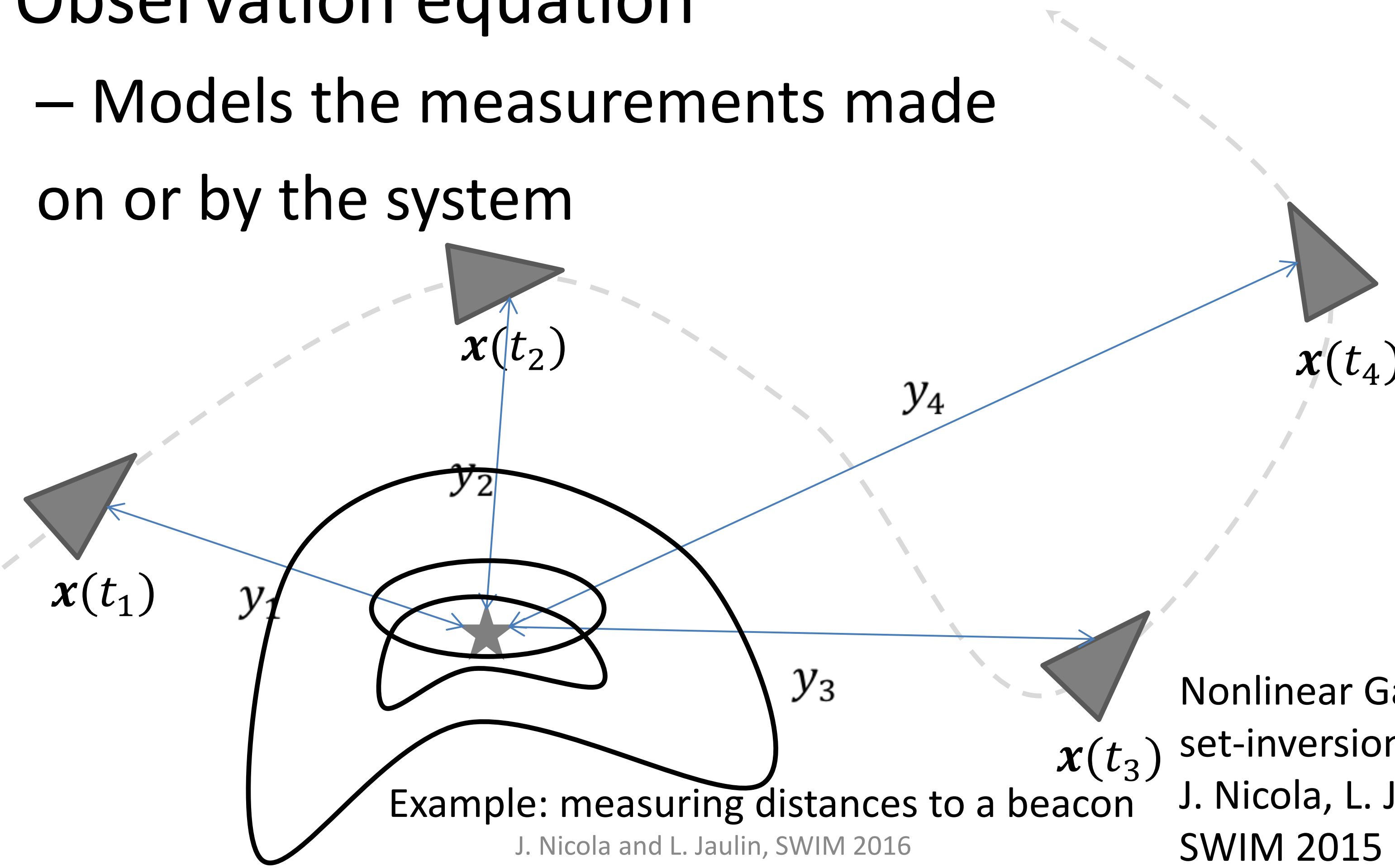
Example: measuring distances to a beacon

- Observation equation
  - – Models the measurement made on or by the system



Example: measuring distances to a beacon

- Observation equation
  – Models the measurements made on or by the system



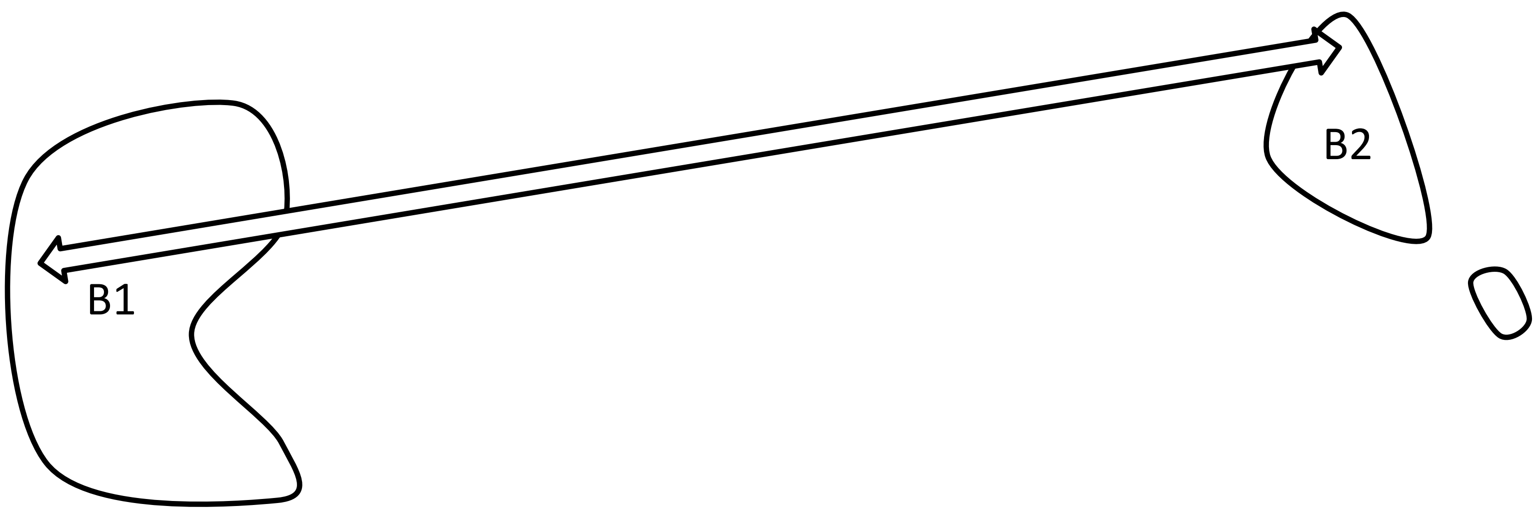
Example: measuring distances to a beacon

Nonlinear Gaussian set-inversion
J. Nicola, L. Jaulin
SWIM 2015

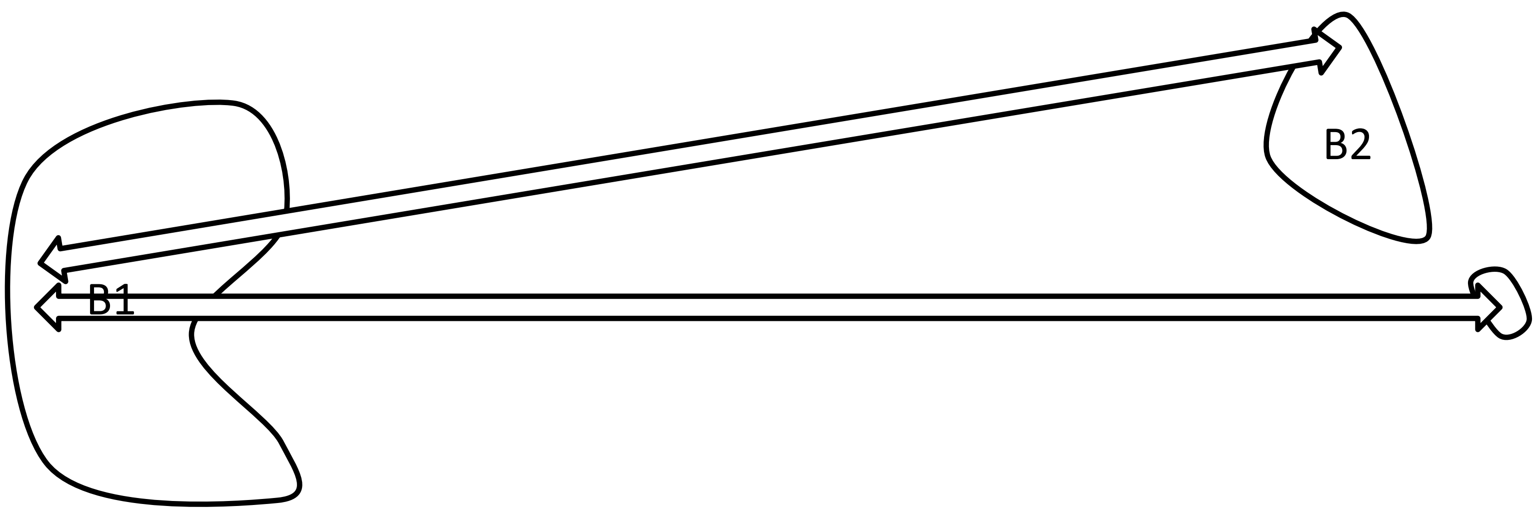
- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2

- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2

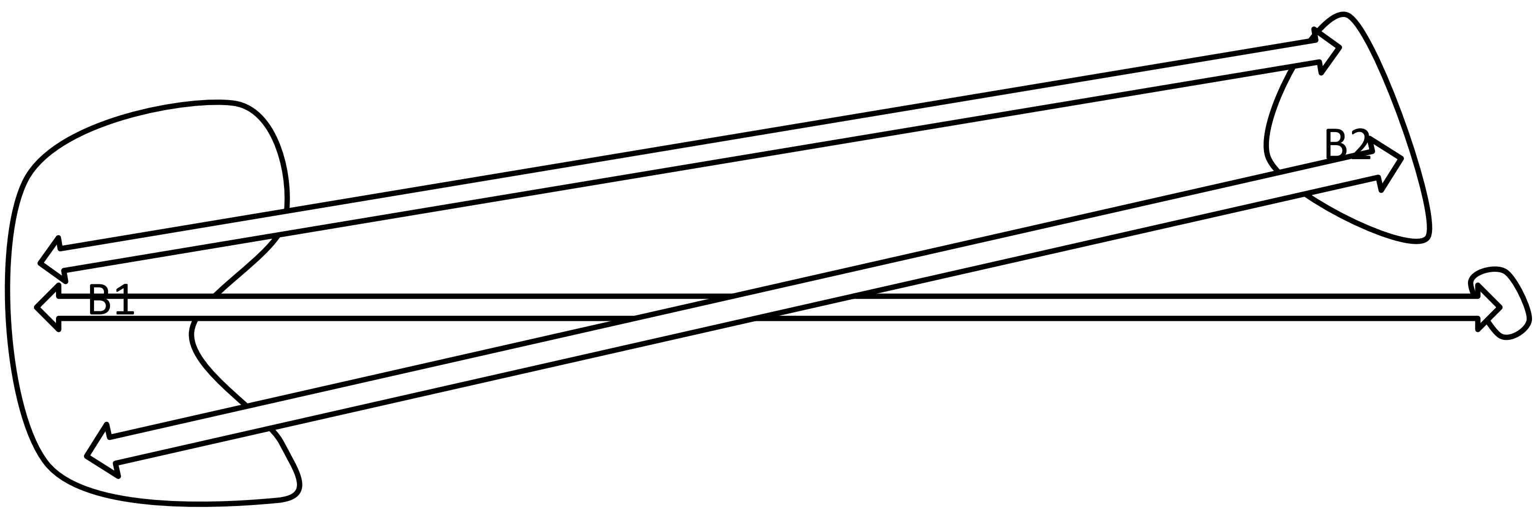


- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2

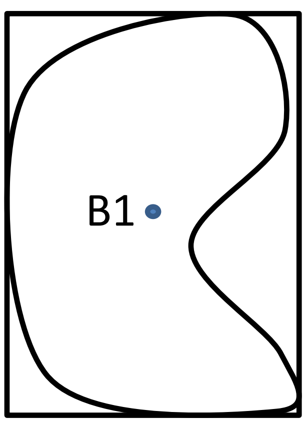


J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

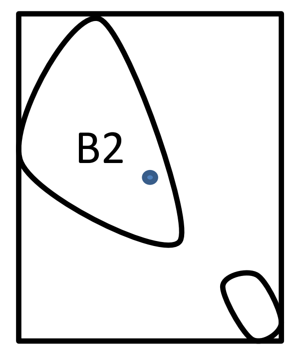
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



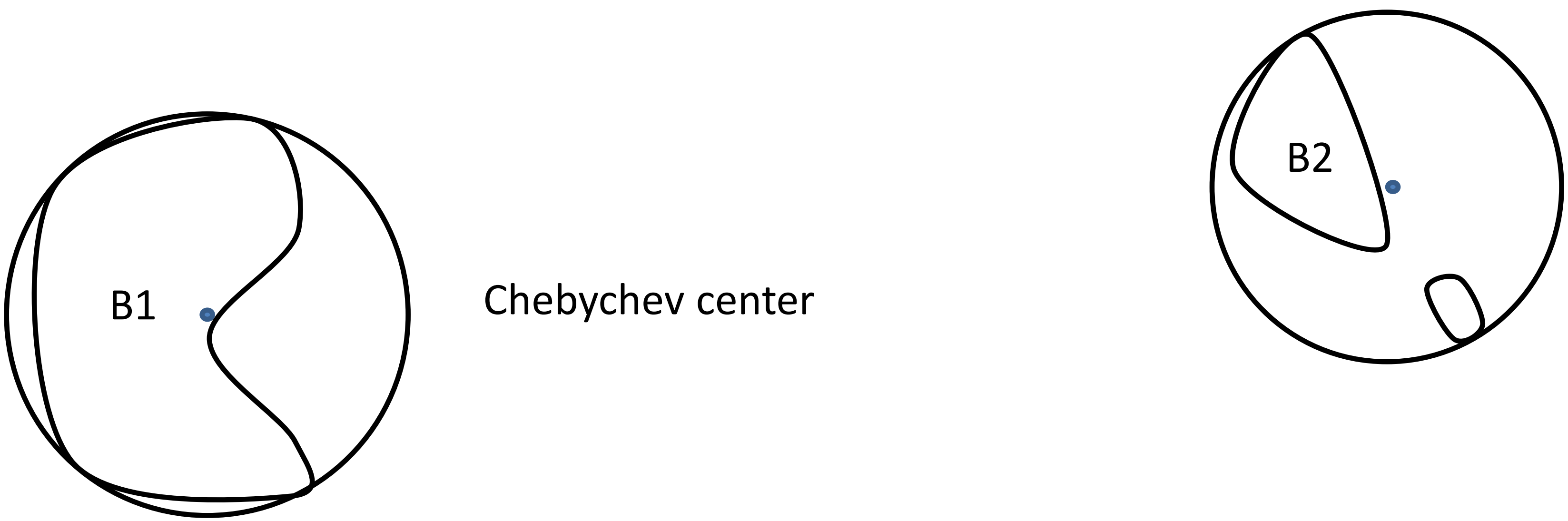
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



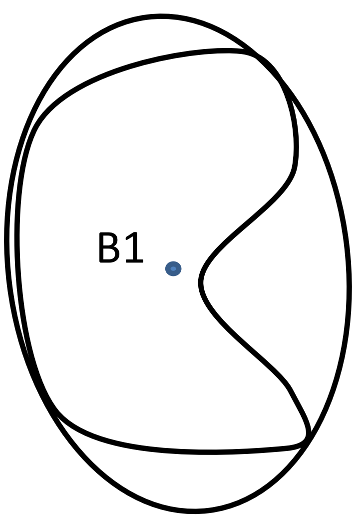
Center of the box-hull



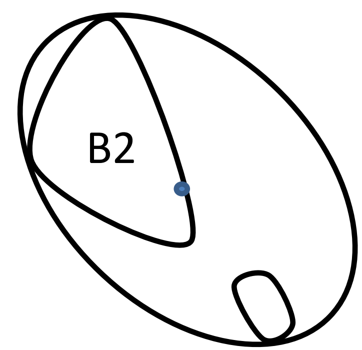
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



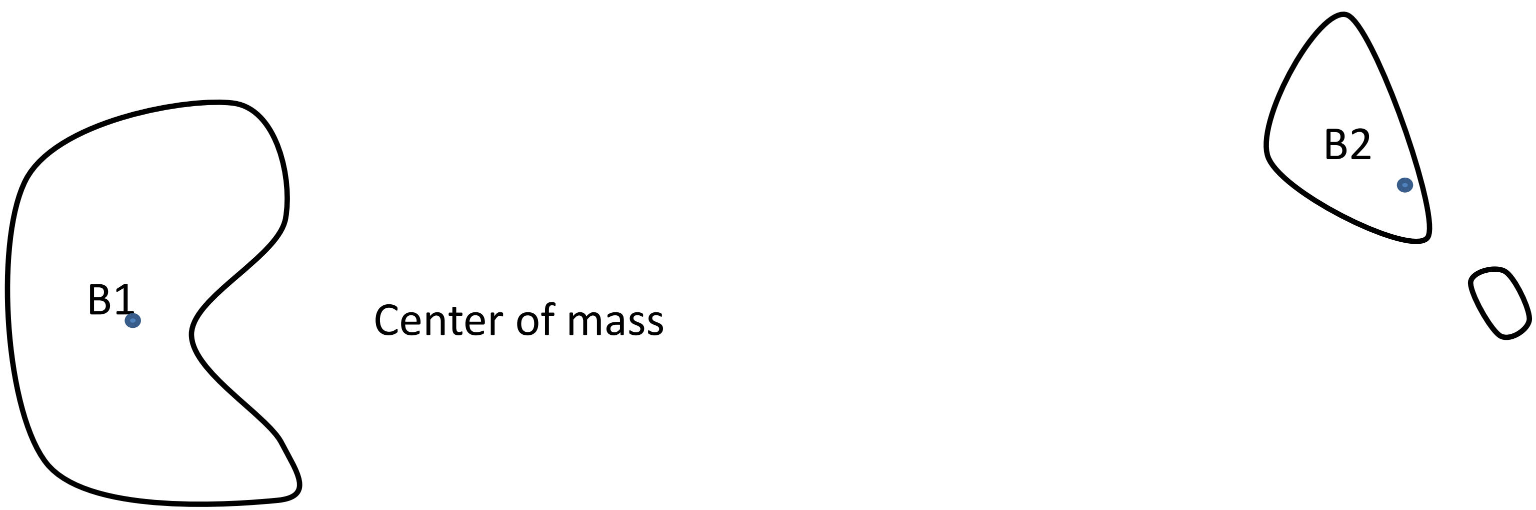
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



Center of the minimum trace/volume enclosing ellipsoid

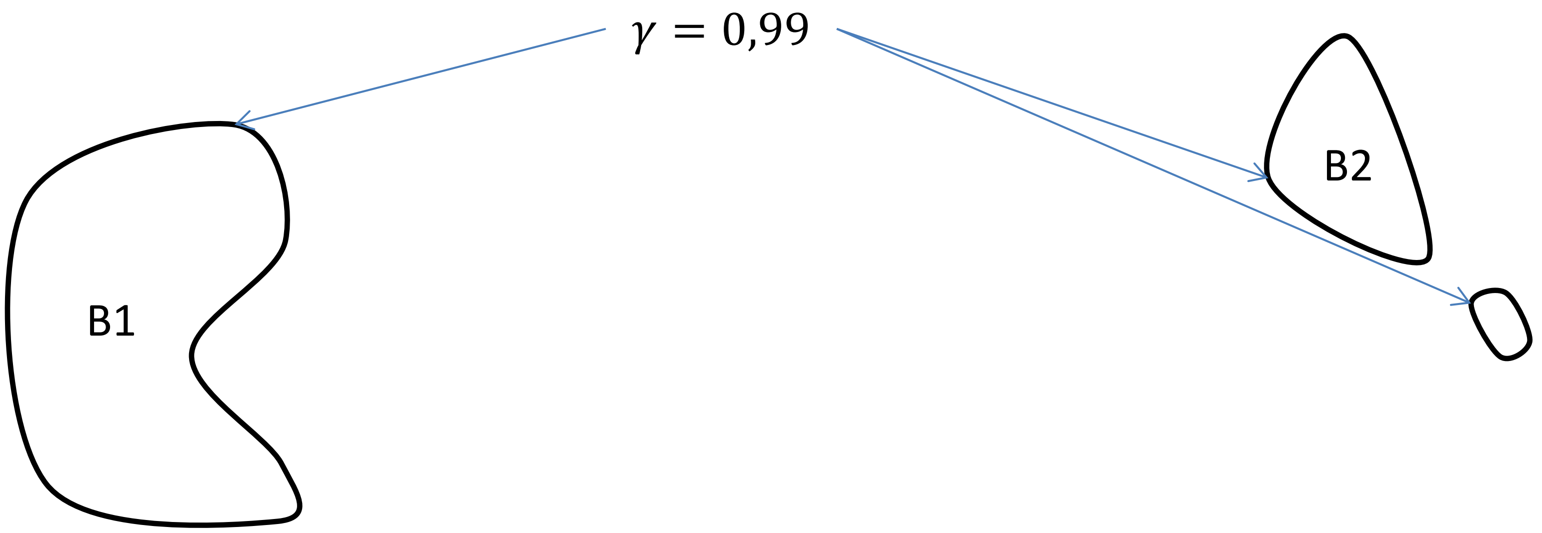


- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2

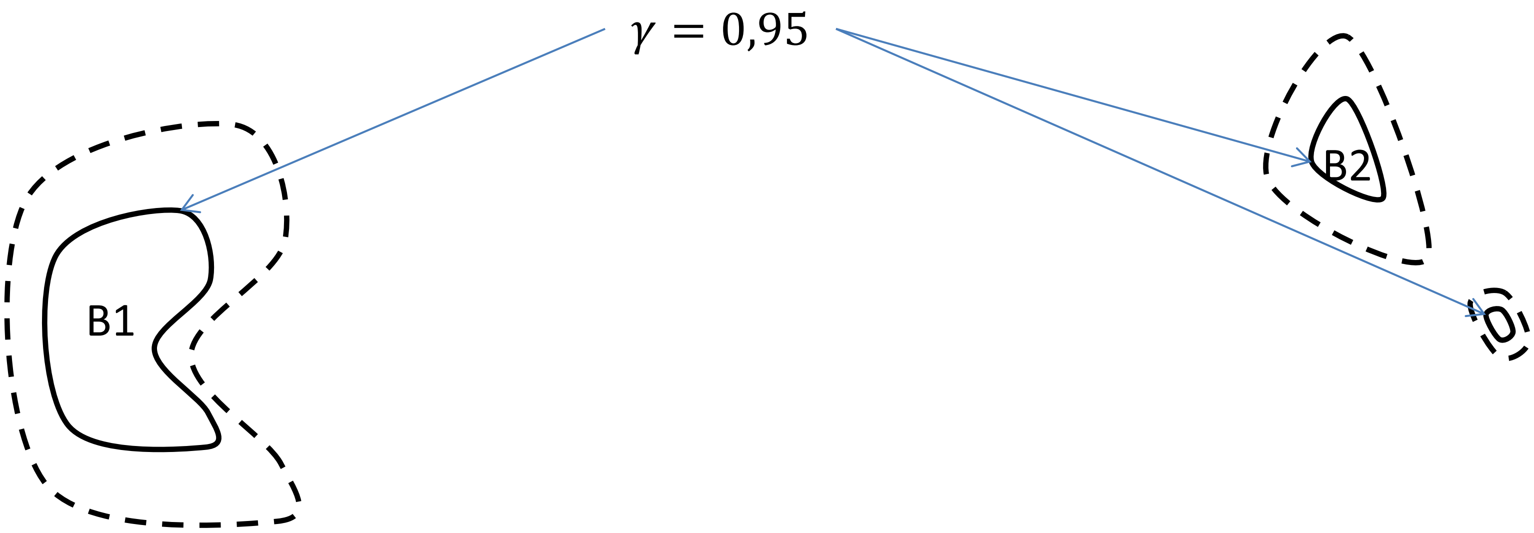


- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

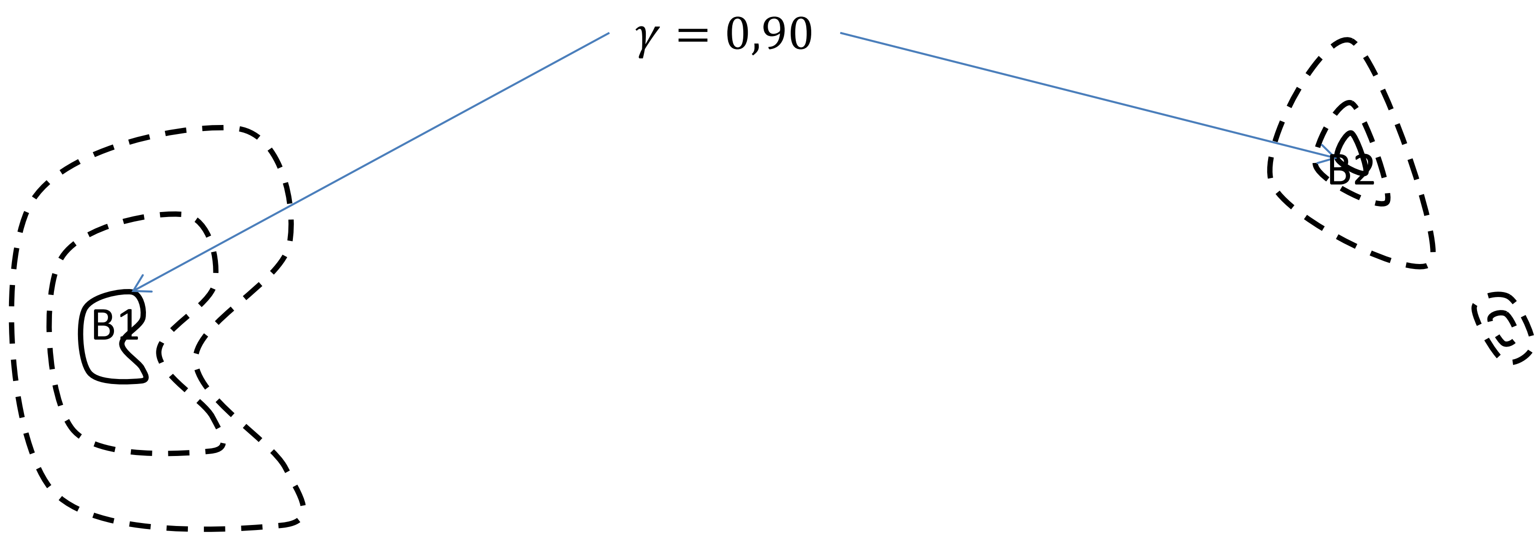
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



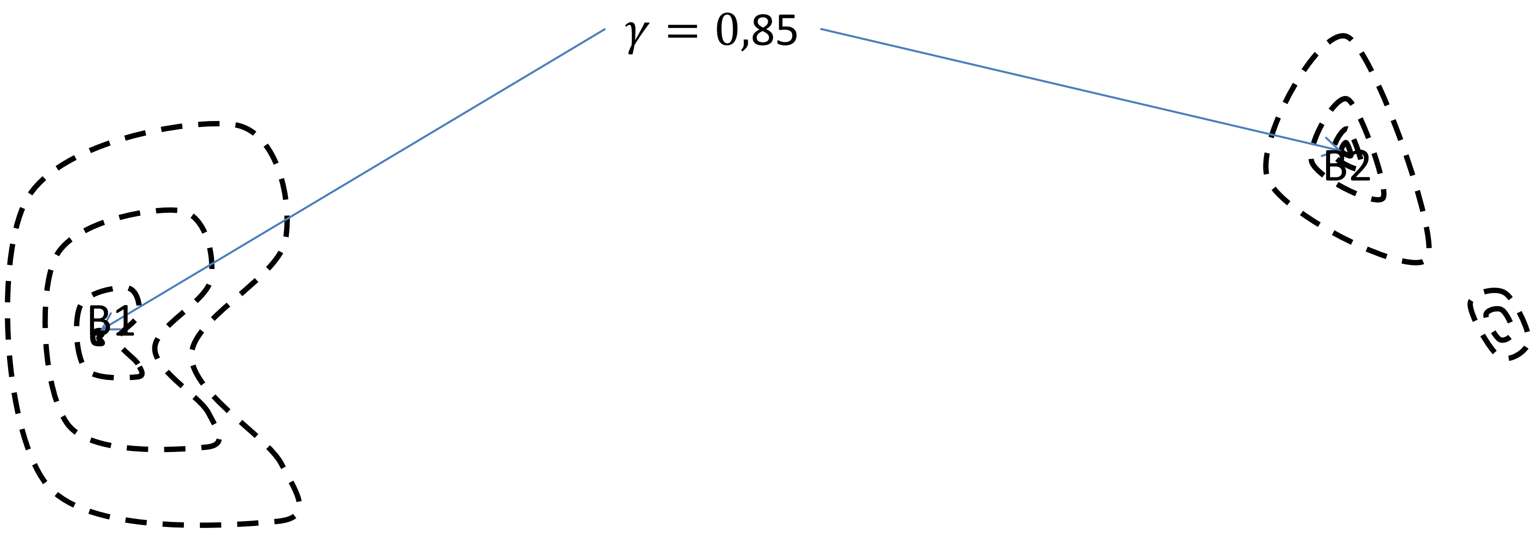
- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2

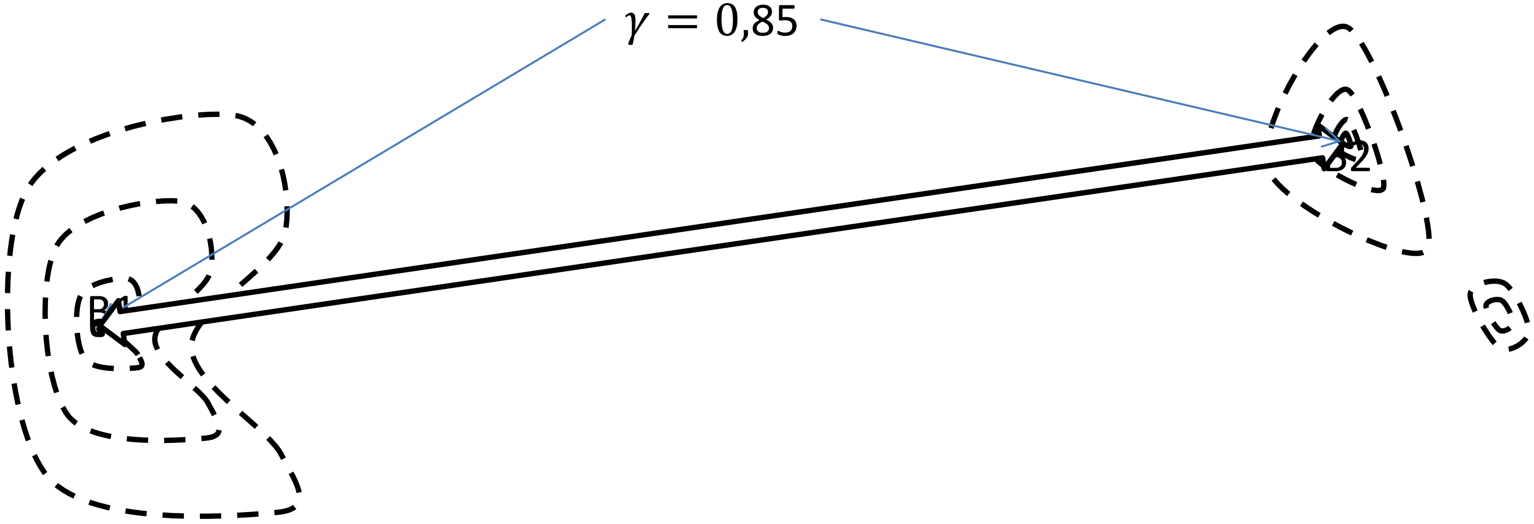


- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

- Practicality of a set-estimation
  - Design a pipe joining beacon 1 & beacon 2



- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

- Find the parameter that maximizes the Likelihood function
  - $L(\boldsymbol{y}|\boldsymbol{p}) = \prod \pi(y_i|\boldsymbol{p})$

- Find the parameter that maximizes the Likelihood function
  - $L(\boldsymbol{y}|\boldsymbol{p}) = \prod \pi(y_i|\boldsymbol{p}) \propto \prod e^{-(\psi_i(\boldsymbol{p})-y_i)^2}$

- Find the parameter that maximizes the Likelihood function
  - $L(\boldsymbol{y}|\boldsymbol{p}) = \prod \pi(y_i|\boldsymbol{p}) \propto \prod e^{-(\psi_i(\boldsymbol{p})-y_i)^2}$
  - $\arg min\left(\sum(\psi_i(\boldsymbol{p}) - y_i)^2\right)$

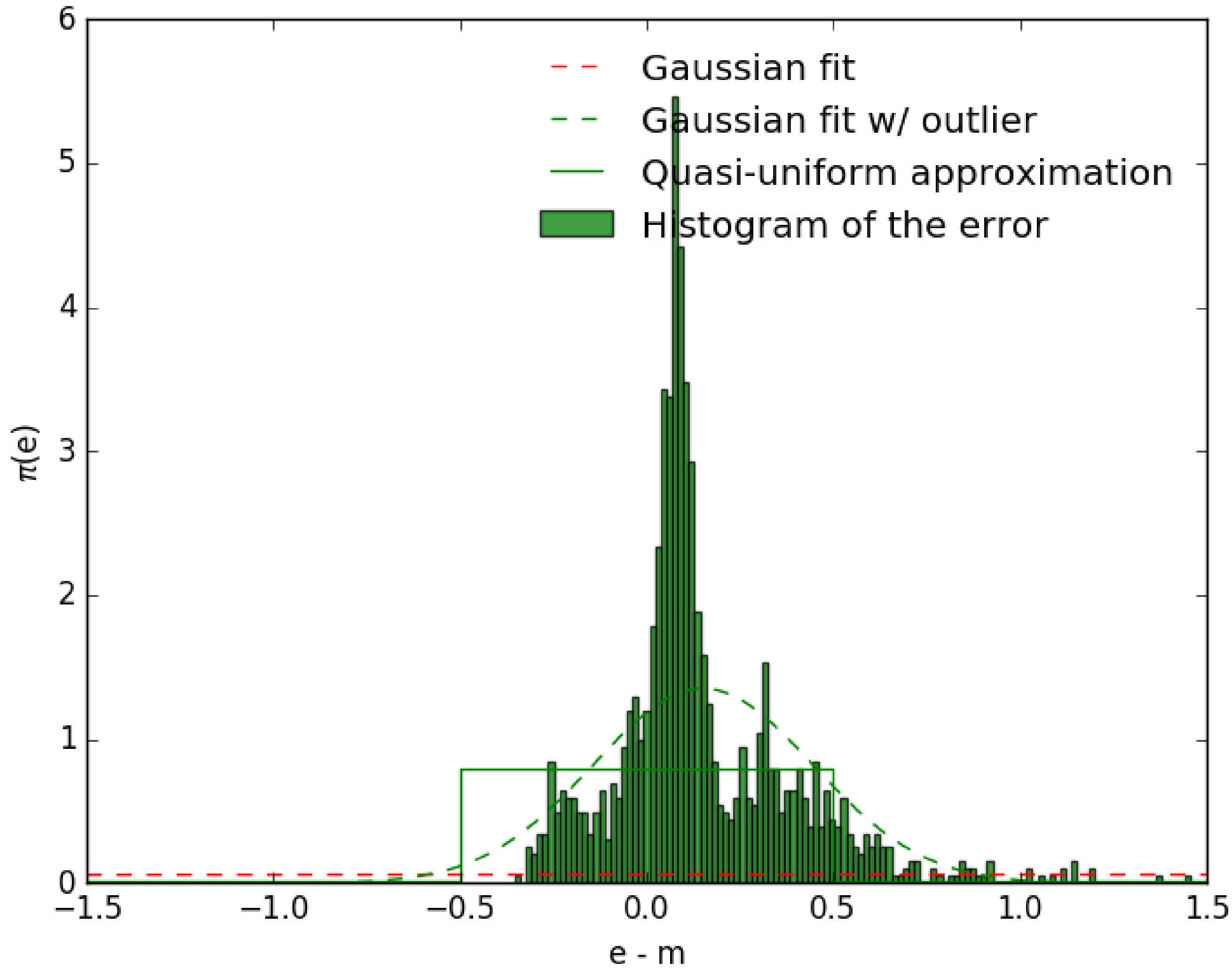
- Find the parameter that maximizes the Likelihood function
  - $L(\boldsymbol{y}|\boldsymbol{p}) = \prod \pi(y_i|\boldsymbol{p}) \propto \prod e^{-(\psi_i(\boldsymbol{p})-y_i)^2}$
  - $\arg min \left(\sum(\psi_i(\boldsymbol{p}) - y_i)^2\right)$
    - This is a classical nonlinear least squares problem

- Why the Gaussian assumption?

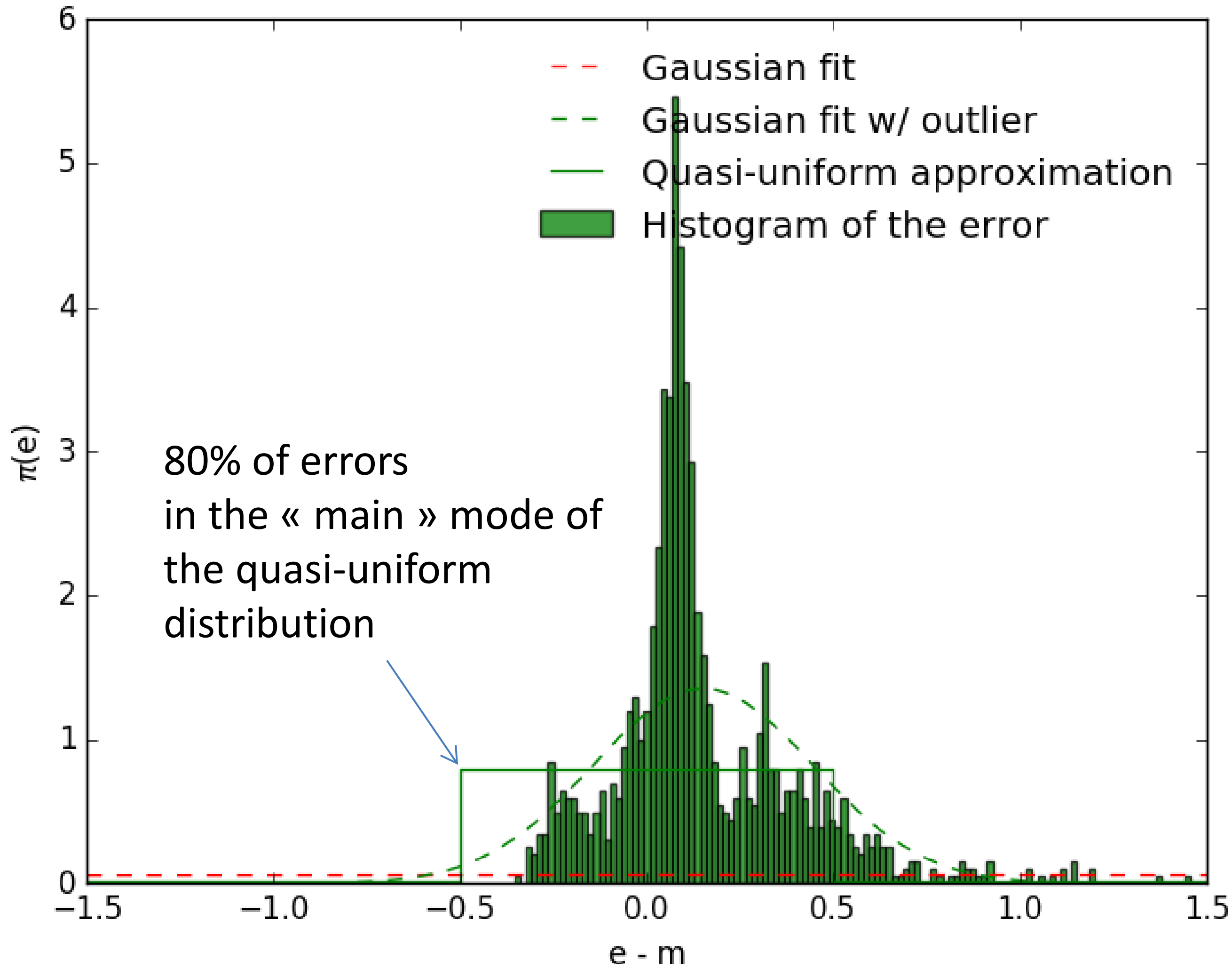
- Why the Gaussian assumption?
  - The distribution is easy to parametrize

- Why the Gaussian assumption?
  - The distribution is easy to parametrize
  - It conveniently reduces the Maximum Likelihood Estimation problem to a (nonlinear) least-squares estimation

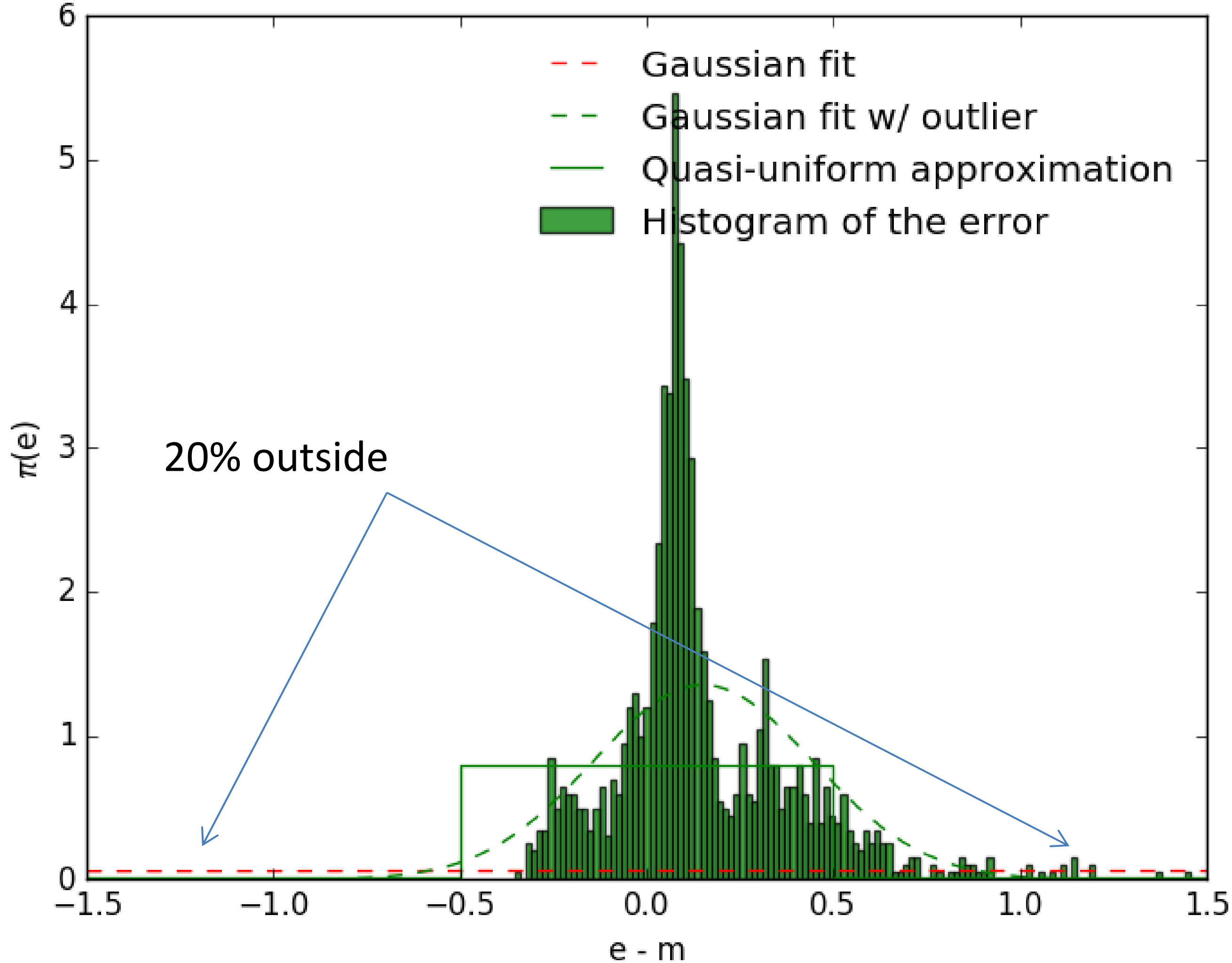
- Is the noise really Gaussian?



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

- Modeling the problem

- Modeling the problem
- Consider the following static parameter estimation model
  - $\boldsymbol{y} = \boldsymbol{\psi}(\boldsymbol{p}) + \boldsymbol{e}$
    - $\boldsymbol{y}$ is the measurement vector
    - $\boldsymbol{\psi}$ is the model
    - $\boldsymbol{p}$ is the parameter vector to be estimated
    - $\boldsymbol{e}$ is the noise

- Modeling the problem
- Consider the following static parameter estimation model
  - $\boldsymbol{y} = \boldsymbol{\psi}(\boldsymbol{p}) + \boldsymbol{e}$
    - $\boldsymbol{y}$ is the measurement vector
    - $\boldsymbol{\psi}$ is the model
    - $\boldsymbol{p}$ is the parameter vector to be estimated
    - $\boldsymbol{e}$ is the noise
  - Define the function
    - $\boldsymbol{f}\,(\boldsymbol{p}) = \boldsymbol{e} = \boldsymbol{y} - \boldsymbol{\psi}(\boldsymbol{p})$

- Modeling the problem
- Given an error interval $[e]$ that is supposed to contain the error e_i if the corresponding data y_i is an inlier

- Modeling the problem
- Given an error interval $[e]$ that is supposed to contain the error e_i if the corresponding data y_i is an inlier
- The OMNE estimator returns the set of all $\boldsymbol{p}$ such as the property $f_i(\boldsymbol{p}) \in [e]$ is satisfied for a maximal number of data

- Modeling the noise by quasi-uniform distributions
  - Assume that the error vector **e** is white (the e_i's are independant), the e_i's are identically distributed with the probability density function π_e which is quasi uniform
    - $\pi_e(e_i) = a$ if $e_i \in [e]$, $\pi_e(e_i) = b < a$ otherwise

- Modeling the noise by quasi-uniform distributions
  - Assume that the error vector **e** is white (the e_i's are independant), the e_i's are identically distributed with the probability density function π_e which is quasi uniform
    - $\pi_e(e_i) = a$ if $e_i \in [e]$, $\pi_e(e_i) = b < a$ otherwise
  - Then the Maximum Likelihood Estimator is OMNE

- Modeling the noise by quasi-uniform distributions
  - Compatible with an infinity of p.d.f
    - Including heavy-tailed distributions
  - Well suited for interval methods
    - Using the q-relaxed intersection we are able to efficiently represent a confidence region of the quasi-uniformly distributed error

– The probability to have more than q measurements outside of the « main mode » of the quasi-uniform distribution is

- $\gamma(q,N,v)=\frac{1}{2}(1+\operatorname{erf}(\frac{N(1-v)-q-1}{\sqrt{2Nv(1-v)}}))$
  - N: number of measurements (N>>1)
  - v: probability of having a measurement in the main mode

– The probability to have more than q measurements outside of the « main mode » of the quasi-uniform distribution is

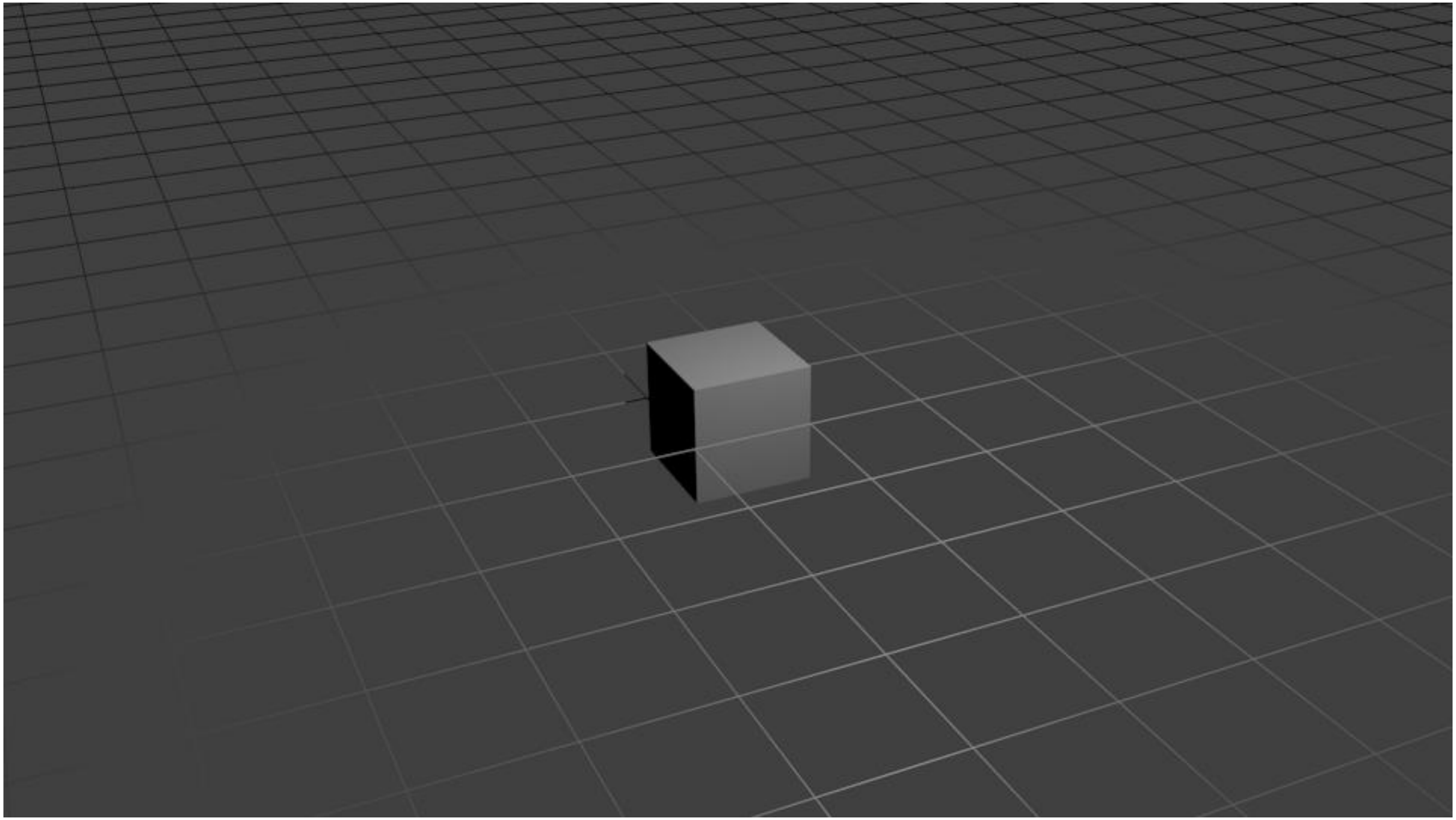
- $\gamma(q,N,v) = \frac{1}{2}(1 + \mathrm{erf}(\frac{N(1-v)-q-1}{\sqrt{2Nv(1-v)}}))$
  – N: number of measurements (N>>1)
  – v: probability of having a measurement in the main mode

– An expression for q is:

$$q(N,v,\gamma) = N(1-v) - 1 - \sqrt{2Nv(1-v)}\,\mathrm{erf}(2\gamma - 1)^{-1}$$

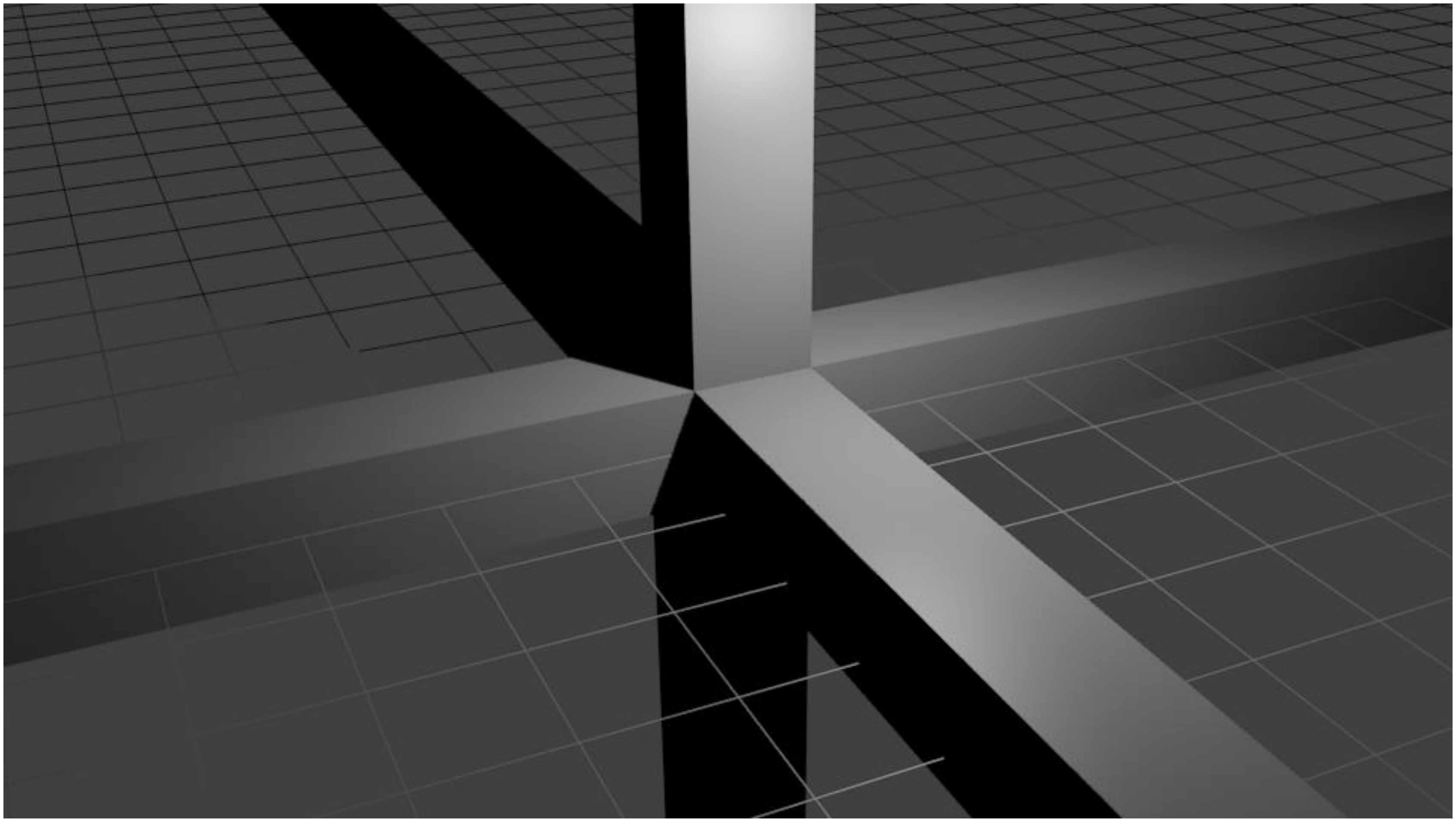
0-relaxed box

$$[x]^{\{0\}} = [x_1] \times [x_2] \times [x_3]$$



1-relaxed box

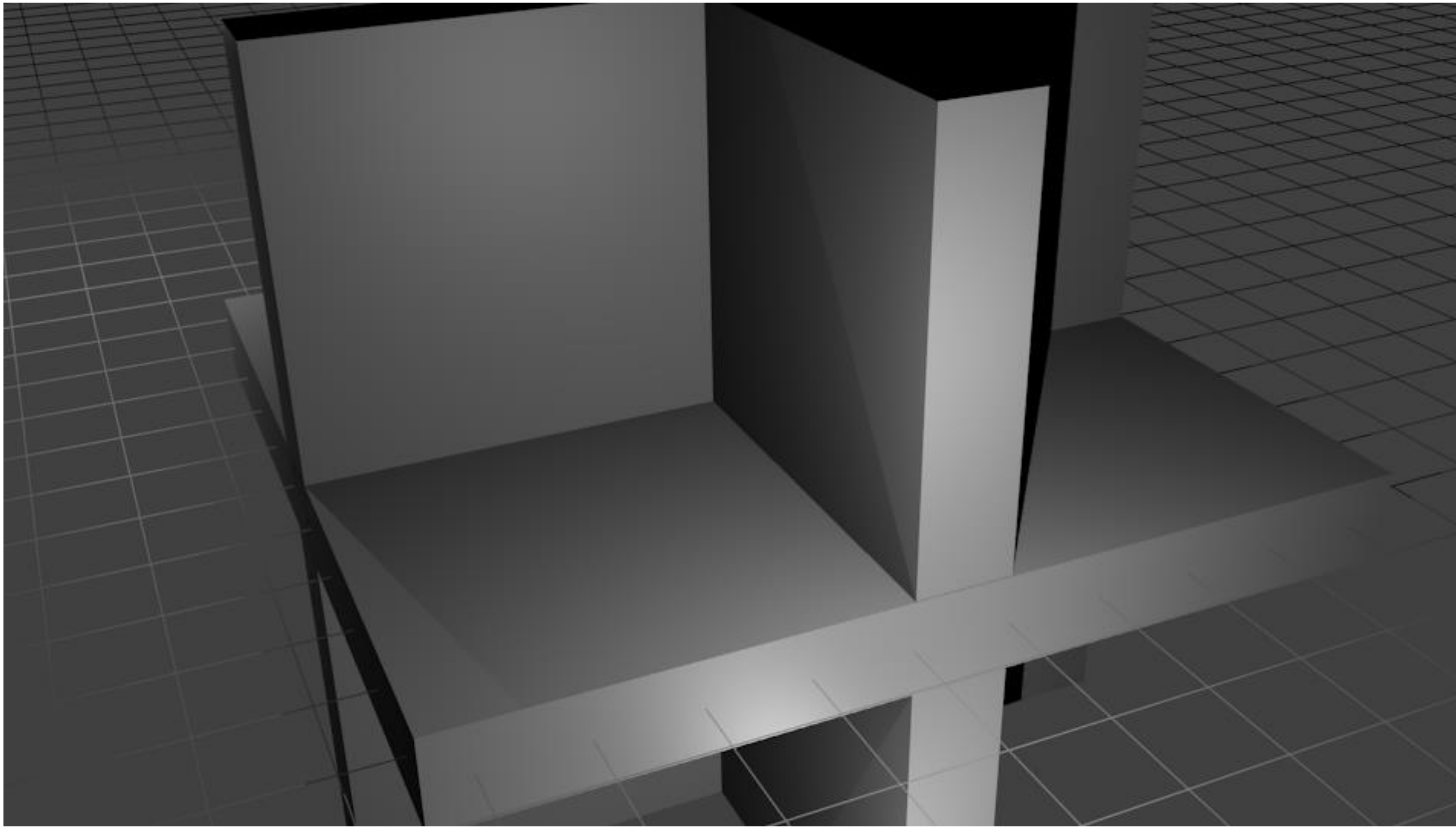
$$[x]^{\{1\}} = [-\infty,\infty] \times [x_2] \times [x_3] \cup [x_1] \times [-\infty,\infty] \times [x_3] \cup [x_1] \times [x_2] \times [-\infty,\infty]$$



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

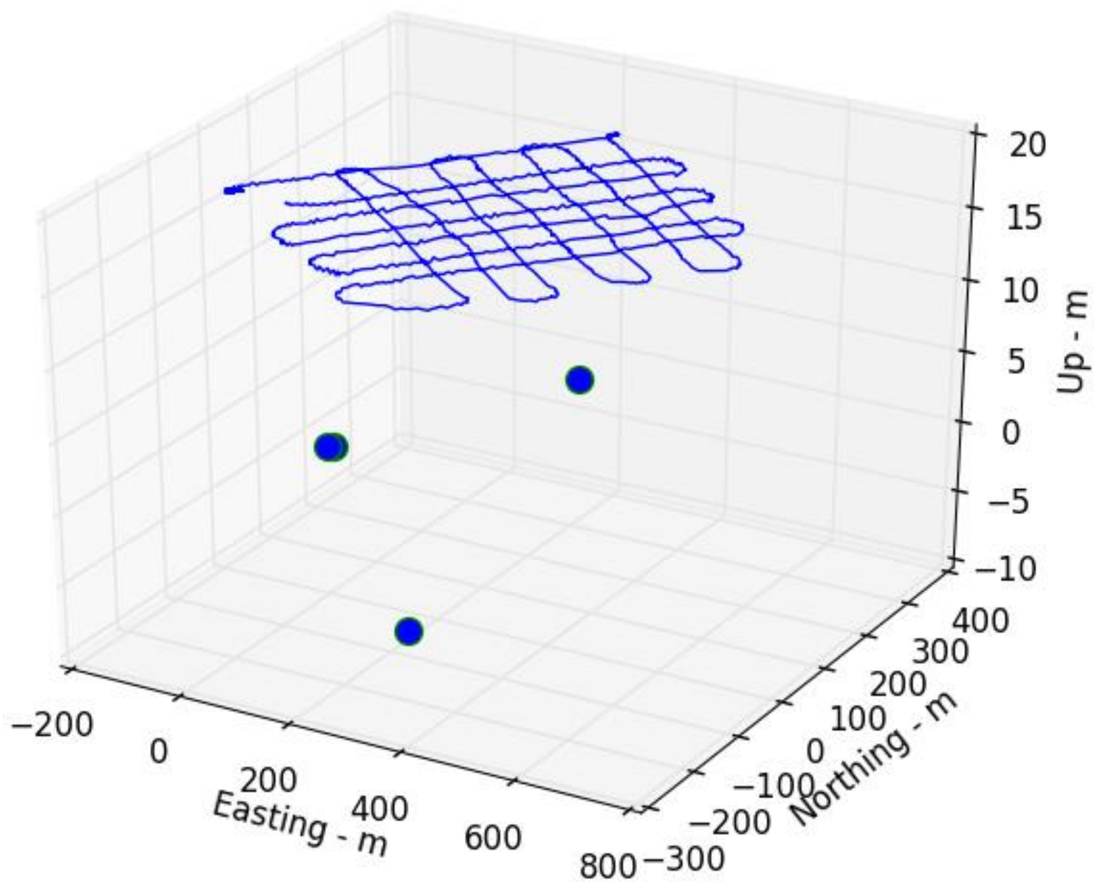
2-relaxed box (truncated)

$$[x]^{\{2\}} = [-\infty,\infty]^{\times 2} \times [x_3] \cup [x_1] \times [-\infty,\infty]^{\times 2} \cup [-\infty,\infty] \times [x_2] \times [-\infty,\infty]$$

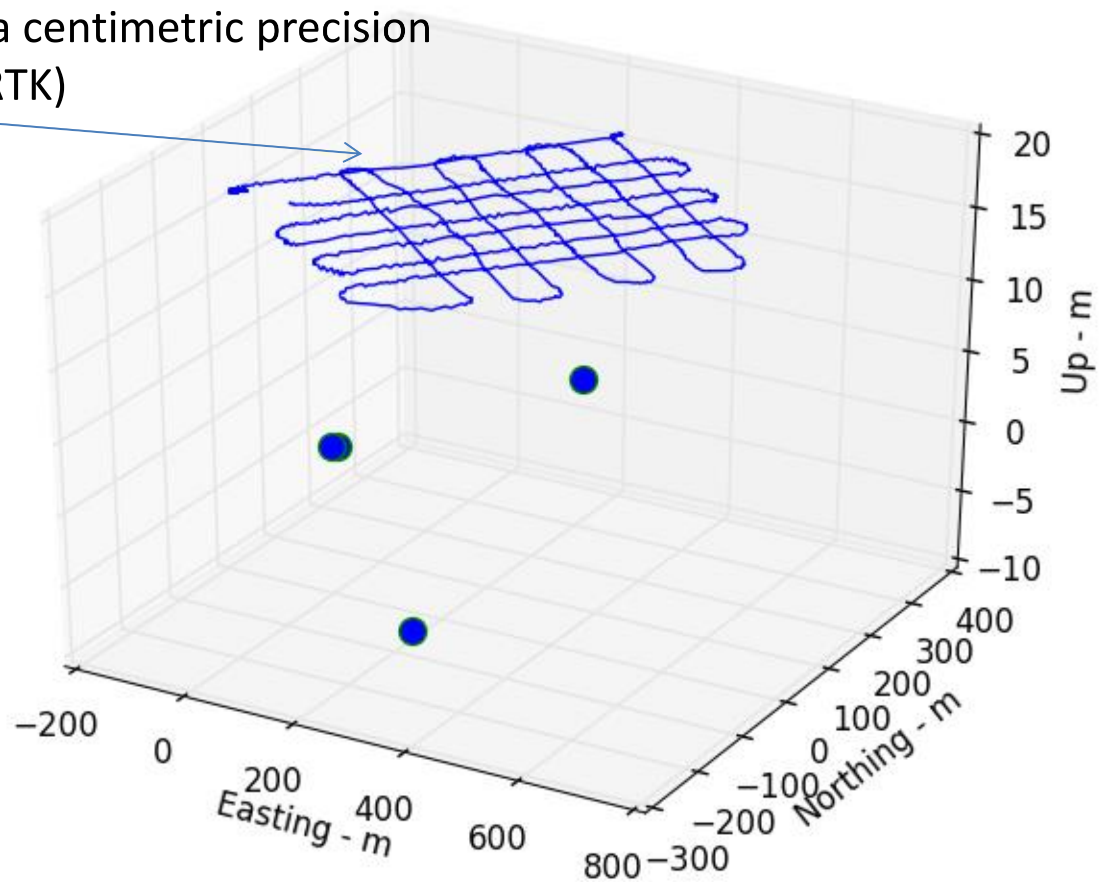


J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

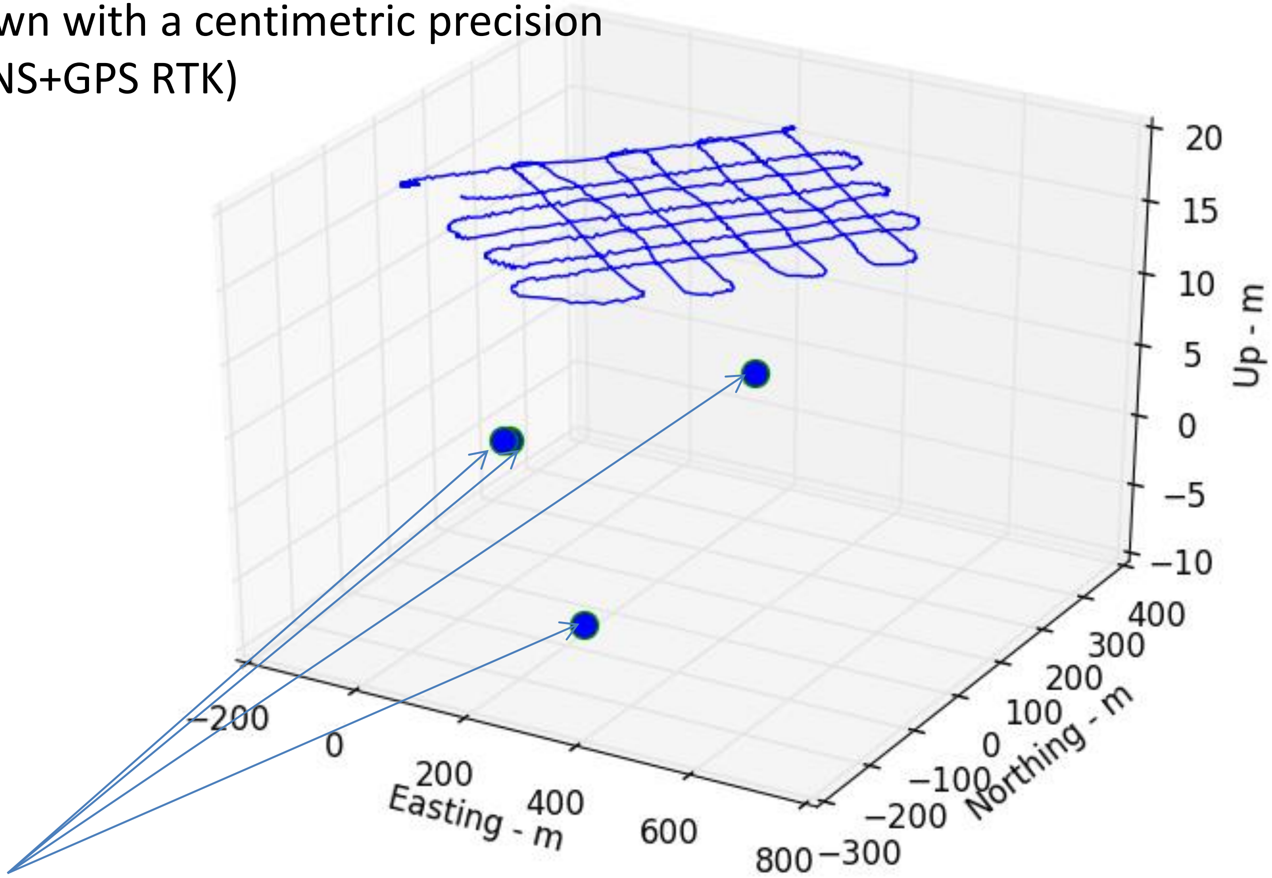
- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion


Easting - m
Northing - m
Up - m
−200
0
200
400
600
800
−300
−200
−100
0
100
200
300
400
−10
−5
0
5
10
15
20

Trajectory known with a centimetric precision
(High quality INS+GPS RTK)

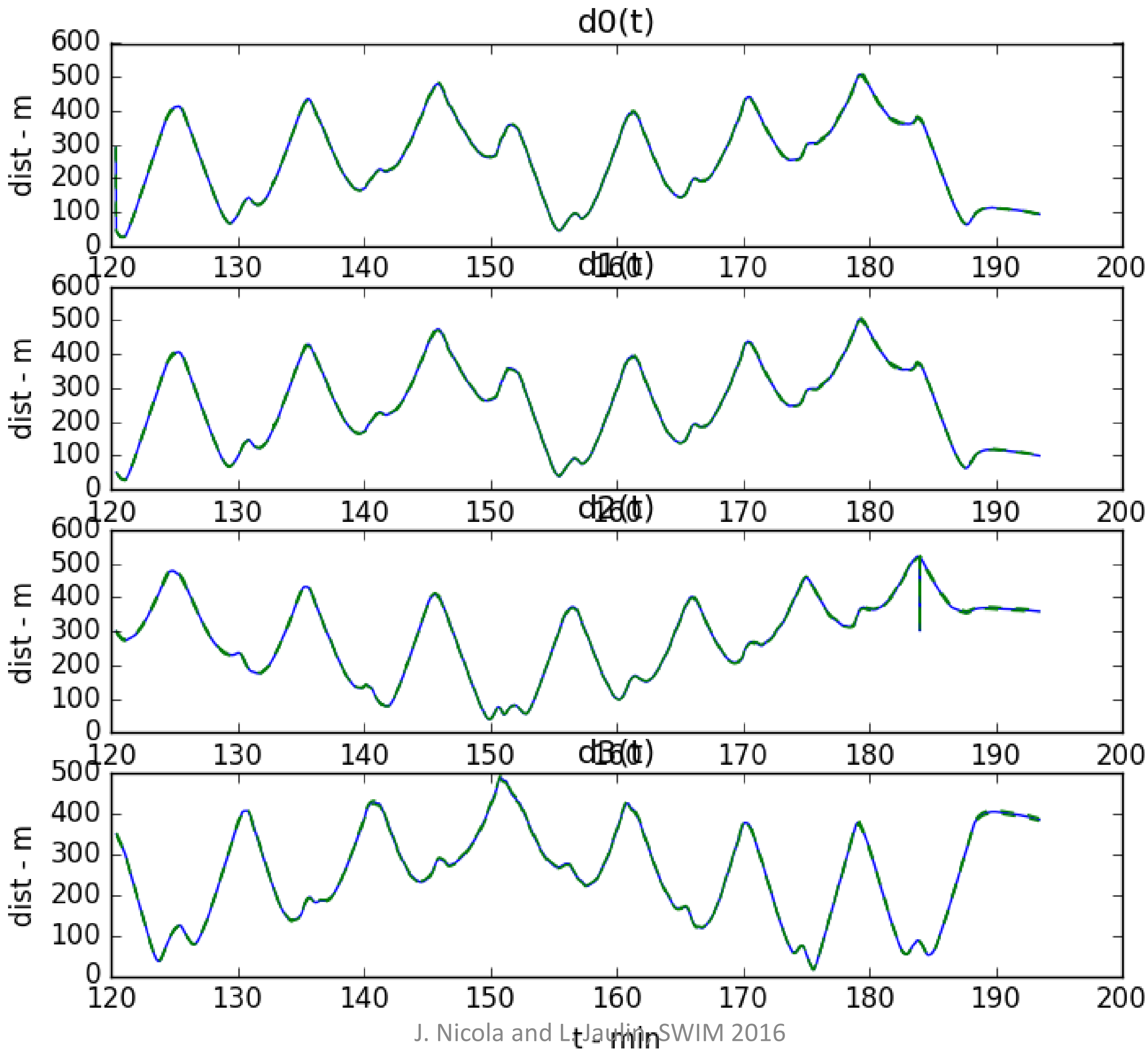


Trajectory known with a centimetric precision
(High quality INS+GPS RTK)



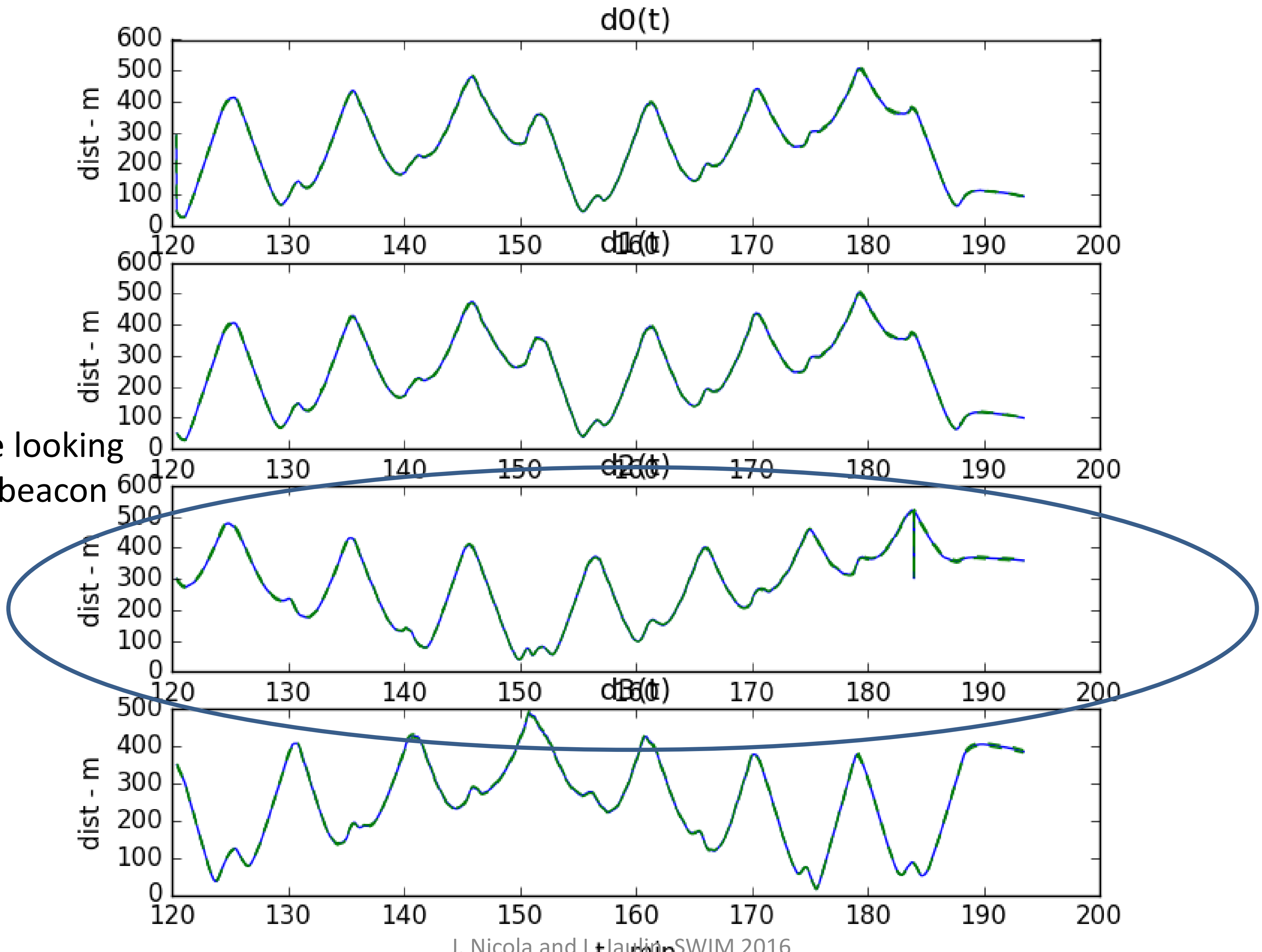
4 beacons we want to localize

Range signals received for each beacon



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

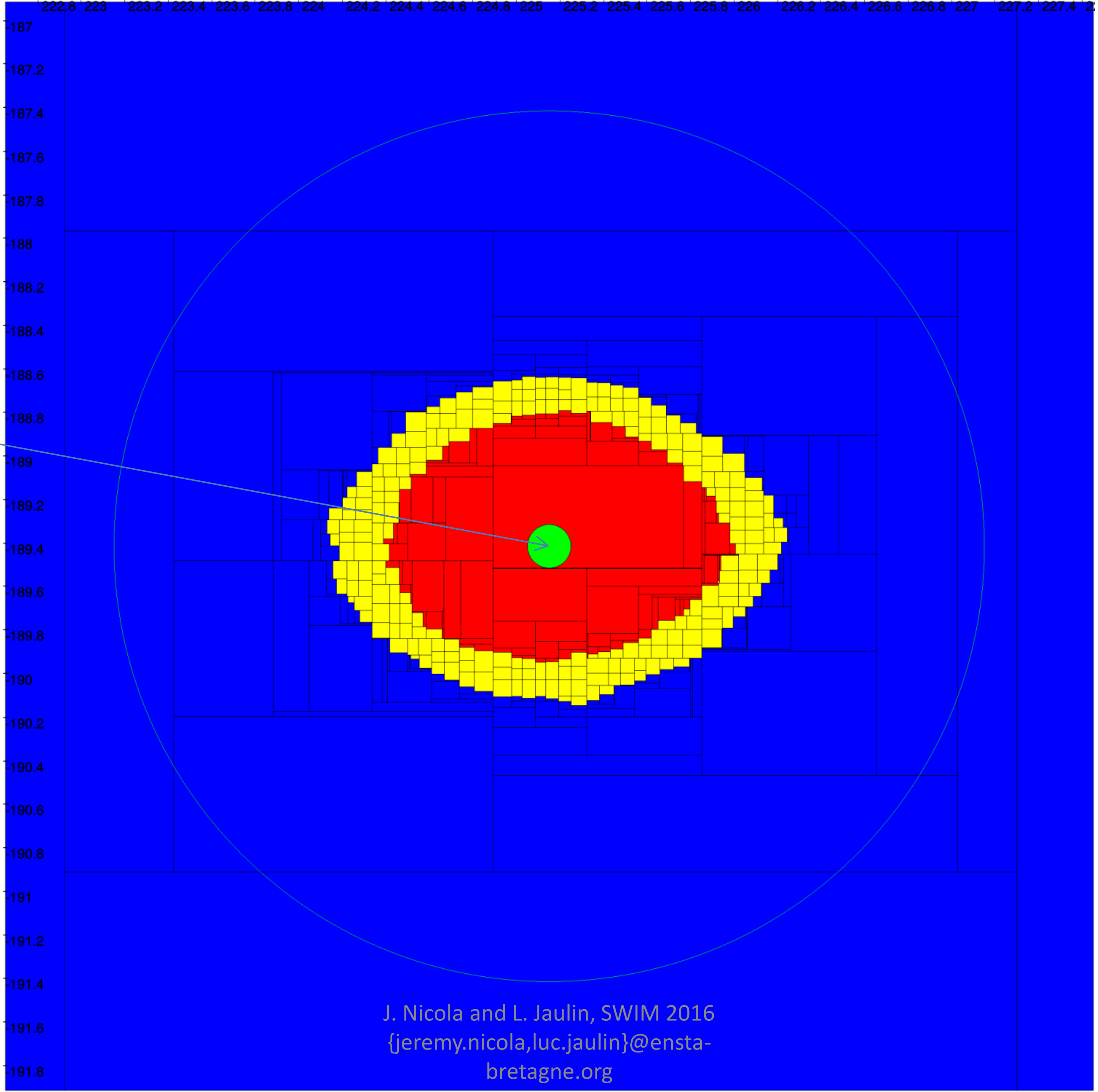
We’ll be looking for this beacon



– For easier illustration, we assume that the z-position of the beacon is exactly known

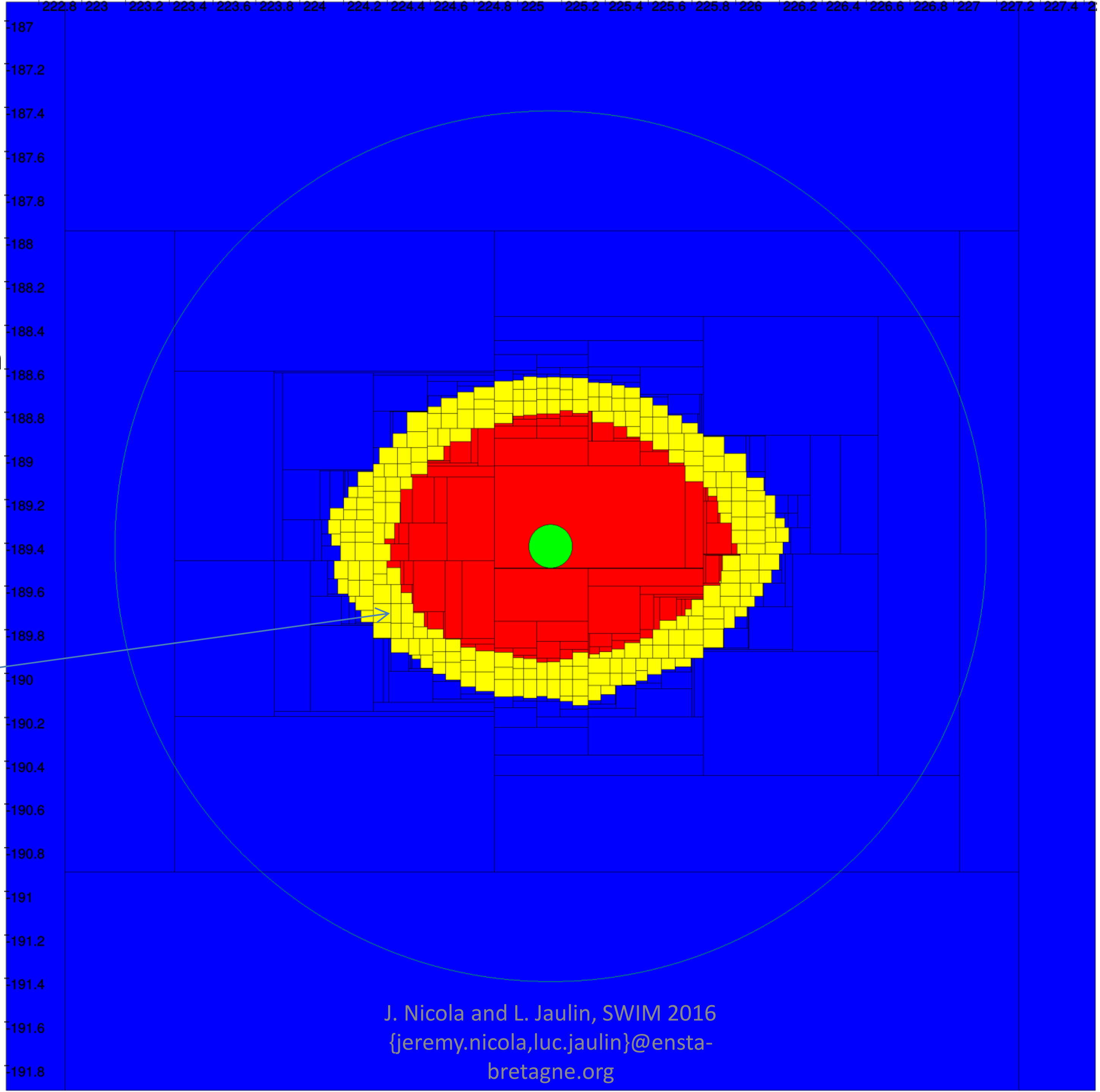
- In practice, the z-component is easily found in an underwater context

Real beacon position

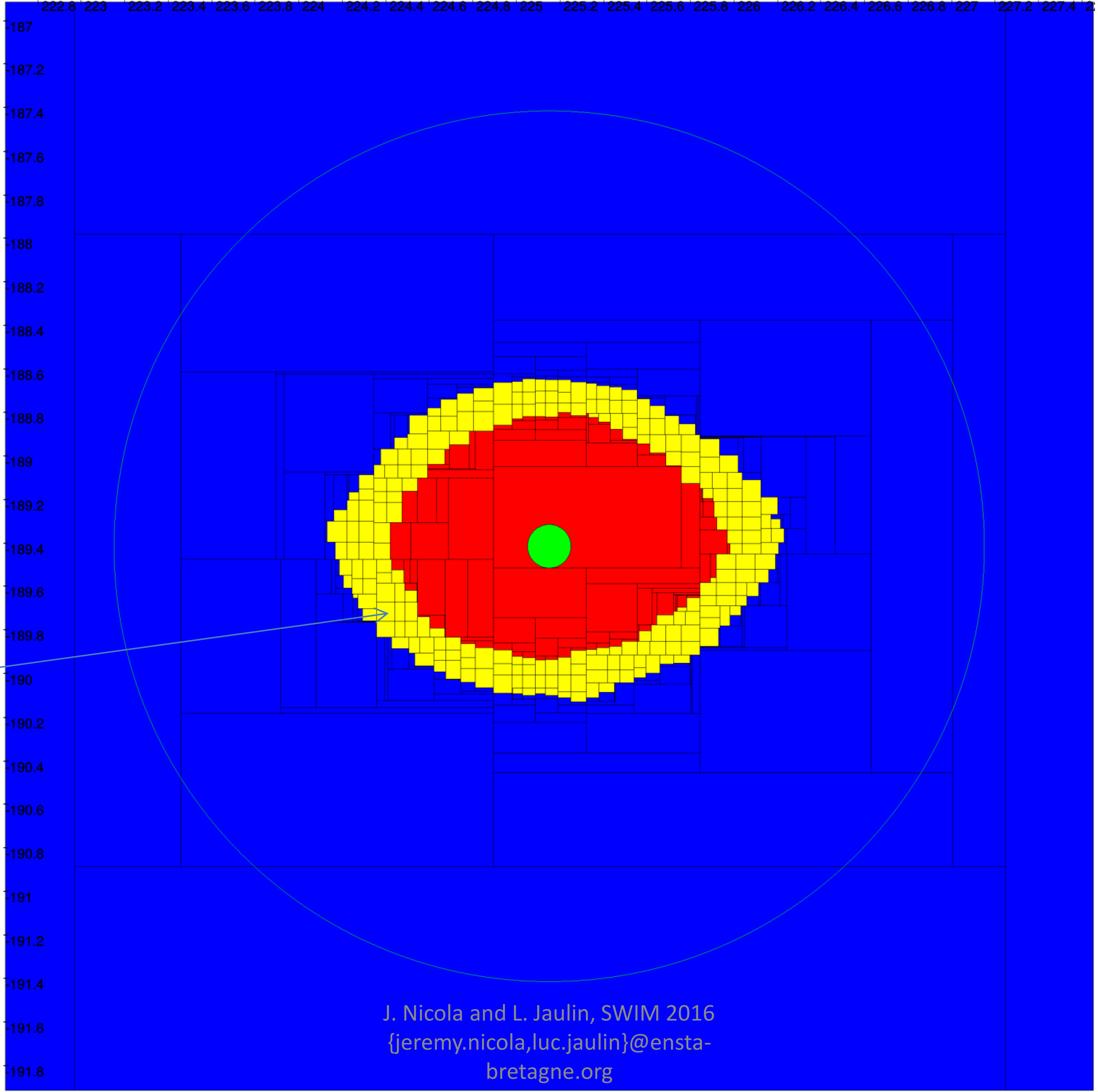


Real beacon position

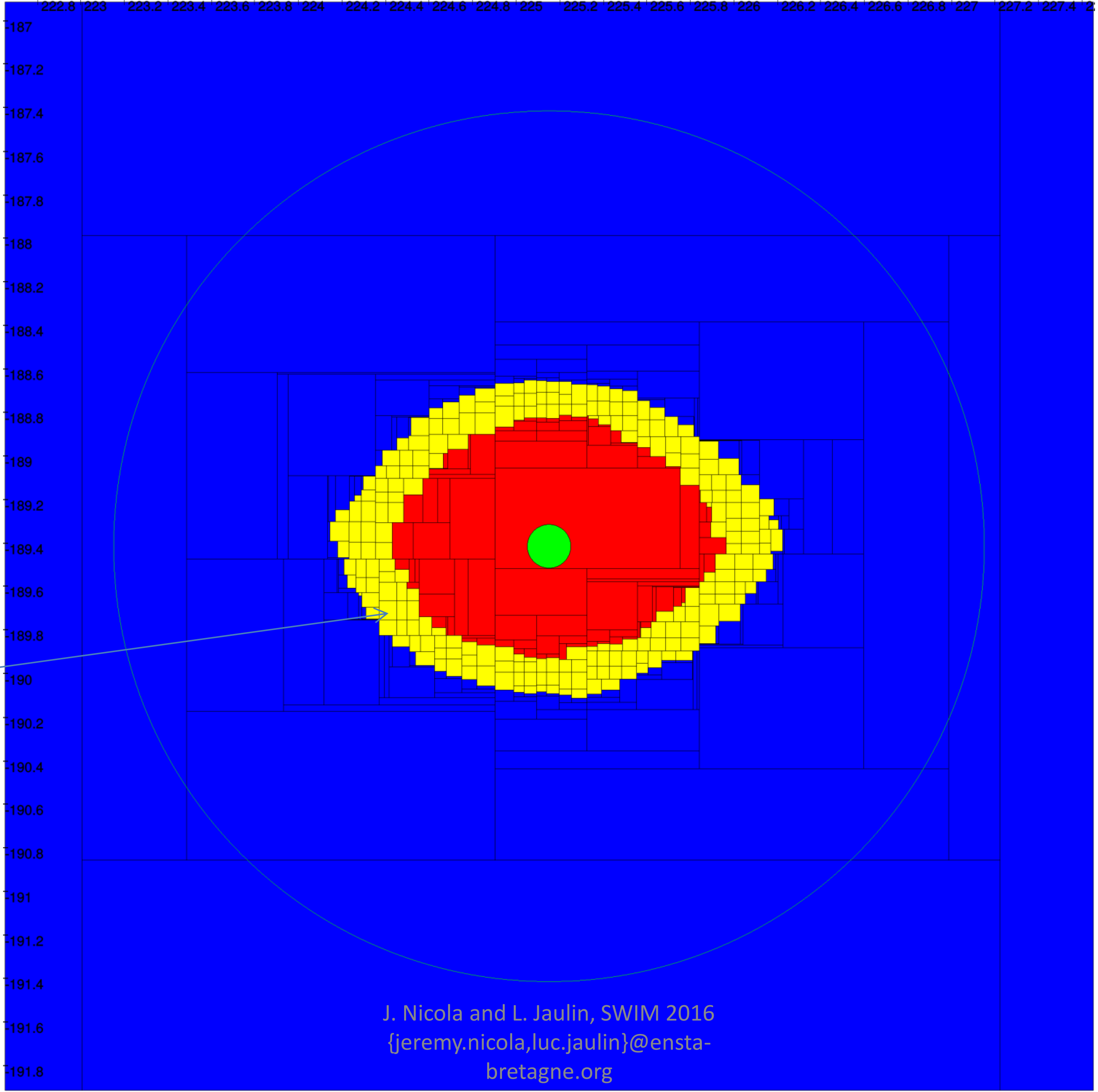
50% confidence region

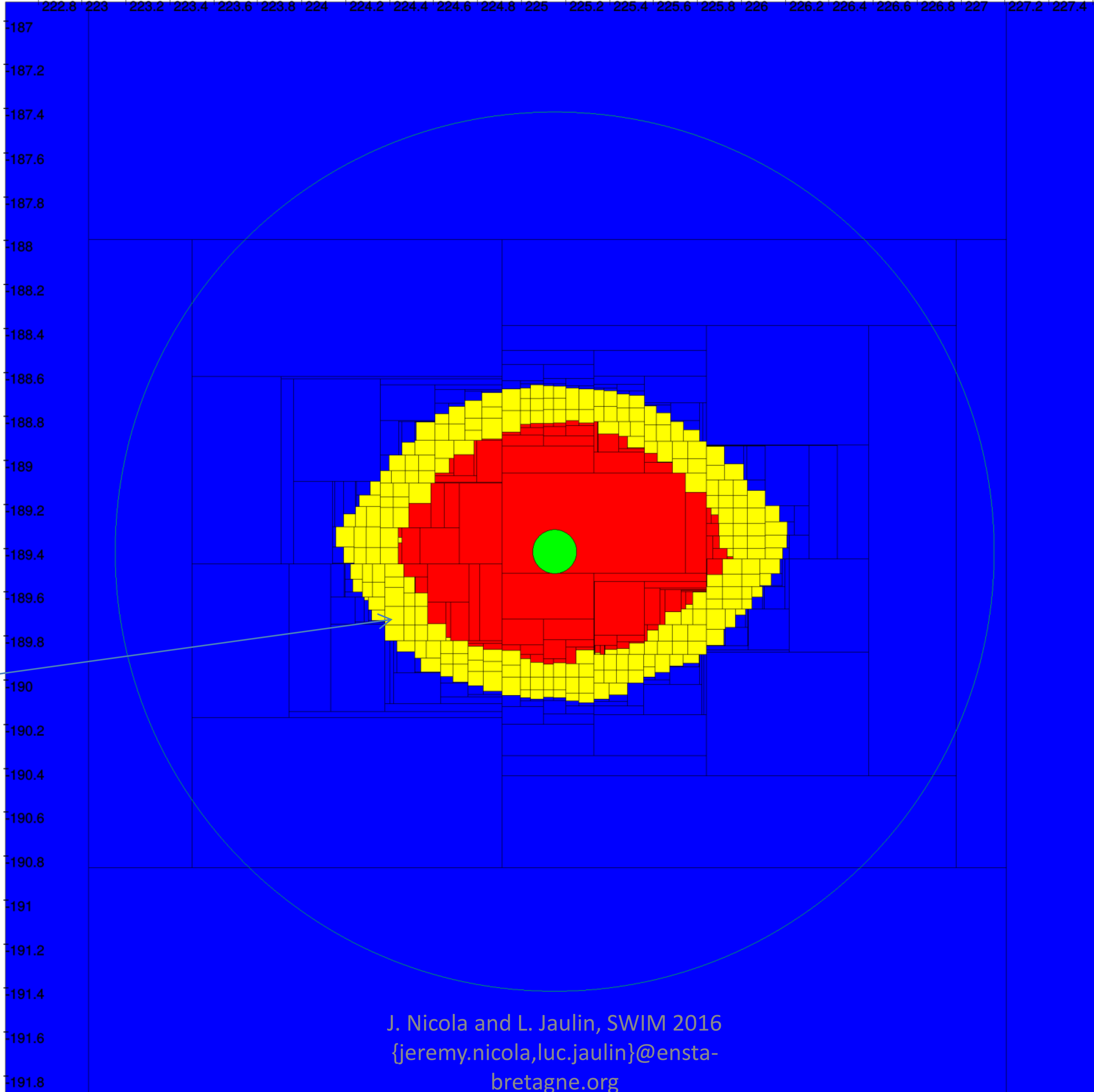


25%
confidence
region



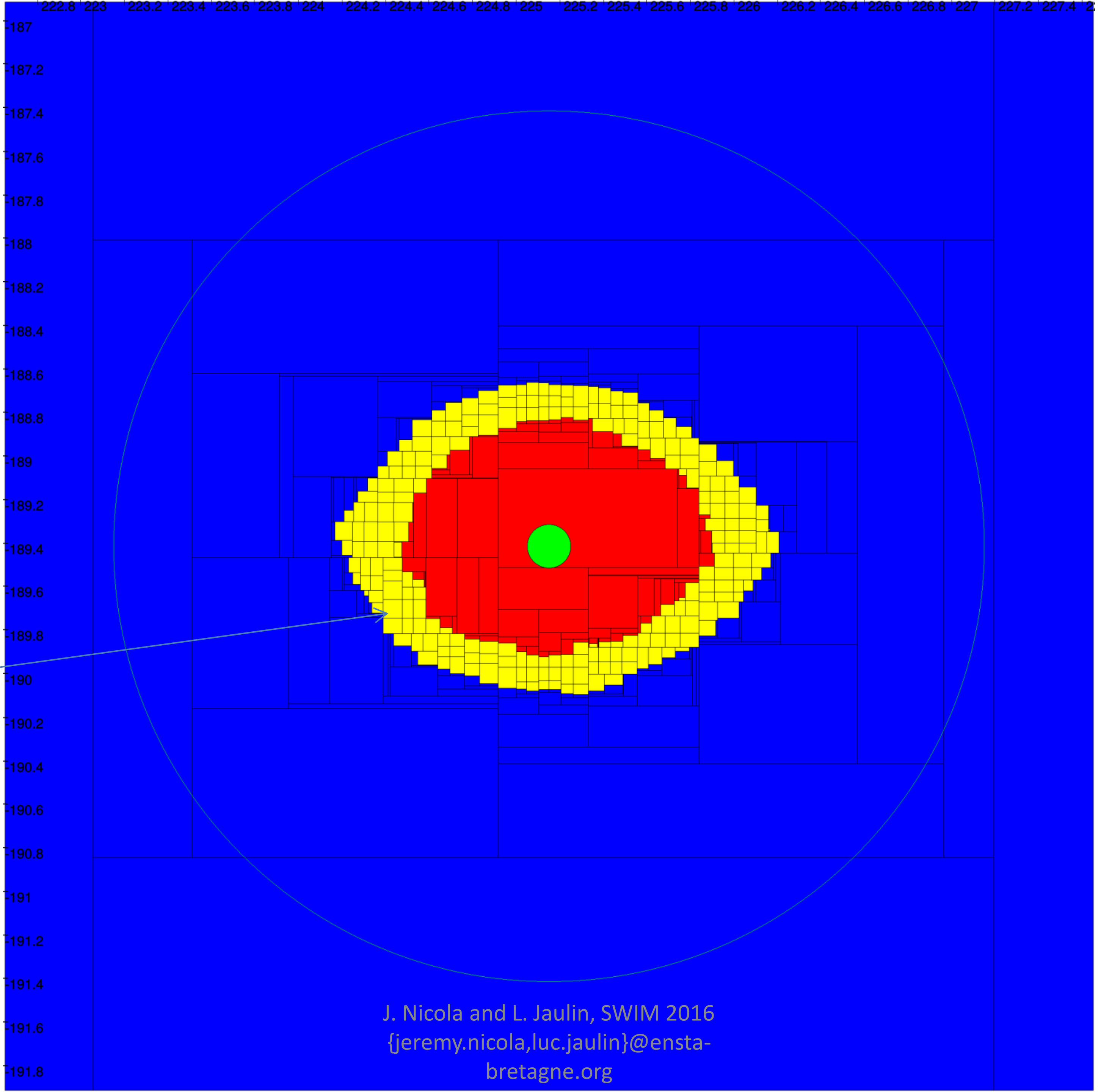
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org


12.5%
confidence
region



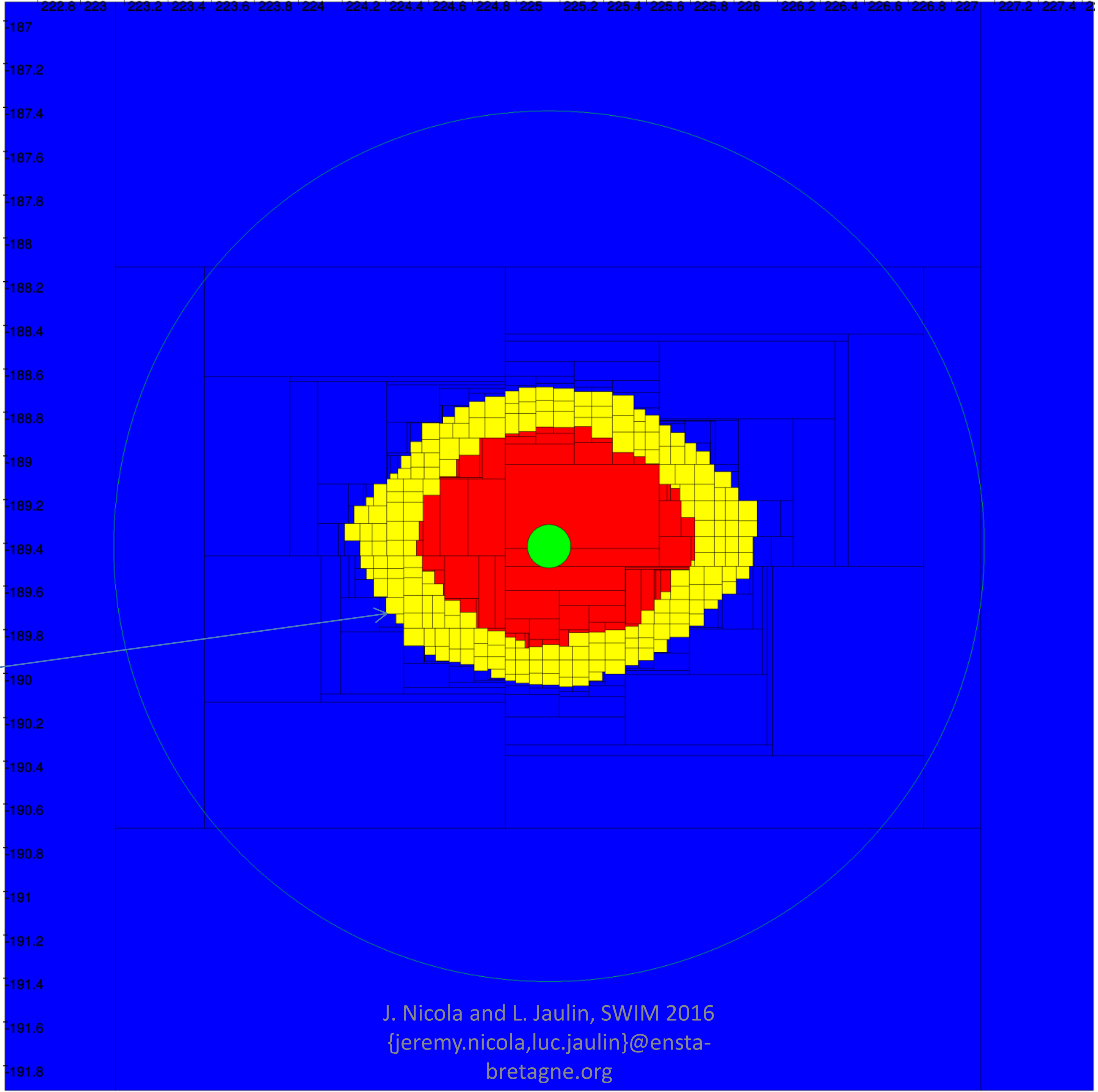
6.25% confidence region

J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



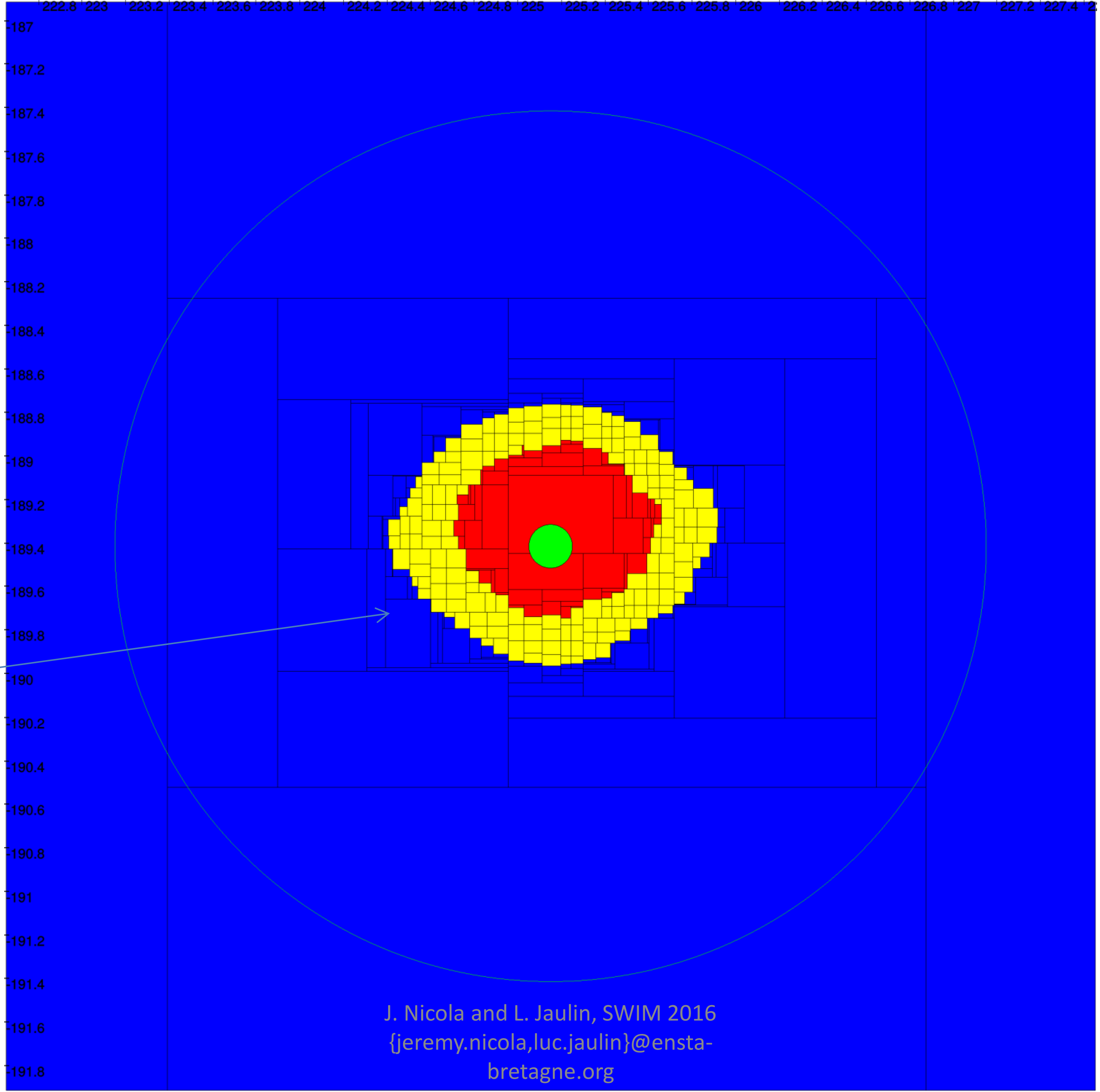
3.125% confidence region

J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



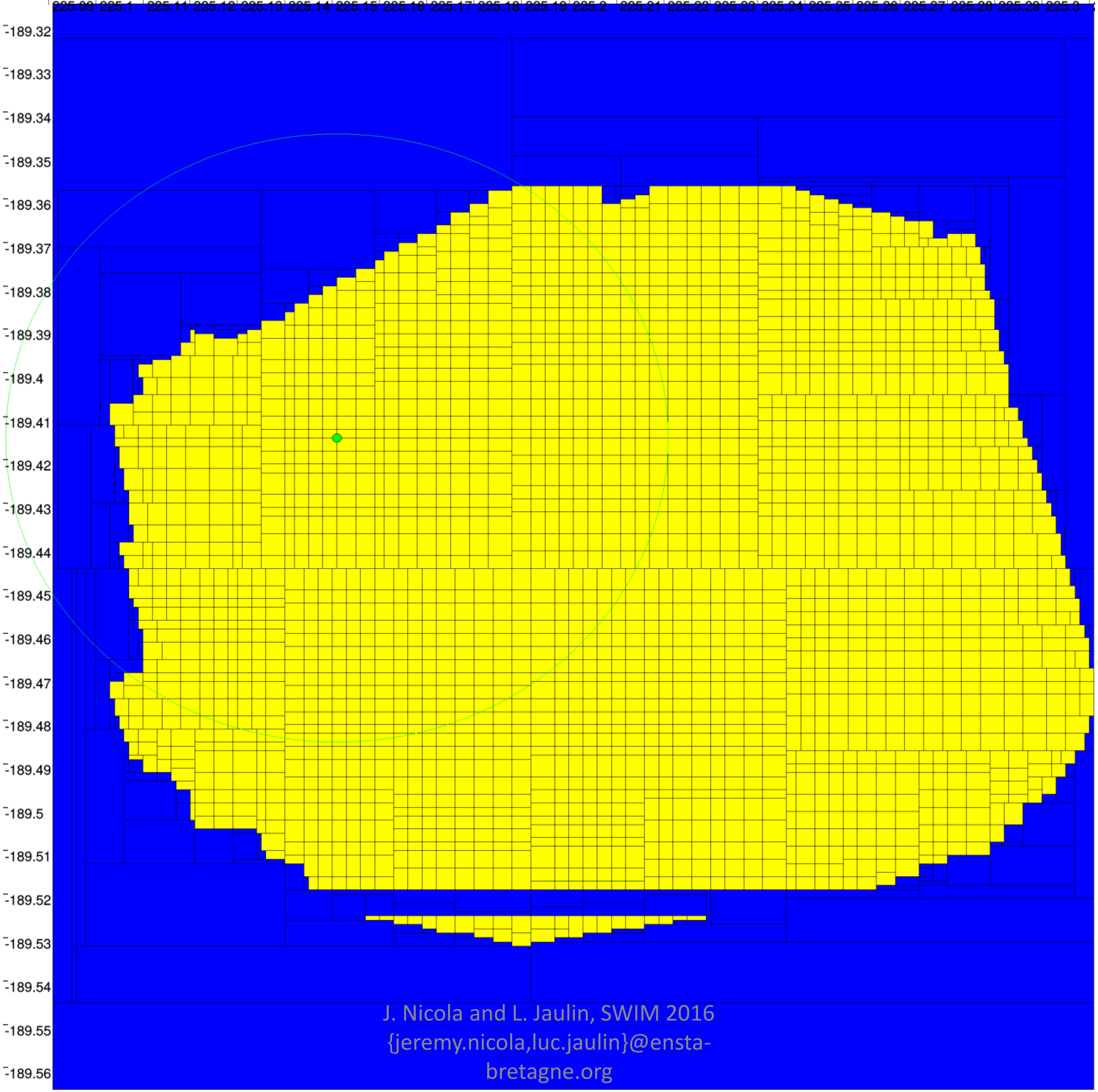
1.5625% confidence region

J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

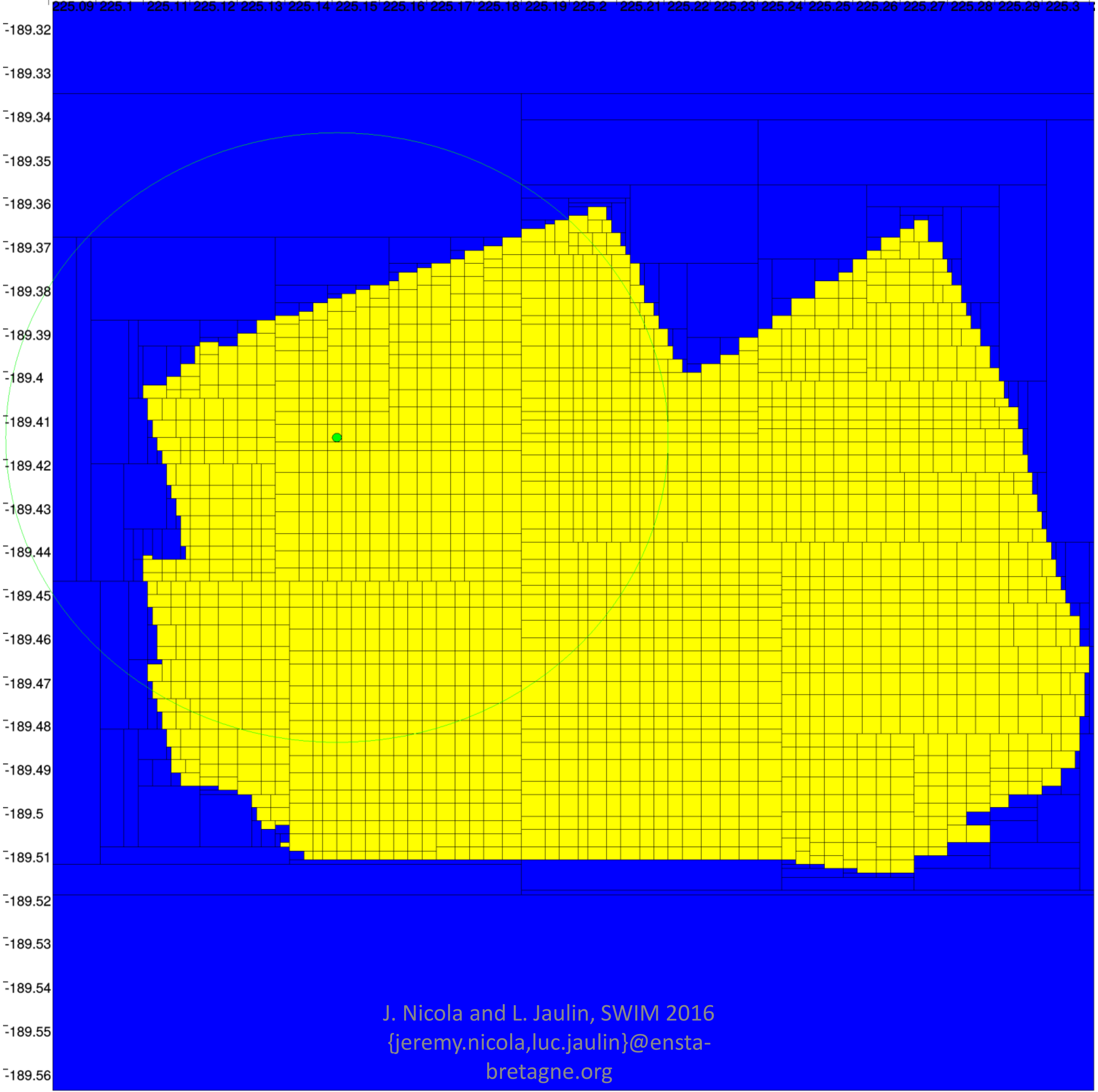


~0.8% confidence region

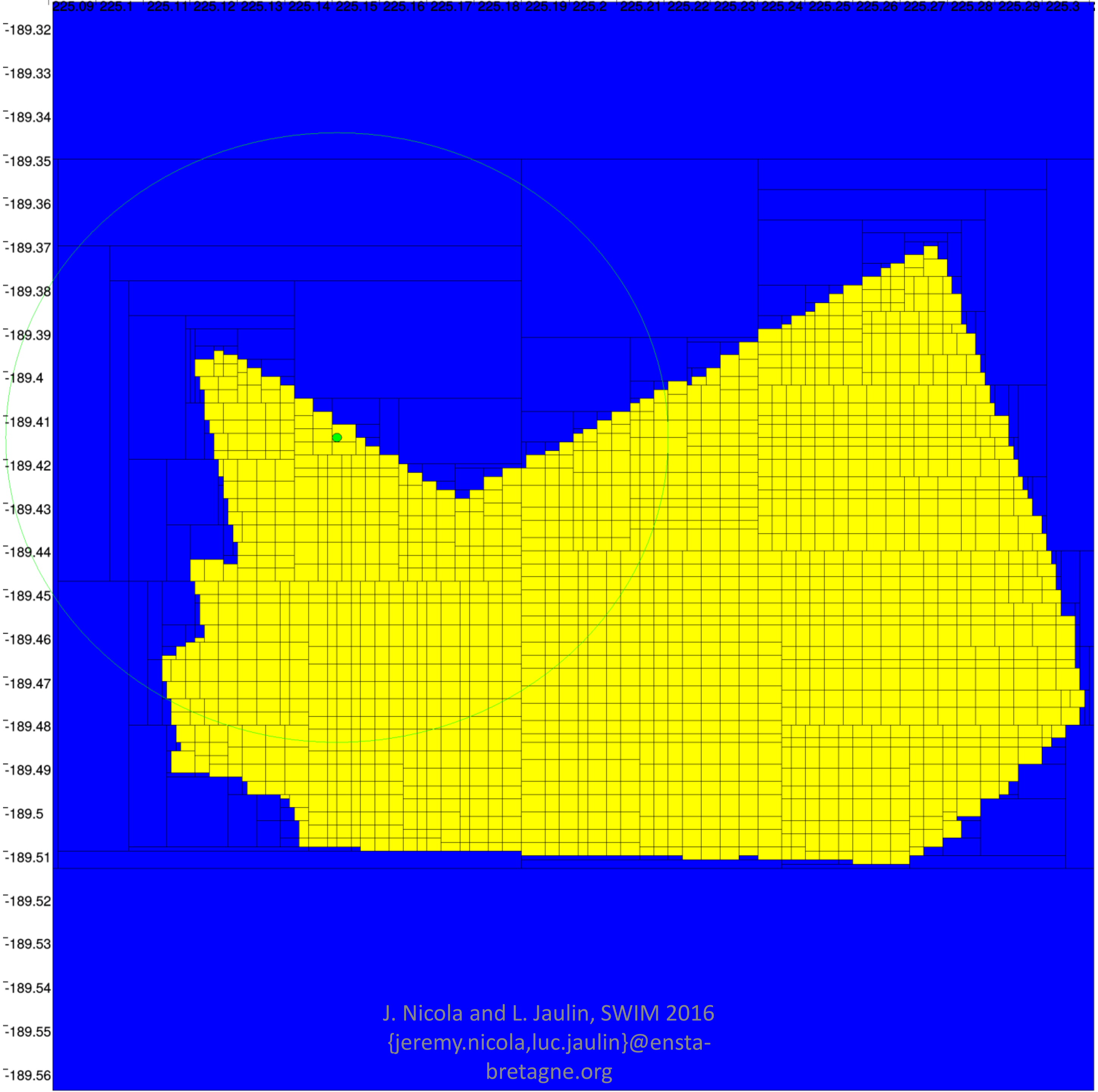
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



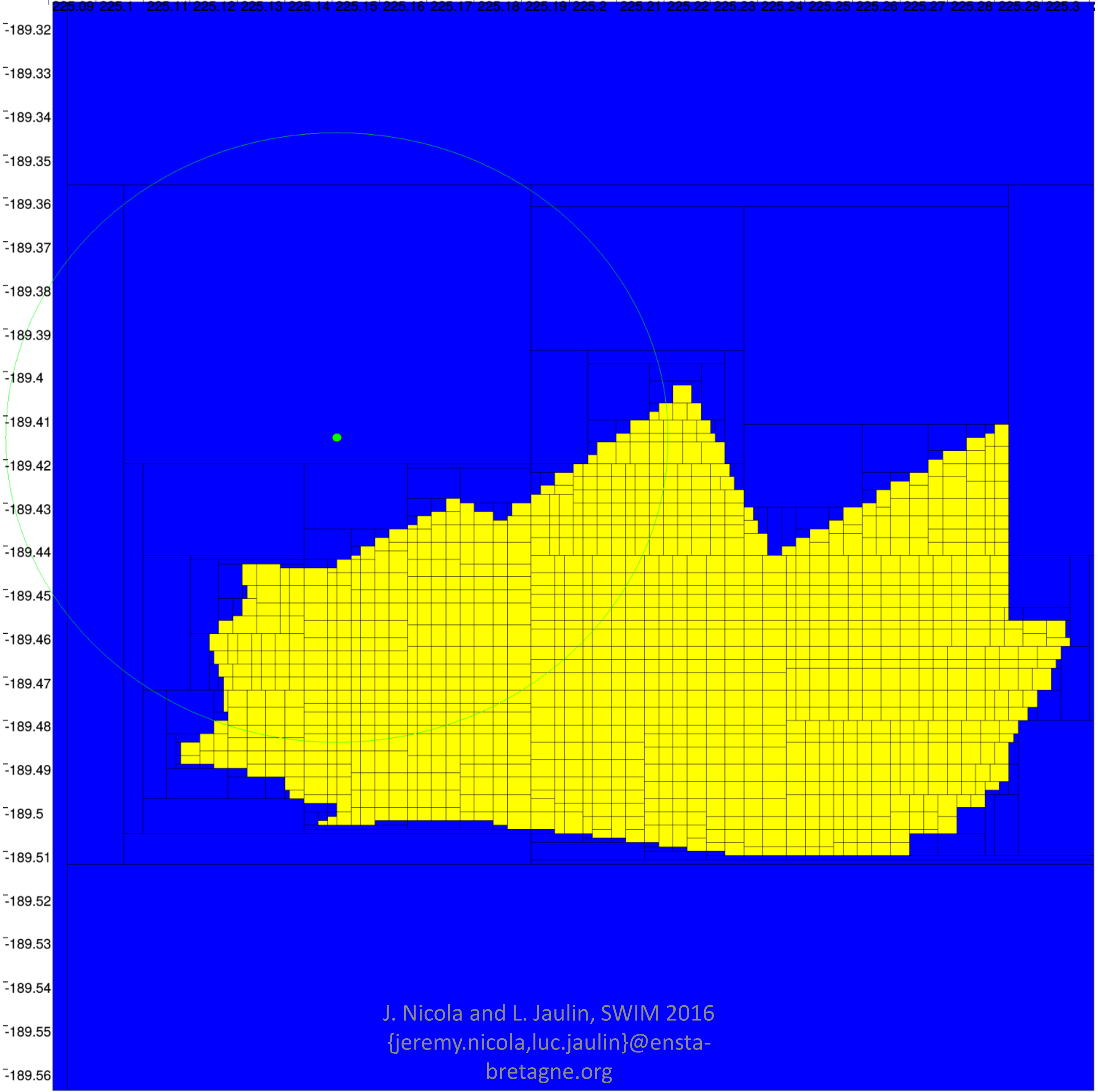
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org



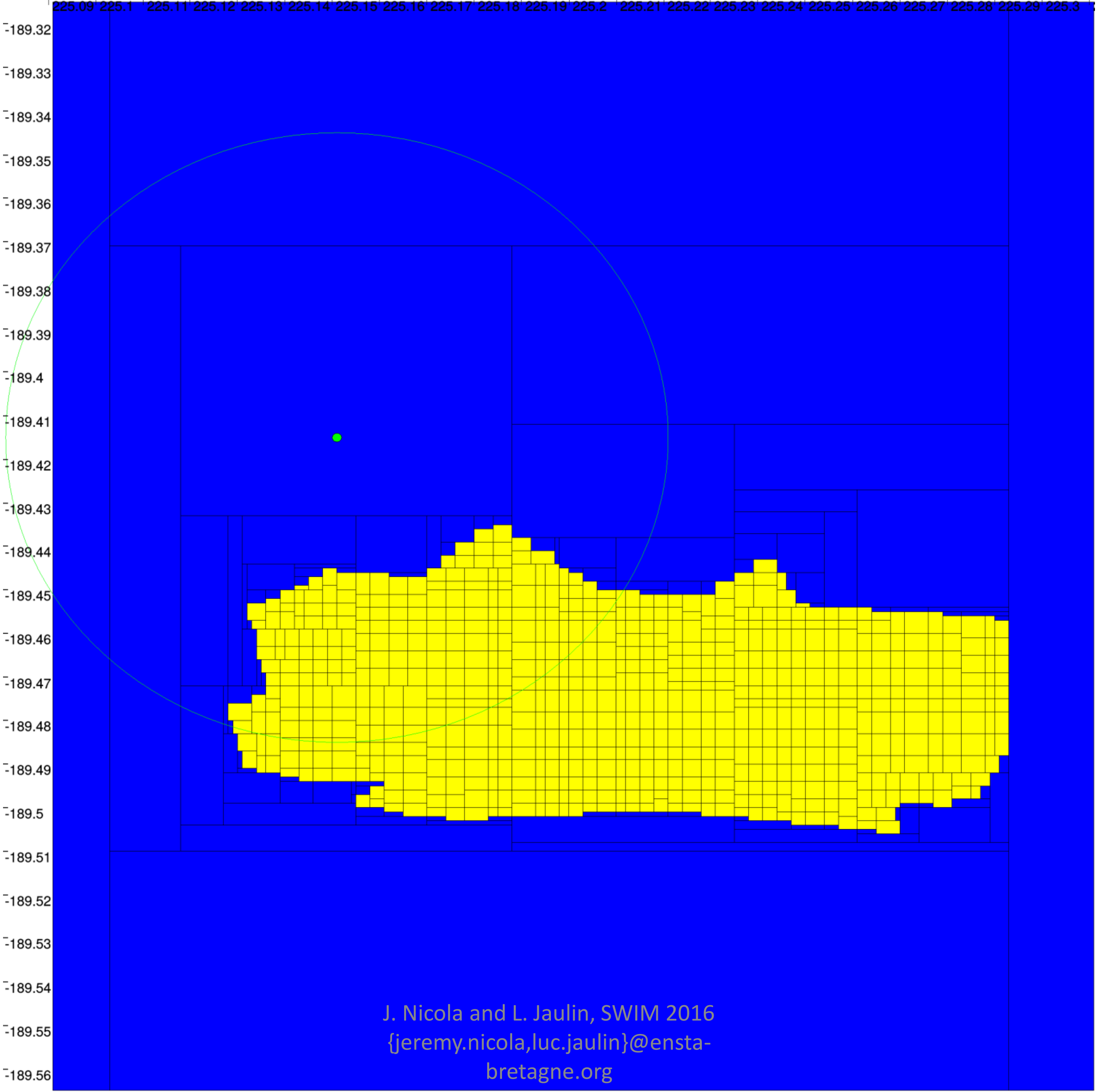
J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

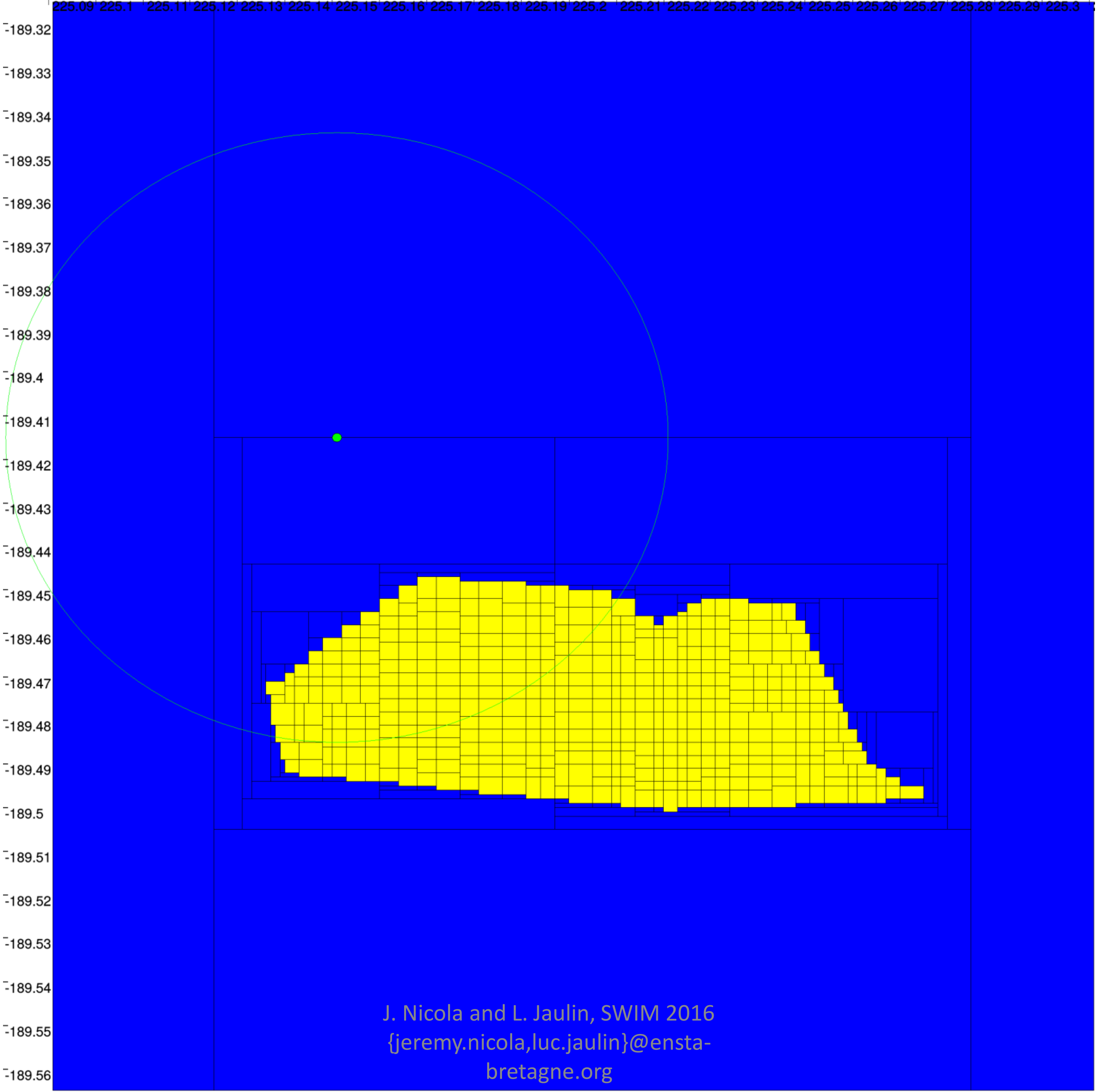


J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

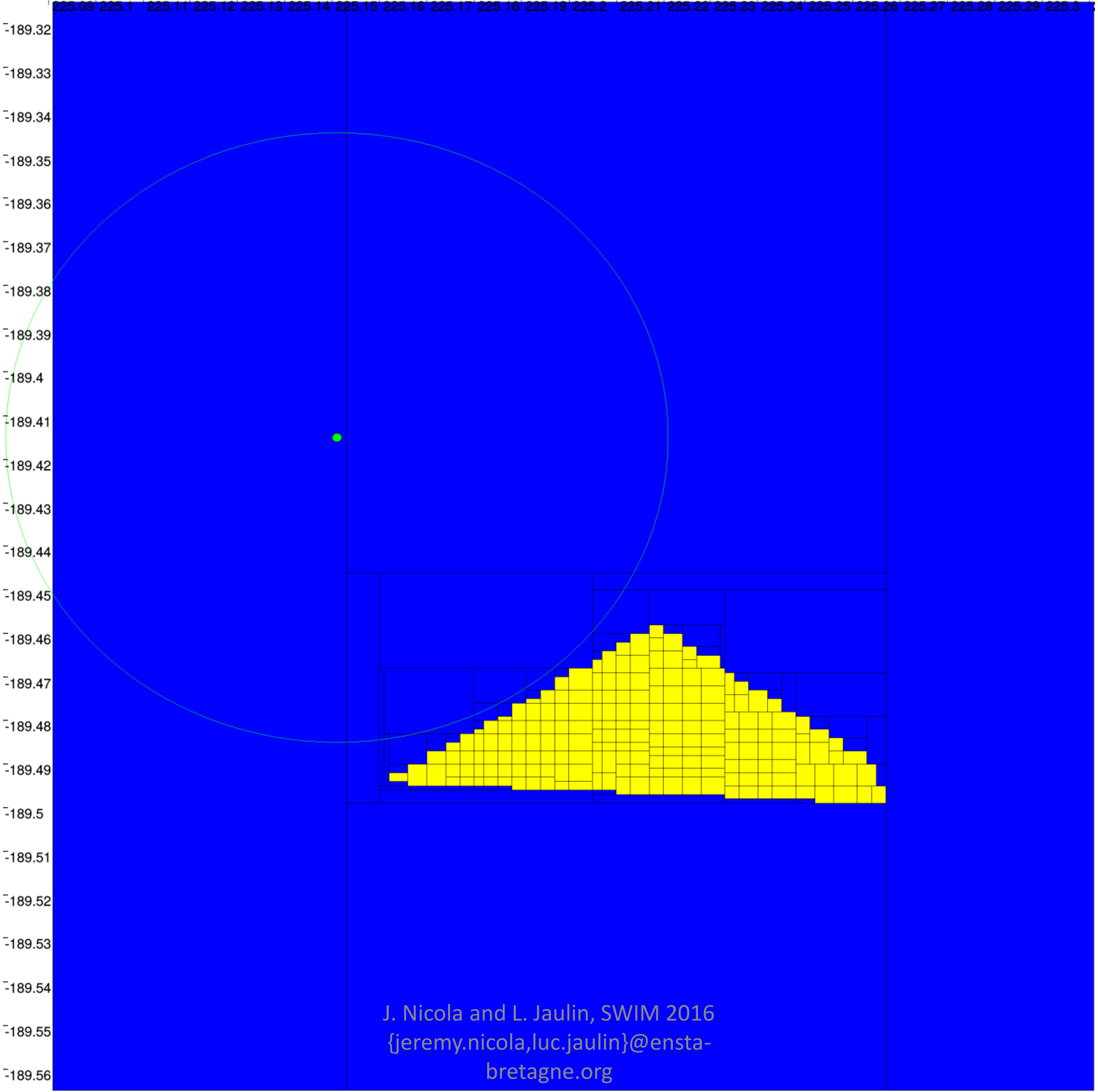


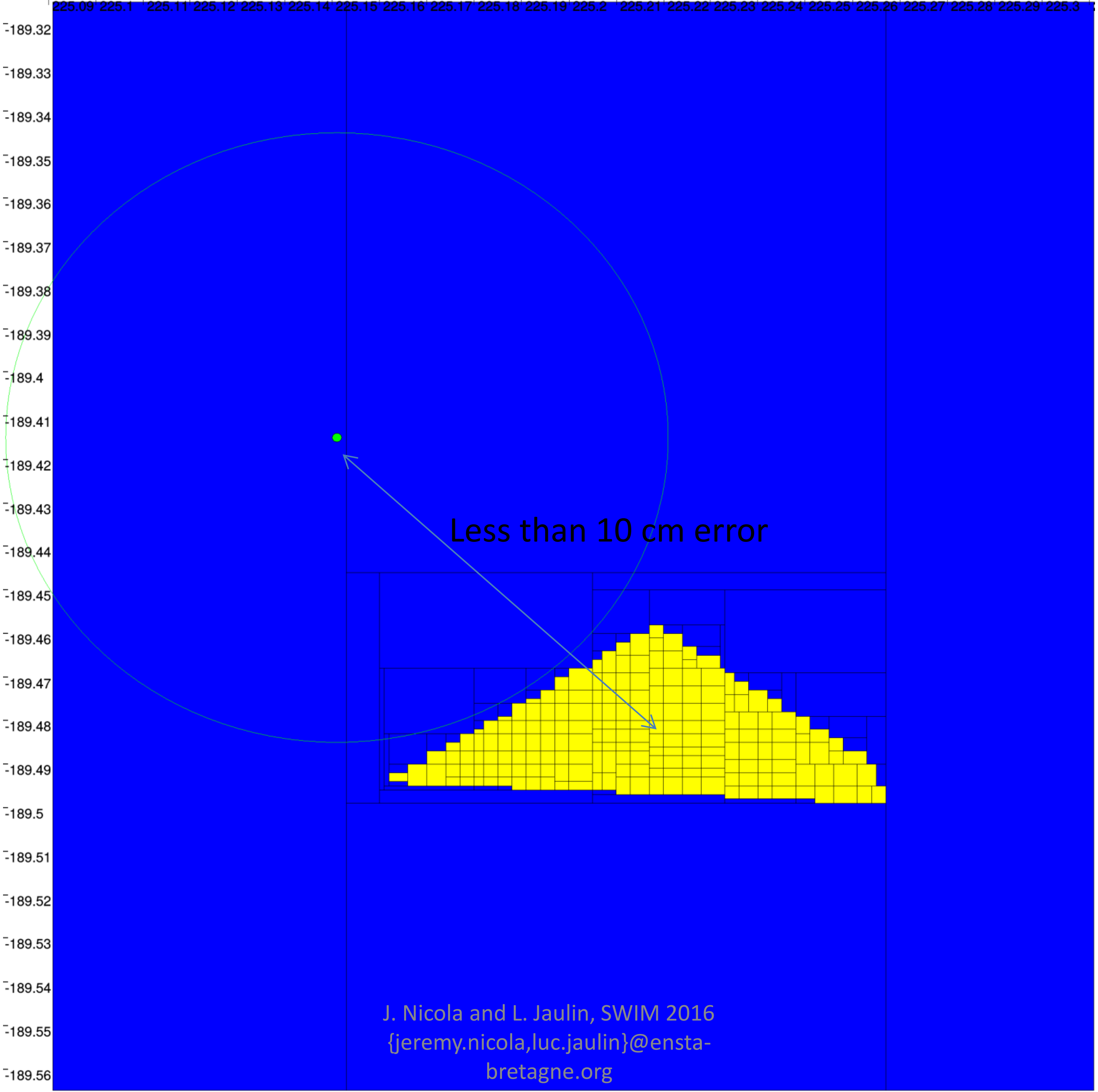
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org


225.09 225.1 225.11 225.12 225.13 225.14 225.15 225.16 225.17 225.18 225.19 225.2 225.21 225.22 225.23 225.24 225.25 225.26 225.27 225.28 225.29 225.3
-189.32
-189.33
-189.34
-189.35
-189.36
-189.37
-189.38
-189.39
-189.4
-189.41
-189.42
-189.43
-189.44
-189.45
-189.46
-189.47
-189.48
-189.49
-189.5
-189.51
-189.52
-189.53
-189.54
-189.55
-189.56



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-bretagne.org

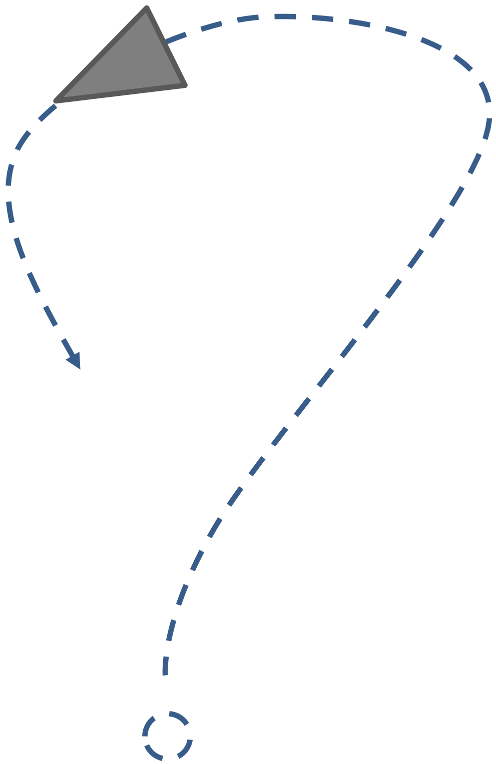

225.09 225.1 225.11 225.12 225.13 225.14 225.15 225.16 225.17 225.18 225.19 225.2 225.21 225.22 225.23 225.24 225.25 225.26 225.27 225.28 225.29 225.3
-189.32
-189.33
-189.34
-189.35
-189.36
-189.37
-189.38
-189.39
-189.4
-189.41
-189.42
-189.43
-189.44
-189.45
-189.46
-189.47
-189.48
-189.49
-189.5
-189.51
-189.52
-189.53
-189.54
-189.55
-189.56


Less than 10 cm error

- Context
- Problem
- Solution intuition
- Classical method
- Proposed method
- Application
- Conclusion

- If the measurement error is assumed to be distributed quasi-uniformly, OMNE / GOMNE is a Maximum Likelihood Estimator
- Maximum Likelihood Estimation in an interval framework enables us to compute estimates precise enough for industrial applications

- Probabilistic and set-membership methods don't have to be considered as different, incompatible representations of uncertainties
- Probabilistic uncertainties can be cast as geometrical constraints, and we can take advantage of them in a set-membership framework
  - Without approximations (reliability)
  - With fault tolerant tools (robustness)
  - With a precision suitable for industrial applications (precision)



J. Nicola and L. Jaulin, SWIM 2016
{jeremy.nicola,luc.jaulin}@ensta-
bretagne.org