Interval Methods for Robust Variable-Structure Control with One- and Two-Sided State Constraints

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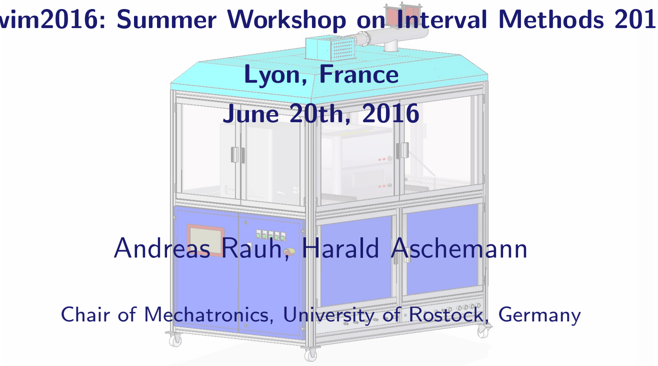

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Contents

- First- and second-order sliding mode control for systems in nonlinear controller canonical form (restricted to the single-input single-output case)
- Extensions to dynamic systems with interval uncertainty
- Consideration of one-sided state constraints (upper bounds on selected state/ output variables)
- Two-sided state constraints
- Simulation results
  - Control of an accelerated mass with non-negligible actuator dynamics
  - Control of the non-stationary thermal behavior of Solid Oxide Fuel Cell modules (with further input range and input rate constraints)
  - Control of an inverted pendulum
- Conclusions and outlook on future work

First-Order vs. Second-Order Sliding Mode Control (1)

System in nonlinear controller canonical form

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) & \ldots & \dot{x}_{n-1}(t) & \dot{x}_n(t) \end{bmatrix}^T$$
$$= \begin{bmatrix} x_2(t) & \ldots & x_n(t) & a\left(\mathbf{x}(t), \mathbf{p}\right) + b\left(\mathbf{x}(t), \mathbf{p}\right) \cdot v(t) \end{bmatrix}^T$$

with the state vector $\mathbf{x}(t) \in \mathbb{R}^n$

Requirement for controllability

$b\left(\mathbf{x}(t), \mathbf{p}\right) \neq 0$ for any possible operating point and system parameter

Feedback linearizing control law for the output $y(t) = x_1(t)$

$$v(t) = \frac{-a\left(\mathbf{x}(t), \mathbf{p}\right) + u(t)}{b\left(\mathbf{x}(t), \mathbf{p}\right)} \in \mathbb{R}$$

First-Order vs. Second-Order Sliding Mode Control (1)

System in nonlinear controller canonical form

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Feedback linearizing control law for the output $y(t) = x_1(t)$

$$v(t) = \frac{-a\left(\mathbf{x}(t), \mathbf{p}\right) + u(t)}{b\left(\mathbf{x}(t), \mathbf{p}\right)} \in \mathbb{R}$$

System becomes a pure integrator chain of length n for perfect system and state information (trivially differentially flat system)

First-Order vs. Second-Order Sliding Mode Control (2)

n-th order integrator chain model with the output $y(t) = x_1(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_{n-1}(t) \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ \vdots \\ x_n(t) \\ u(t) \end{bmatrix}$$

Definition of the tracking error and its r-th time derivative

$$\tilde{\xi}_1^{(r)}(t) = x_1^{(r)}(t) - x_{1,\mathrm{d}}^{(r)}(t) \qquad \text{with} \qquad r \in \{0, 1, \dots, n\}$$

First-order sliding mode (Hurwitz polynomial of order $n-1$)

$$s := s(t) = \sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r)}(t) \ , \quad \alpha_{n-1} = 1 \qquad \Longrightarrow \qquad s \to 0$$

First-Order vs. Second-Order Sliding Mode Control (2)

n-th order integrator chain model with the output $y(t) = x_1(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \vdots \\ \dot{x}_{n-1}(t) \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ \vdots \\ x_n(t) \\ u(t) \end{bmatrix}$$

Definition of the tracking error and its r-th time derivative

$$\tilde{\xi}_1^{(r)}(t) = x_1^{(r)}(t) - x_{1,\mathrm{d}}^{(r)}(t) \qquad \text{with} \qquad r \in \{0, 1, \ldots, n\}$$

Second-order sliding mode (integral component for $\alpha_{-1} \neq 0$)

$$\gamma_1 \dot{s} + \gamma_0 s = \alpha_{-1} \int_0^t \tilde{\xi}_1(\tau)\mathrm{d}\tau + \sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r)}(t) \;, \quad \begin{matrix} \gamma_0 > 0 \\ \gamma_1 > 0 \end{matrix} \quad \Longrightarrow \quad \begin{matrix} s \to 0 \\ \dot{s} \to 0 \end{matrix}$$

Derivation of the Control Law (1)

Lyapunov function candidate (first-order sliding mode)

$$V^{\langle \mathrm{I} \rangle} = \frac{1}{2} s^2 > 0 \quad \text{for} \quad s \neq 0$$

Lyapunov function candidate (second-order sliding mode)

$$V^{\langle \mathrm{II} \rangle} = \frac{1}{2} \cdot \left(s^2 + \lambda \dot{s}^2 \right) > 0 \quad \text{with the scaling factor} \quad \lambda > 0$$

Derivation of the Control Law (2)

Stability requirement (first-order sliding mode)

$$\dot{V}^{\langle \mathrm{I} \rangle} = s \cdot \dot{s} = \left(\sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r)}(t) \right) \cdot \left(\sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r+1)}(t) \right) < 0 \quad \text{for} \quad s \neq 0$$

Stability requirement (second-order sliding mode), $\lambda = \gamma_1 > 0$

$$\begin{aligned} \dot{V}^{\langle \mathrm{II} \rangle} &= s \cdot \dot{s} + \lambda \cdot \dot{s} \cdot \ddot{s} \\ &= s \cdot \dot{s} + \dot{s} \cdot \left(-\frac{\lambda \gamma_0}{\gamma_1} \dot{s} + \frac{\lambda}{\gamma_1} \sum_{r=0}^{n} \alpha_{r-1} \tilde{\xi}_1^{(r)}(t) \right) < 0 \\ & \qquad \text{for} \quad s \neq 0 \quad \text{and/ or} \quad \dot{s} \neq 0 \end{aligned}$$

Derivation of the Control Law (2)

Stability requirement (first-order sliding mode)

$$\dot{V}^{\langle \mathrm{I} \rangle} = s \cdot \dot{s} = \left(\sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r)}(t) \right) \cdot \left(\sum_{r=0}^{n-1} \alpha_r \tilde{\xi}_1^{(r+1)}(t) \right) < -\eta \cdot |s|$$

Stability requirement (second-order sliding mode), $\lambda = \gamma_1 > 0$

$$\dot{V}^{\langle \mathrm{II} \rangle} = s \cdot \dot{s} + \dot{s} \cdot \left(-\gamma_0 \dot{s} + \sum_{r=0}^{n-1} \alpha_{r-1} \tilde{\xi}_1^{(r)}(t) + \alpha_{n-1} \cdot \left(u(t) - x_{1,\mathrm{d}}^{(n)}(t) \right) \right) < 0$$

Derivation of the Control Law (2)

Stability requirement (first-order sliding mode)

$$\left(\sum_{r=0}^{n-1}\alpha_r\tilde{\xi}_1^{(r)}(t)\right)\cdot\left(\sum_{r=0}^{n-1}\alpha_r\tilde{\xi}_1^{(r+1)}(t)\right) < -\eta\cdot\left(\sum_{r=0}^{n-1}\alpha_r\tilde{\xi}_1^{(r)}(t)\right)\cdot\operatorname{sign}(s)$$

Stability requirement (second-order sliding mode), $\lambda = \gamma_1 > 0$

$$\dot{V}^{\langle \mathrm{II}\rangle} < -\eta_1\cdot|\dot{s}| - \eta_2\cdot|s|\cdot|\dot{s}| = -\dot{s}\cdot\operatorname{sign}(\dot{s})\cdot(\eta_1+\eta_2\cdot|s|)$$

Derivation of the Control Law (3)

Control law (first-order sliding mode)

$$u(t) = u^{\langle \mathrm{I} \rangle}(t) = x_{1,\mathrm{d}}^{(n)}(t) - \sum_{r=0}^{n-2} \alpha_r \tilde{\xi}_1^{(r+1)}(t) - \tilde{\eta} \cdot \mathrm{sign}\,(s)$$

Questions

- What are necessary extensions for the interval case?
- What are the implementation requirements for an *interval-valued control signal*?
- Why/ How to generalize *preferably* the first-order case?

Derivation of the Control Law (3)

Control law (second-order sliding mode)

$$u(t) = u^{\langle \mathrm{II} \rangle}(t) = x_{1,\mathrm{d}}^{(n)}(t)$$
$$+ \frac{1}{\alpha_{n-1}} \cdot \left(\gamma_0 \dot{s} - s - \sum_{r=0}^{n-1} \alpha_{r-1} \tilde{\xi}_1^{(r)}(t) - \mathrm{sign}\,(\dot{s}) \cdot (\tilde{\eta}_1 + \tilde{\eta}_2 \cdot |s|) \right)$$

Questions

- What are necessary extensions for the interval case?
- What are the implementation requirements for an *interval-valued control signal*?
- Why/ How to generalize *preferably* the first-order case?

Interval-Based Sliding Mode Control (1)

Definition of tracking error signals and sliding surface

- Specification of a sufficiently smooth desired output trajectory $y_{\mathrm{d}} = x_{1,\mathrm{d}}$
- Interval definition of the tracking error and its derivatives

$$\tilde{\xi}_1^{(r)} \in \left[\tilde{\xi}_1^{(r)}\right] = \left[x_1^{(r)}\right] - x_{1,\mathrm{d}}^{(r)} , \quad r \in \{0, 1, \ldots, n\}$$

- As before: Desired operating points are located on the *sliding surface*

$$s := \tilde{\xi}_1^{(n-1)}(t) + \sum_{r=0}^{n-2} \alpha_r \cdot \tilde{\xi}_1^{(r)}(t) = 0$$

- $\alpha_0, \ldots, \alpha_{n-2}$ are coefficients of a Hurwitz polynomial of order $n-1$

Interval-Based Sliding Mode Control (1)

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- As before: Desired operating points are located on the *sliding surface*

$$s := \tilde{\xi}_1^{(n-1)}(t) + \sum_{r=0}^{n-2} \alpha_r \cdot \tilde{\xi}_1^{(r)}(t) = 0$$

Guaranteed stabilizing control: Lyapunov function candidate

$$V = \frac{1}{2}s^2 > 0 \quad \text{with} \quad \dot{V} = s \cdot \dot{s} < 0 \quad \text{for} \quad s \neq 0$$

Interval-Based Sliding Mode Control (2)

Guaranteed stabilization despite uncertainty: Interval formulation of a variable-structure control law

$$\left[v^{\langle \mathrm{I} \rangle}\right] := \frac{-a\left([\mathbf{x}], [\mathbf{p}]\right) + x_{1,\mathrm{d}}^{(n)} - \sum_{r=0}^{n-2} \alpha_r \cdot \left[\tilde{\xi}_1^{(r+1)}\right] - \tilde{\eta} \cdot \mathrm{sign}\left([s]\right)}{b\left([\mathbf{x}], [\mathbf{p}]\right)}$$

with a suitably chosen parameter $\tilde{\eta} > 0$ and $0 \notin b\left([\mathbf{x}], [\mathbf{p}]\right)$

Guaranteed stabilizing control: Extraction of suitable point values

$$\mathcal{V} := \{\underline{v} - \epsilon, \underline{v} + \epsilon, \overline{v} - \epsilon, \overline{v} + \epsilon\}$$

with $\underline{v} := \inf\{[v]\}$, $\overline{v} := \sup\{[v]\}$ and some small $\epsilon > 0 \Longrightarrow \dot{V} < 0$ needs to be satisfied with certainty

Interval-Based Sliding Mode Control (2)

Guaranteed stabilization despite uncertainty: Interval formulation of a variable-structure control law

$$\left[v^{\langle \mathrm{I} \rangle}\right] := \frac{-a\left([\mathbf{x}],[\mathbf{p}]\right) + x_{1,\mathrm{d}}^{(n)} - \sum\limits_{r=0}^{n-2} \alpha_r \cdot \left[\tilde{\xi}_1^{(r+1)}\right] - \tilde{\eta} \cdot \mathrm{sign}\left([s]\right)}{b\left([\mathbf{x}],[\mathbf{p}]\right)}$$

with a suitably chosen parameter $\tilde{\eta} > 0$ and $0 \notin b\left([\mathbf{x}],[\mathbf{p}]\right)$

Guaranteed stabilizing control: Extraction of suitable point values

- Guaranteed stabilization of system dynamics
- Extension: Guaranteed state constraints in terms of strict one- and two-sided barrier functions
- Inclusion of bounds on input variation rates (reduction of the effect of chattering)

Sliding Mode Control with One-Sided State Constraints (1)

Specification of an upper state constraint

$$x_1 < \bar{x}_{1,\max} := x_{1,\mathrm{d}} + \Delta x_{1,\max} \quad \text{with} \quad \Delta x_{1,\max} > 0$$

Extension of the Lyapunov function candidate by a one-sided barrier function (repelling potential)

Extended ansatz for a Lyapunov function candidate

$$V^{\langle j,\mathrm{A}\rangle} = V^{\langle j\rangle} + V^{\langle \mathrm{A}\rangle} > 0 \quad \text{for} \quad s \neq 0 \quad \text{with}$$

$$V^{\langle \mathrm{A}\rangle} = \rho_{\mathrm{V}} \cdot \ln\left(\frac{\sigma_{\mathrm{V}} \cdot \bar{x}_{1,\max}}{\bar{x}_{1,\max} - x_1}\right) \text{ and } x_1 < \bar{x}_{1,\max} \text{ for both } j \in \{\mathrm{I}, \mathrm{II}\}$$

Sliding Mode Control with One-Sided State Constraints (1)

Specification of an upper state constraint

$$x_1 < \bar{x}_{1,\max} := x_{1,\mathrm{d}} + \Delta x_{1,\max} \quad \text{with} \quad \Delta x_{1,\max} > 0$$

Computation of the corresponding time derivative

Extended ansatz for a Lyapunov function candidate

$$\dot{V}^{\langle j,\mathrm{A}\rangle} = \dot{V}^{\langle j\rangle} + \dot{V}^{\langle \mathrm{A}\rangle} < 0 \quad \text{with}$$

$$\dot{V}^{\langle \mathrm{A}\rangle} = \frac{\rho_{\mathrm{V}}}{\bar{x}_{1,\max}} \cdot \left(\frac{-x_1 \cdot \dot{\bar{x}}_{1,\max} + \dot{x}_1 \cdot \bar{x}_{1,\max}}{\bar{x}_{1,\max} - x_1} \right) , \quad \rho_{\mathrm{V}} > 0 , \quad \sigma_{\mathrm{V}} > 0$$

Note

Dominating influence of $\dot{V}^{\langle j\rangle}$ in the neighborhood of $s = 0$ must be given

Sliding Mode Control with One-Sided State Constraints (1)

Specification of an upper state constraint

$$x_1 < \bar{x}_{1,\max} := x_{1,\mathrm{d}} + \Delta x_{1,\max} \quad \text{with} \quad \Delta x_{1,\max} > 0$$

Modified stability requirement (first-order case)

$$s \cdot \underbrace{\left(\sum_{r=0}^{n-2} \alpha_r \tilde{\xi}_1^{(r+1)} + u - x_{1,\mathrm{d}}^{(n)} + \eta \cdot \mathrm{sign}\,(s) + \frac{1}{s} \cdot \dot{V}^{\langle \mathrm{A} \rangle} \right)}_{-\beta \cdot \mathrm{sign}(s)} < 0$$

Sliding Mode Control with One-Sided State Constraints (2)

New control signal for the first-order case

$$u = u^{\langle \mathrm{I,A} \rangle} = u^{\langle \mathrm{I} \rangle} - \frac{s}{s^2 + \tilde{\epsilon}} \cdot \dot{V}^{\langle \mathrm{A} \rangle}$$

New control signal for the second-order case

$$u = u^{\langle \mathrm{II,A} \rangle} = u^{\langle \mathrm{II} \rangle} - \frac{1}{\alpha_{n-1}} \cdot \frac{\dot{s}}{\dot{s}^2 + \tilde{\epsilon}} \cdot \dot{V}^{\langle \mathrm{A} \rangle}$$

Note

- The approximations $\frac{1}{s} \approx \frac{s}{s^2+\tilde{\epsilon}}$ and $\frac{1}{\dot{s}} \approx \frac{\dot{s}}{\dot{s}^2+\tilde{\epsilon}}$ are only necessary for $|s| \gg 0$ and $|\dot{s}| \gg 0$.
- The variable-structure part is deactivated in a close vicinity of $|s| = 0$ and $|\dot{s}| = 0$ in the following interval case. For $0 \in [s]$, $0 \in [\dot{s}]$, the sign of s and $\dot{s}$ cannot be determined unambiguously.

Sliding Mode Control with Two-Sided State Constraints (1)

Specification of worst-case state deviations

$$|x_1 - x_{1,\mathrm{d}}| \geq \bar{\chi} > 0 \ , \quad \bar{\chi} = \mathrm{const}$$

Extension of the Lyapunov function candidate by a two-sided barrier function

Extended ansatz for a Lyapunov function candidate

$$V^{\langle j,\mathrm{B}\rangle} = V^{\langle j\rangle} + V^{\langle \mathrm{B}\rangle} > 0 \quad \text{for} \quad s \neq 0 \quad \text{with}$$

$$V^{\langle \mathrm{B}\rangle} = \rho_{\mathrm{V}} \cdot \ln\left(\frac{\bar{\chi}^{2l}}{\bar{\chi}^{2l} - (x_1 - x_{1,\mathrm{d}})^{2l}}\right) \text{ and } l \in \mathbb{N} \text{ for both } j \in \{\mathrm{I}, \mathrm{II}\}$$

Sliding Mode Control with Two-Sided State Constraints (1)

Specification of worst-case state deviations

$$|x_1 - x_{1,\mathrm{d}}| \geq \bar{\chi} > 0 \ , \quad \bar{\chi} = \mathrm{const}$$

Computation of the corresponding time derivative

Extended ansatz for a Lyapunov function candidate

$$\dot{V}^{\langle j,\mathrm{B}\rangle} = \dot{V}^{\langle j\rangle} + \dot{V}^{\langle \mathrm{B}\rangle} < 0 \quad \text{with}$$

$$\dot{V}^{\langle \mathrm{B}\rangle} = \rho_{\mathrm{V}} \cdot \frac{2l \cdot \left(x_1 - x_{1,\mathrm{d}}\right)^{2l-1} \cdot \left(\dot{x}_1 - \dot{x}_{1,\mathrm{d}}\right)}{\bar{\chi}^{2l} - \left(x_1 - x_{1,\mathrm{d}}\right)^{2l}} \ , \quad \rho_{\mathrm{V}} > 0$$

Note

Dominating influence of $\dot{V}^{\langle j\rangle}$ in the neighborhood of $s = 0$ must be given

Sliding Mode Control with Two-Sided State Constraints (2)

New control signal for the first-order case

$$u^{\langle \mathrm{I,B} \rangle} = u^{\langle \mathrm{I} \rangle} - s^{-1} \cdot \dot{V}^{\langle \mathrm{B} \rangle}$$

New control signal for the second-order case

$$u^{\langle \mathrm{II,B} \rangle} = u^{\langle \mathrm{II} \rangle} - (\alpha_{n-1} \cdot \dot{s})^{-1} \cdot \dot{V}^{\langle \mathrm{B} \rangle}$$

Note

The same approximations $\frac{1}{s} \approx \frac{s}{s^2+\tilde{\epsilon}}$ and $\frac{1}{\dot{s}} \approx \frac{\dot{s}}{\dot{s}^2+\tilde{\epsilon}}$ are necessary as before for $|s| \gg 0$ and $|\dot{s}| \gg 0$.

Interval-Based Sliding Mode Control (continued)

Guaranteed stabilization despite uncertainty: Interval formulation of a variable-structure control law

$$\left[v^{\langle \mathrm{I}\rangle}\right] := \frac{-a\left([\mathbf{x}],[\mathbf{p}]\right) + x_{1,\mathrm{d}}^{(n)} - \sum\limits_{r=0}^{n-2} \alpha_r \cdot \left[\tilde{\xi}_1^{(r+1)}\right] - \tilde{\eta}\cdot \mathrm{sign}\left([s]\right)}{b\left([\mathbf{x}],[\mathbf{p}]\right)}$$

Extension in the case of one-sided state constraints

$$\left[v^{\langle \mathrm{I,A}\rangle}\right] = \left[v^{\langle \mathrm{I}\rangle}\right] - \frac{1}{b\left([\mathbf{x}],[\mathbf{p}]\right)} \cdot \frac{[s]}{[s]^2 + \tilde{\epsilon}} \cdot \left[\dot{V}^{\langle \mathrm{A}\rangle}\right]$$

Interval-Based Sliding Mode Control (continued)

Guaranteed stabilization despite uncertainty: Interval formulation of a variable-structure control law

$$\left[v^{\langle \mathrm{I} \rangle}\right] := \frac{-a\left([\mathbf{x}],[\mathbf{p}]\right) + x_{1,\mathrm{d}}^{(n)} - \sum_{r=0}^{n-2} \alpha_r \cdot \left[\tilde{\xi}_1^{(r+1)}\right] - \tilde{\eta} \cdot \mathrm{sign}\left([s]\right)}{b\left([\mathbf{x}],[\mathbf{p}]\right)}$$

Extension in the case of one-sided state constraints

$$\left[v^{\langle \mathrm{I,A} \rangle}\right] = \left[v^{\langle \mathrm{I} \rangle}\right] - \frac{1}{b\left([\mathbf{x}],[\mathbf{p}]\right)} \cdot \frac{[s]}{[s]^2 + \tilde{\epsilon}} \cdot \left[\dot{V}^{\langle \mathrm{A} \rangle}\right]$$

Extension in the case of two-sided state constraints

$$\left[v^{\langle \mathrm{I,B} \rangle}\right] = \left[v^{\langle \mathrm{I} \rangle}\right] - \frac{1}{b\left([\mathbf{x}],[\mathbf{p}]\right)} \cdot \frac{[s]}{[s]^2 + \tilde{\epsilon}} \cdot \left[\dot{V}^{\langle \mathrm{B} \rangle}\right]$$

Velocity Control of a Point Mass

System model

- Position: x_1
- Velocity: x_2
- Input force: x_3 (mass normalized to 1)

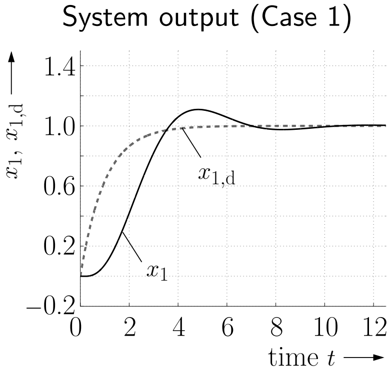
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ p_1 x_1 + p_2 x_2 + p_3 x_3 + p_4 v \end{bmatrix}$$

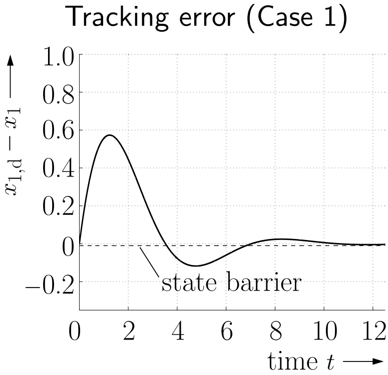
with the uncertain parameters $p_i \in [-0.1\ ;\ 0.1]$, $i \in \{1, 2, 3\}$, and $p_4 = 1$, containing both asymptotically stable and unstable realizations

Control Parameterizations

	Lyapunov function V	Barrier	System parameters	Measurement tolerance for x_1	Variable-structure gain
Case 1	$V^{\langle \mathrm{I} \rangle}$, $\alpha_0 = 1$, $\alpha_1 = 0.9$	—	$p_i = 0$, $i = \{1, 2, 3\}$, $p_4 = 1$	—	$\tilde{\eta} = 20$
Case 2	$V^{\langle \mathrm{I,A} \rangle}$, $\alpha_0 = 1$, $\alpha_1 = 0.9$	$\rho_{\mathrm{V}} = 0.5$, $\sigma_{\mathrm{V}} = 1$, $\Delta x_{1,\max} = 0.01$	$p_i = 0$, $i = \{1, 2, 3\}$, $p_4 = 1$	—	$\tilde{\eta} = 20$
Case 3	$V^{\langle \mathrm{I,A} \rangle}$, $\alpha_0 = 15$, $\alpha_1 = 0.9$	$\rho_{\mathrm{V}} = 0.75$, $\sigma_{\mathrm{V}} = 1$, $\Delta x_{1,\max} = 0.01$	$p_i \in [-0.1 \ ; \ 0.1]$, $i = \{1, 2, 3\}$, $p_4 = 1$	$0.0025 \cdot [-1; 1]$	$\tilde{\eta} = 20$
Case 4	$V^{\langle \mathrm{I} \rangle}$, $\alpha_0 = 15$, $\alpha_1 = 0.9$	—	$p_i \in [-0.1 \ ; \ 0.1]$, $i = \{1, 2, 3\}$, $p_4 = 1$	$0.0025 \cdot [-1; 1]$	$\tilde{\eta} = 400$
Case 5	$V^{\langle \mathrm{I,B} \rangle}$, $\alpha_0 = 15$, $\alpha_1 = 0.9$	$\rho_{\mathrm{V}} = 5$, $l = 1$, $\bar{\chi} = 0.165$	$p_i \in [-0.1 \ ; \ 0.1]$, $i = \{1, 2, 3\}$, $p_4 = 1$	$0.0025 \cdot [-1; 1]$	$\tilde{\eta} = 400$

Simulation Results

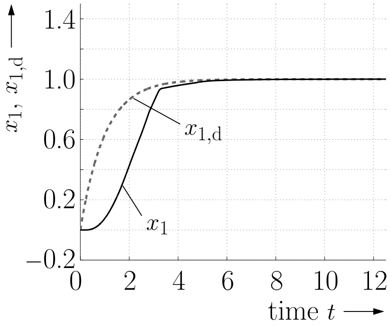




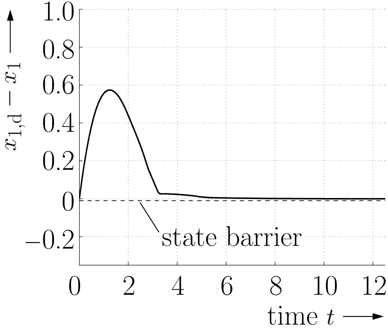
Violation of one-sided state constraint $\Longrightarrow$ Barrier function is **deactivated**, parameters are assumed to be **exactly known**

Simulation Results

System output (Case 2)

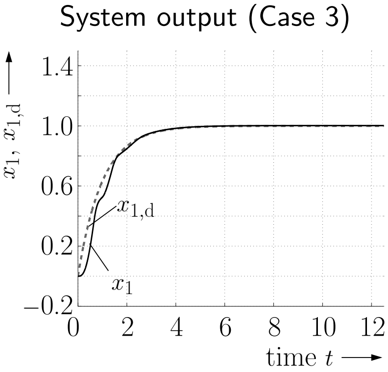


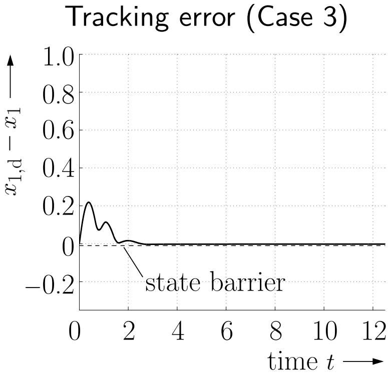
Tracking error (Case 2)



No violation of one-sided state constraint $\Longrightarrow$ Barrier function is **activated**, parameters are assumed to be **exactly known**

Simulation Results

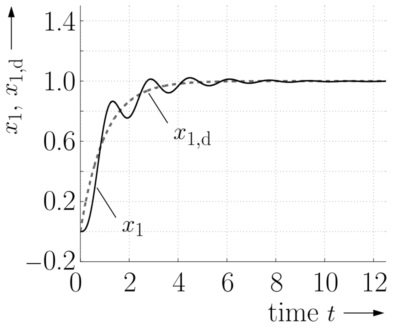




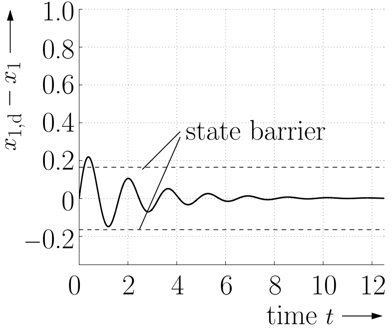
No violation of one-sided state constraint $\Longrightarrow$ Barrier function is **activated**, parameters and measured states are **uncertain**

Simulation Results

System output (Case 4)

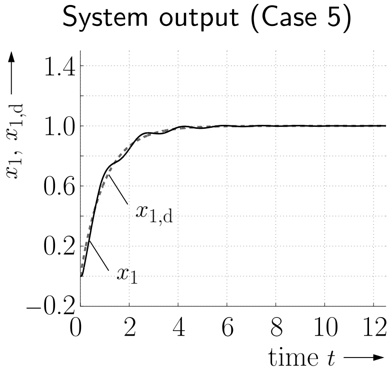


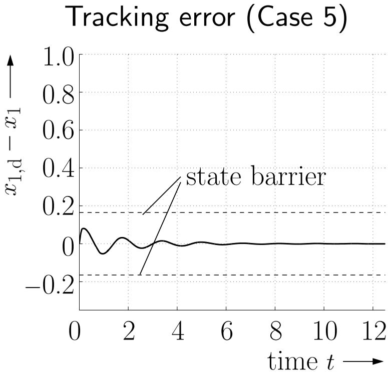
Tracking error (Case 4)



Violation of two-sided state constraint $\Longrightarrow$ Barrier function is **deactivated**, parameters and measured states are **uncertain**

Simulation Results



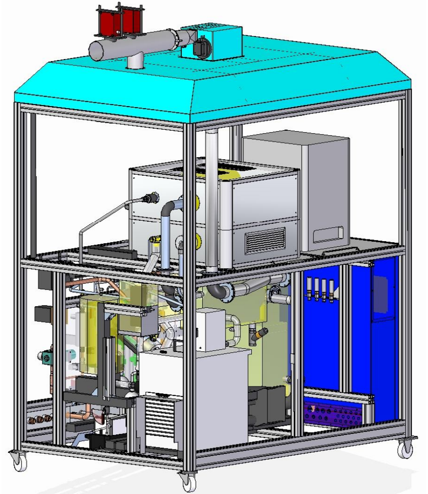


No violation of two-sided state constraint $\Longrightarrow$ Barrier function is **activated**, parameters and measured states are **uncertain**

Control-Oriented Modeling of SOFC Systems (1)

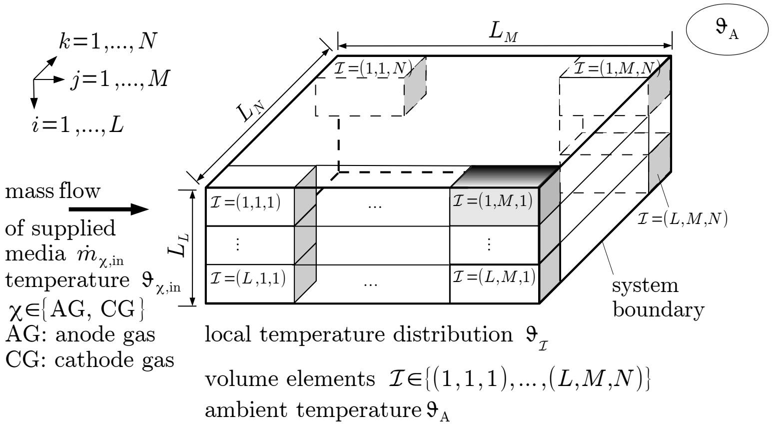
Configuration of the SOFC test rig at the Chair of Mechatronics

- Supply of fuel gas (hydrogen and/or mixture of methane, carbon monoxide, water vapor)
- Supply of air
- Independent preheaters for fuel gas and air
- Stack module containing fuel cells in electric series connection
- Electric load as disturbance



Control-Oriented Modeling of SOFC Systems (2)

Spatial semi-discretization of the fuel cell stack module

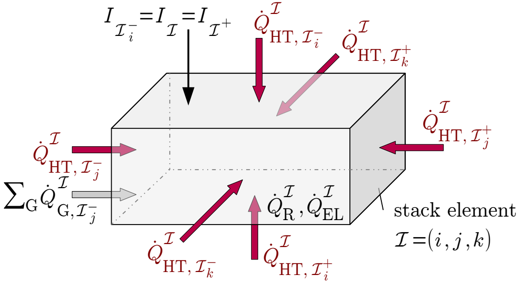


Control-Oriented Modeling of SOFC Systems (3)

Mathematical representation of the piecewise homogeneous temperature distribution $\Longrightarrow$ spatial semi-discretization

$$\dot{\vartheta}_{\mathcal{I}}(t) = \frac{1}{c_{\mathcal{I}} m_{\mathcal{I}}} \left(\dot{Q}^{\mathcal{I}}_{\mathrm{HT}}(t) + \sum_{\mathrm{G}\in\{\mathrm{AG},\mathrm{CG}\}} \dot{Q}^{\mathcal{I}}_{\mathrm{G},\mathcal{I}_j^-}(t) + \dot{Q}^{\mathcal{I}}_{\mathrm{R}}(t) + \dot{Q}^{\mathcal{I}}_{\mathrm{EL}}(t) \right)$$

1. HT: Heat transfer (heat conduction and convection)
2. G: Enthalpy flows of supplied gases
3. R: Exothermic reaction enthalpy
4. EL: Ohmic losses



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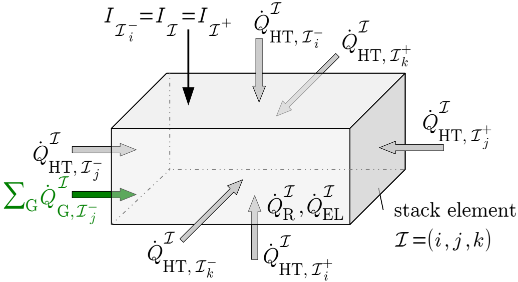

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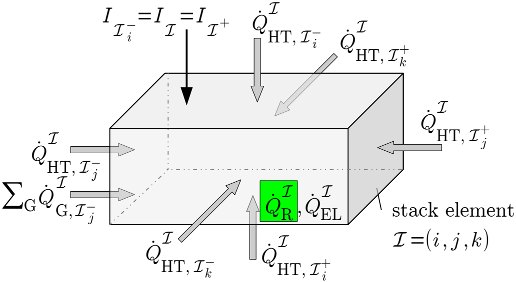

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Mathematical representation of the piecewise homogeneous temperature distribution $\Longrightarrow$ spatial semi-discretization

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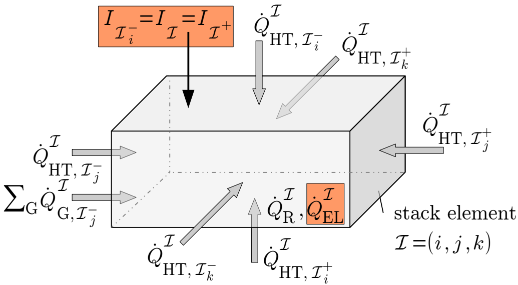
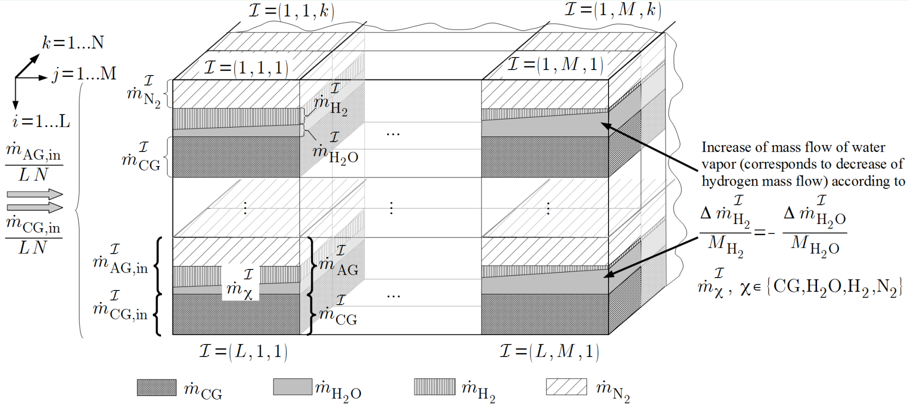


Control-Oriented Modeling of SOFC Systems (4)

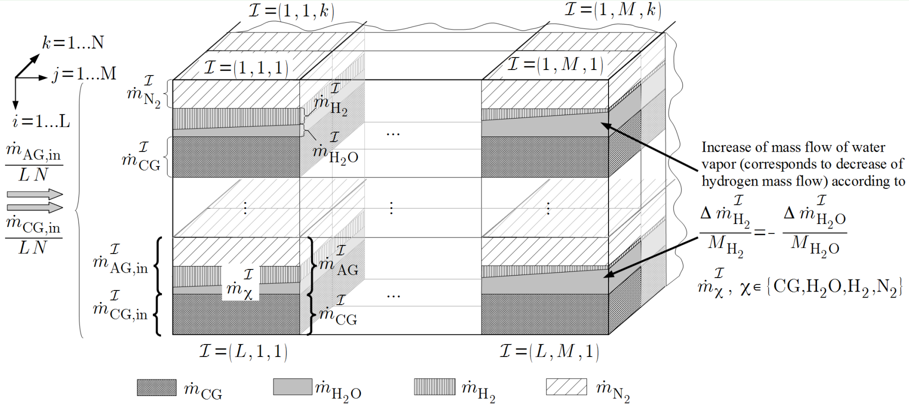
Local mass flow balances in the semi-discretized fuel cell stack module



Anode gas composition: $\dot{m}_{AG,in}(t) = \dot{m}_{H_2,in}(t) + \dot{m}_{N_2,in}(t) + \dot{m}_{H_2O,in}(t)$

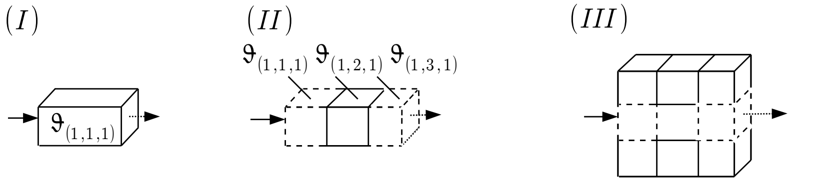
Control-Oriented Modeling of SOFC Systems (4)

Local mass flow balances in the semi-discretized fuel cell stack module



Anode gas composition: Air $\dot{m}_{CG,in}(t)$

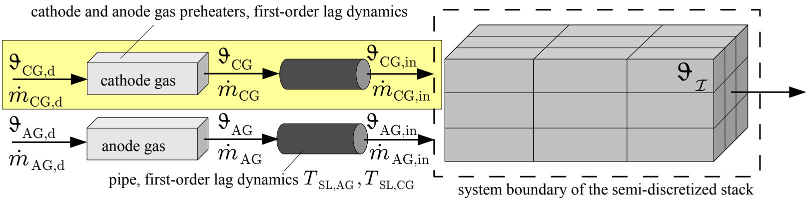
Different Variants of the Finite Volume Model (1)



$$x_{\mathrm{FC}} = \vartheta_{(1,1,1)} \qquad \mathbf{x}_{\mathrm{FC}}^T = [\vartheta_{(1,1,1)}, \vartheta_{(1,2,1)}, \vartheta_{(1,3,1)}] \qquad \mathbf{x}_{\mathrm{FC}}^T = [\vartheta_{(1,1,1)}, \ldots, \vartheta_{(3,3,1)}]$$

- Configuration (I): Typical for synthesizing a controller that is only applied during the system's heating phase
- Configuration (II): Simplest option for preventing local overtemperatures: Differentially flat or non-flat scenarios, depending on the choice of the system output $\vartheta_{\mathcal{I}^*}$
- Configuration (III): Generally non-flat configuration

Different Variants of the Finite Volume Model (2)

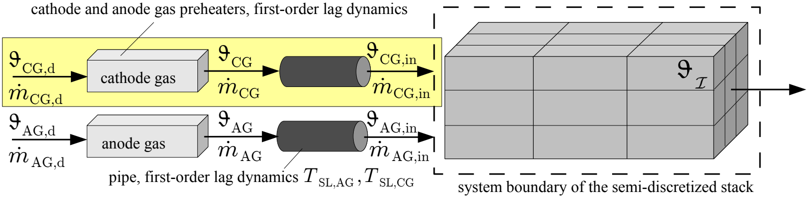


$\vartheta_{\mathrm{AG,d}}, \vartheta_{\mathrm{CG,d}}$	- desired temp. of anode and cathode gas	$\dot{m}_{\mathrm{AG}}, \dot{m}_{\mathrm{CG}}$	- mass flow (anode, cathode) at preheater outlet
$\vartheta_{\mathrm{AG,in}}, \vartheta_{\mathrm{CG,in}}$	- temperatures in the inlet gas manifold	$\dot{m}_{\mathrm{AG,d}}, \dot{m}_{\mathrm{CG,d}}$	- desired mass flows of anode and cathode gas
$\vartheta_{\mathrm{AG}}, \vartheta_{\mathrm{CG}}$	- temperatures at the preheater outlet	$T_{\mathrm{AG}}, T_{\mathrm{CG}}$	- time constants of the preheaters

System input: Cathode gas enthalpy flow (single-input single-output formulation) in configuration (II), preheater dynamics neglected

$$v_{\mathrm{CG,in}}(t) = \dot{m}_{\mathrm{CG,in}}(t) \cdot \vartheta_{\mathrm{CG,in}}(t)$$

Different Variants of the Finite Volume Model (2)

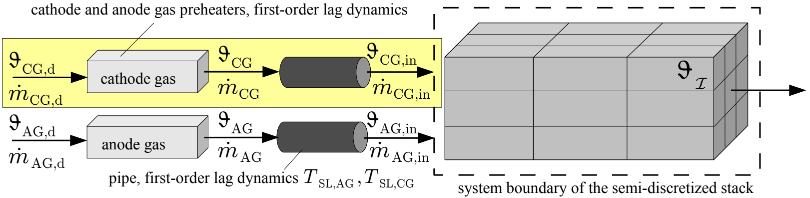


$\vartheta_{\mathrm{AG,d}}, \vartheta_{\mathrm{CG,d}}$	- desired temp. of anode and cathode gas	$\dot{m}_{\mathrm{AG}}, \dot{m}_{\mathrm{CG}}$	- mass flow (anode, cathode) at preheater outlet
$\vartheta_{\mathrm{AG,in}}, \vartheta_{\mathrm{CG,in}}$	- temperatures in the inlet gas manifold	$\dot{m}_{\mathrm{AG,d}}, \dot{m}_{\mathrm{CG,d}}$	- desired mass flows of anode and cathode gas
$\vartheta_{\mathrm{AG}}, \vartheta_{\mathrm{CG}}$	- temperatures at the preheater outlet	$T_{\mathrm{AG}}, T_{\mathrm{CG}}$	- time constants of the preheaters

System input: Cathode gas enthalpy flow (single-input single-output formulation) in configuration (II), preheater dynamics included

$$v_{\mathrm{CG,d}}(t) = \dot{m}_{\mathrm{CG,d}}(t) \cdot \vartheta_{\mathrm{CG,d}}(t) \; , \; \; \dot{m}_{\mathrm{CG,d}}(t) \approx \dot{m}_{\mathrm{CG,in}}(t)$$

Different Variants of the Finite Volume Model (2)



$\vartheta_{\mathrm{AG,d}}, \vartheta_{\mathrm{CG,d}}$	- desired temp. of anode and cathode gas	$\dot{m}_{\mathrm{AG}}, \dot{m}_{\mathrm{CG}}$	- mass flow (anode, cathode) at preheater outlet
$\vartheta_{\mathrm{AG,in}}, \vartheta_{\mathrm{CG,in}}$	- temperatures in the inlet gas manifold	$\dot{m}_{\mathrm{AG,d}}, \dot{m}_{\mathrm{CG,d}}$	- desired mass flows of anode and cathode gas
$\vartheta_{\mathrm{AG}}, \vartheta_{\mathrm{CG}}$	- temperatures at the preheater outlet	$T_{\mathrm{AG}}, T_{\mathrm{CG}}$	- time constants of the preheaters

Vector representation of the input (multi-input single-output formulation)

$$\mathbf{u}_{\mathrm{CG,d}}(t) = \begin{bmatrix} \dot{m}_{\mathrm{CG,d}}(t) \\ \vartheta_{\mathrm{CG,d}}(t) \end{bmatrix}$$

Transformation into Nonlinear Controller Canonical Form (1)

Input-affine state-space representation

$$\dot{\mathbf{x}}(t) = \mathbf{f}\left(\mathbf{x}(t), \mathbf{p}, v_{\mathrm{CG,d}}(t), \mathbf{v}_{\mathrm{AG,d}}(t)\right)$$

Computation of Lie derivatives of the system output $y(t) = h(\mathbf{x}(t)) = \vartheta_{\mathcal{I}^*}$, $\mathbf{x}(t) \in \mathbb{R}^{\mathcal{N}}$

$$\frac{d^r y(t)}{dt^r} = y^{(r)}(t) = L_{\mathbf{f}}^r h(\mathbf{x}(t)) = L_{\mathbf{f}}\left(L_{\mathbf{f}}^{r-1} h(\mathbf{x}(t))\right) \ , \quad r = 1, \ldots, \delta - 1$$

with the relative degree δ defined according to

$$\frac{\partial L_{\mathbf{f}}^r h(\mathbf{x}(t))}{\partial v_{\mathrm{CG,d}}} \equiv 0 \ \text{ for } \ r = 0, \ldots, \delta - 1 \ \text{ and } \ \frac{\partial L_{\mathbf{f}}^\delta h(\mathbf{x}(t))}{\partial v_{\mathrm{CG,d}}} \neq 0$$

Transformation into Nonlinear Controller Canonical Form (2)

Introduction of the new state vector

$$\boldsymbol{\xi} = \left[h(\mathbf{x}), \quad L_{\mathbf{f}} h(\mathbf{x}), \quad \ldots, \quad L_{\mathbf{f}}^{\delta-1} h(\mathbf{x}) \right]^T \in \mathbb{R}^{\delta} \quad \text{with} \quad \xi_1 = y = h(\mathbf{x})$$

New set of state equations (Brunovsky canonical form)

$$\begin{aligned} \left[\; \dot{\boldsymbol{\xi}}^T \; \vdots \; \dot{\boldsymbol{\zeta}}^T \; \right]^T &= \left[\; L_{\mathbf{f}} h(\mathbf{x}), \; \ldots, \; L_{\mathbf{f}}^{\delta} h(\mathbf{x}) \; \vdots \; L_{\mathbf{f}}^{\delta+1} h(\mathbf{x}), \; \ldots, \; L_{\mathbf{f}}^{\mathcal{N}} h(\mathbf{x}) \; \right]^T \\ &= \left[\; \xi_2, \quad \ldots, \quad \xi_{\delta}, \quad \tilde{a}(\mathbf{x}, \mathbf{p}, d) \; \vdots \; \mathbf{a}^{\Diamond}(\mathbf{x}, \mathbf{p}, d)^T \; \right]^T \\ &\quad + \left[\; 0, \quad \ldots, \quad \tilde{b}(\mathbf{x}, \mathbf{p}) \cdot {\color{red} v_{\mathrm{CG,d}}} \; \vdots \; \mathbf{b}^{\Diamond}(\mathbf{x}, \mathbf{p}, d, v_{\mathrm{CG,d}}, \dot{v}_{\mathrm{CG,d}}, \ldots)^T \; \right]^T \end{aligned}$$

with the additive bounded disturbance $d \in [d]$, $d \in \mathbb{R}$, and the interval parameters $\mathbf{p} \in [\mathbf{p}]$, $\mathbf{p} \in \mathbb{R}^{n_{\mathrm{p}}}$

Transformation into Nonlinear Controller Canonical Form (3)

Goal: Accurate trajectory tracking and stabilization of the error dynamics despite the interval uncertainties $d \in [d]$ and $\mathbf{p} \in [\mathbf{p}]$

$$\left[\ \dot{\boldsymbol{\xi}}^T \ \vdots\ \dot{\boldsymbol{\zeta}}^T\ \right]^T = \left[\ \xi_2, \quad \ldots, \quad \xi_\delta, \quad \tilde{a}(\mathbf{x}, \mathbf{p}, d) \ \vdots\ \mathbf{a}^\Diamond(\mathbf{x}, \mathbf{p}, d)^T\ \right]^T$$
$$+ \left[\ 0, \quad \ldots, \quad \tilde{b}(\mathbf{x}, \mathbf{p}) \cdot v_{\mathrm{CG,d}} \ \vdots\ \mathbf{b}^\Diamond(\mathbf{x}, \mathbf{p}, d, v_{\mathrm{CG,d}}, \dot{v}_{\mathrm{CG,d}}, \ldots)^T\ \right]^T$$

- Use of the variable $v_{\mathrm{CG,d}}$ as the control input
- Derivation of an interval-based variable structure control law

Requirements

- Estimation of all state variables $\mathbf{x}$, of the parameters $\mathbf{p}$, the disturbance d, and their corresponding interval bounds in real time
- Note: If $\delta \equiv \mathcal{N}$, the output y coincides with the flat system output
- Otherwise: The bounded states $\boldsymbol{\zeta}$ of the non-controllable internal dynamics act as disturbances onto the system model

Transformation into Nonlinear Controller Canonical Form (3)

Goal: Accurate trajectory tracking and stabilization of the error dynamics despite the interval uncertainties $d \in [d]$ and $\mathbf{p} \in [\mathbf{p}]$

$$\left[\ \dot{\boldsymbol{\xi}}^T \ \vdots\ \dot{\boldsymbol{\zeta}}^T\ \right]^T = \left[\ \xi_2, \quad \ldots, \quad \xi_\delta, \quad \tilde{a}(\mathbf{x},\mathbf{p},d)\ \vdots\ \mathbf{a}^\Diamond(\mathbf{x},\mathbf{p},d)^T\ \right]^T$$
$$+ \left[\ 0, \quad \ldots, \quad \tilde{b}(\mathbf{x},\mathbf{p}) \cdot v_{\mathrm{CG,d}}\ \vdots\ \mathbf{b}^\Diamond(\mathbf{x},\mathbf{p},d,v_{\mathrm{CG,d}},\dot{v}_{\mathrm{CG,d}},\ldots)^T\ \right]^T$$

- Use of the variable $v_{\mathrm{CG,d}}$ as the control input
- Derivation of an interval-based variable structure control law

Possible estimation approaches

- Linear gain-scheduled state observer (Luenberger-like structure)
- Sensitivity-based estimation: Receding horizon approach (online minimization of a quadratic error measure)
- Observer in controller canonical form
- Robustification by linear matrix inequalities possible

Handling of Input Rate Limitations

Extension of the system input by a further lag element

$$T_{\mathrm{r}} \cdot \dot{\tilde{v}}_{\mathrm{CG,d}} + \tilde{v}_{\mathrm{CG,d}} = \check{v}_{\mathrm{CG,d}}$$

with the new system input $\check{v}_{\mathrm{CG,d}}$ and the fixed time constant $T_{\mathrm{r}} > 0$

Guaranteed compatibility of the actual system input with the rate constraints

$$\left|\dot{\tilde{v}}_{\mathrm{CG,d}}\right| \leq {T_{\mathrm{r}}}^{-1} \cdot \left(\sup\{[v_{\mathrm{CG,max}}]\} - \inf\{[v_{\mathrm{CG,max}}]\}\right)$$

under the prerequisite

$$\begin{aligned}\inf\{[v_{\mathrm{CG,max}}]\} &\equiv \inf\{[\tilde{v}_{\mathrm{CG,max}}]\} \equiv \inf\{[\check{v}_{\mathrm{CG,max}}]\}\,,\\ \sup\{[v_{\mathrm{CG,max}}]\} &\equiv \sup\{[\tilde{v}_{\mathrm{CG,max}}]\} \equiv \sup\{[\check{v}_{\mathrm{CG,max}}]\}\end{aligned}$$

Control Parameterization: Basic Approach (Excerpt)

Control signal feasible?

Yes

Break, apply the control for the time step t_k, and proceed with the subsequent discretization step

No

Adaption of $\tilde{\eta}$ (Alternative: adapt the parameters α_r in definition of sliding surface)

Input saturation exceeded

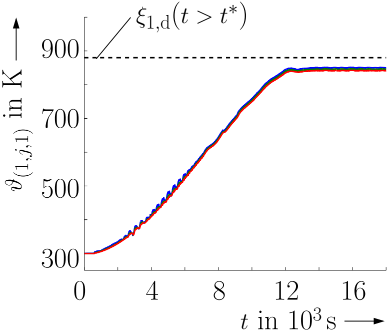
a) $\tilde{v}_{\mathrm{CG,d}}(t_k) < \inf\{[v_{\mathrm{CG,max}}]\}$	b) $\tilde{v}_{\mathrm{CG,d}}(t_k) > \sup\{[v_{\mathrm{CG,max}}]\}$
Increase $\tilde{\eta}$ if $\dfrac{\partial \tilde{v}_{\mathrm{CG,d}}}{\partial \tilde{\eta}} > 0$ **Decrease** $\tilde{\eta}$ if $\dfrac{\partial \tilde{v}_{\mathrm{CG,d}}}{\partial \tilde{\eta}} < 0$	**Increase** $\tilde{\eta}$ if $\dfrac{\partial \tilde{v}_{\mathrm{CG,d}}}{\partial \tilde{\eta}} < 0$ **Decrease** $\tilde{\eta}$ if $\dfrac{\partial \tilde{v}_{\mathrm{CG,d}}}{\partial \tilde{\eta}} > 0$

Control Parameterization: Extension for Online Gain Scheduling

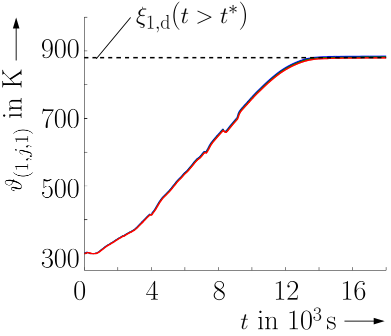
- Case 1: Offline parameterization with cutoff for control signal
- Case 2: Online parameterization
  1. Define a desired eigenvalue λ_r of multiplicity $\delta - 1$ on the sliding surface with corresponding parameters α_r
  2. Initialize $\tilde{\eta}$ with the desired value
  3. Adapt $\tilde{\eta}$ in a line-search approach (fixed number of $N_\eta = 5$ steps) to ensure compatibility of $\tilde{v}_{\mathrm{CG,d}}$ with the control constraints
     - Stop, if admissible control is found
     - If no admissible control is found within N_η steps, adapt the eigenvalue λ_r and restart with Step (2); Break after at most $N_\lambda = 5$ repetitions

- Treatment of input rate constraints: Extension of the system input by a further lag element
- Simulation case study: $L = N = 1$, $M = 3$

Simulation Results: Stack Temperatures

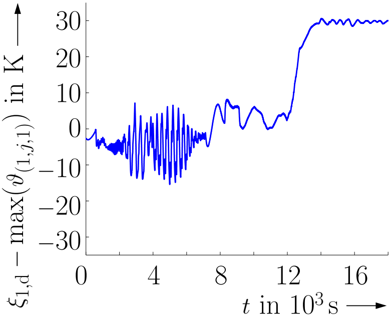


Offline parameterization

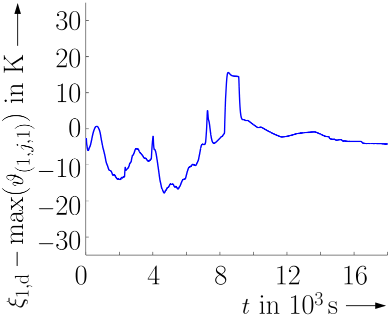


Online parameterization

Simulation Results: Tracking Error

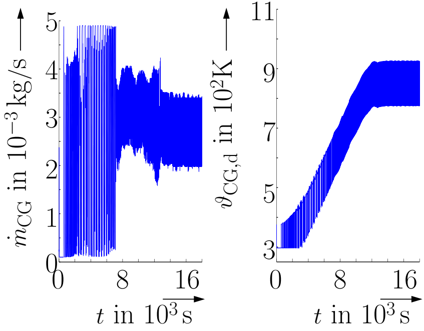


Offline parameterization

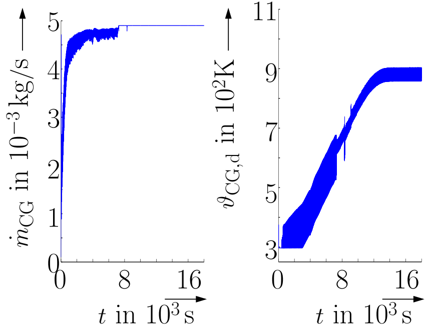


Online parameterization

Simulation Results: CG Preheater Inputs



Offline parameterization



Online parameterization

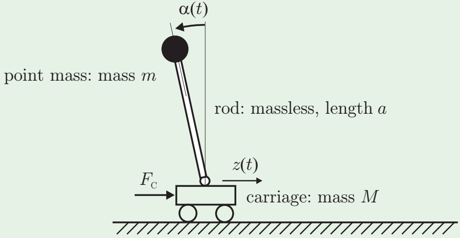
Control of an Inverted Pendulum

System model: Inverted pendulum on a carriage

- State vector $\mathbf{z} = \begin{bmatrix} \alpha & z & \dot{\alpha} & \dot{z} \end{bmatrix}^T$; Control variable: Force F_{C}
- Derivation of a set of first-order state equations with

$$\ddot{\alpha} = \frac{2g\sin(\alpha)\,(M+m) - ma\dot{\alpha}^2\sin(2\alpha) + 2\cos(\alpha)F_{\mathrm{C}}}{a\,(2M + m\,(1-\cos(2\alpha)))} \quad \text{and}$$

$$\ddot{z} = \frac{mg\sin(2\alpha) - 2ma\sin(\alpha)\dot{\alpha}^2 + 2F_{\mathrm{C}}}{2M + m\,(1-\cos(2\alpha))}$$



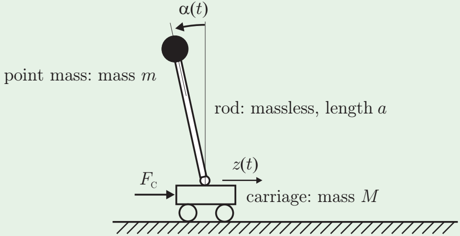
Control of an Inverted Pendulum

System model: Inverted pendulum on a carriage

- State vector $\mathbf{z} = \begin{bmatrix} \alpha & z & \dot{\alpha} & \dot{z} \end{bmatrix}^T$; y: horizontal pendulum tip position
- Derivation of a set of first-order state equations with

$$\ddot{\alpha} = \frac{2g\sin(\alpha)\,(M+m) - ma\dot{\alpha}^2\sin(2\alpha) + 2\cos(\alpha)F_{\mathrm{C}}}{a\,(2M + m\,(1-\cos(2\alpha)))} \quad \text{and}$$

$$\ddot{z} = \frac{mg\sin(2\alpha) - 2ma\sin(\alpha)\dot{\alpha}^2 + 2F_{\mathrm{C}}}{2M + m\,(1-\cos(2\alpha))}$$



Approximate Transformation into Nonlinear Controller Canonical Form (1)

Successive computation of the output's Lie derivatives

$$y = -a\sin(\alpha) + z\ , \quad \dot{y} = -a\cos(\alpha)\dot{\alpha} + \dot{z}$$

$$\begin{aligned}
\ddot{y} &= a\sin(\alpha)\dot{\alpha}^2 + \frac{mg\sin(2\alpha) - 2ma\sin(\alpha)\dot{\alpha}^2 + 2F_{\mathrm{C}}}{2M + m\left(1-\cos(2\alpha)\right)} \\
&\quad - \cos\alpha \frac{2g\sin(\alpha)\left(M+m\right) - ma\dot{\alpha}^2\sin(2\alpha) + 2\cos(\alpha)F_{\mathrm{C}}}{2M + m\left(1-\cos(2\alpha)\right)} \\
&\approx -p_1 g\alpha + p_2 \\
\dddot{y} &= p_1 g\dot{\alpha} + p_3 \\
y^{(4)} &= p_1 g\ddot{\alpha} + p_4 = p_1 g \frac{2g\sin(\alpha)\left(M+m\right) - ma\dot{\alpha}^2\sin(2\alpha)}{a\left(2M + m\left(1-\cos(2\alpha)\right)\right)} + p_4 \\
&\quad + \frac{2p_1 g\cos(\alpha)}{a\left(2M + m\left(1-\cos(2\alpha)\right)\right)} F_{\mathrm{C}} =: a\left(\mathbf{x},\mathbf{p}\right) + b\left(\mathbf{x},\mathbf{p}\right)\cdot v
\end{aligned}$$

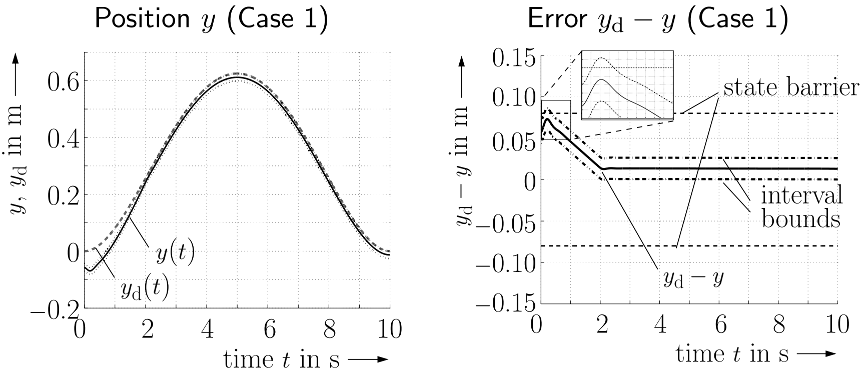
Approximate Transformation into Nonlinear Controller Canonical Form (2)

State-space representation for control design

- State vector after transformation of coordinates $\mathbf{x} = \begin{bmatrix} y & \dot{y} & \ddot{y} & \dddot{y} \end{bmatrix}^T$
- Control input $v := F_{\mathrm{C}}$
- Interval parameters $[p_i]$, $i \in \{1, \ldots, 4\}$ (representation of approximation and modeling errors)
- Error for angle measurement $[-0.01\ ;\ 0.01]\ \mathrm{rad}$ and carriage position measurement $[-0.01\ ;\ 0.01]\ \mathrm{m}$

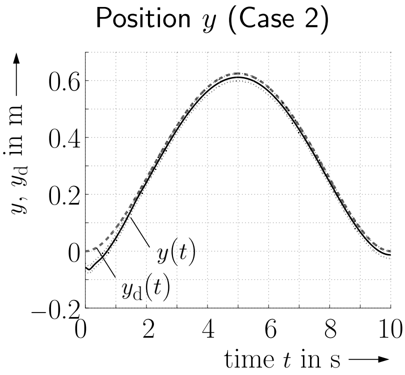
$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a\left(\mathbf{x}, \mathbf{p}\right) + b\left(\mathbf{x}, \mathbf{p}\right) \cdot v
\end{aligned}
$$

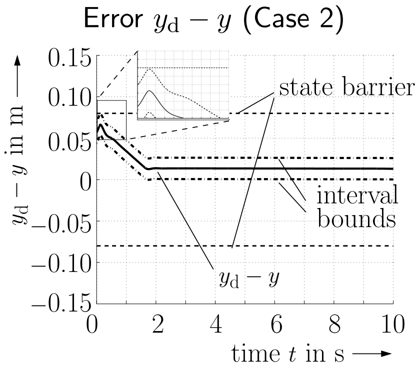
Simulation Results



Violation of two-sided state constraint $\Longrightarrow$ Barrier function is **deactivated**, parameters and measured states are **uncertain**

Simulation Results





No violation of two-sided state constraint $\Longrightarrow$ Barrier function is **activated**, parameters and measured states are **uncertain**

Outlook: Interval-Based Backstepping Control (1)

Interval-based backstepping control

- System model is now given in strict feedback form

$$\begin{aligned}
\dot{x}_1 &= a_1\left(x_1, \mathbf{p}\right) + b_1\left(x_1, \mathbf{p}\right) \cdot x_2 \\
\dot{x}_2 &= a_2\left(x_1, x_2, \mathbf{p}\right) + b_2\left(x_1, x_2, \mathbf{p}\right) \cdot x_3 \\
&\vdots \\
\dot{x}_n &= a_n\left(x_1, \ldots, x_n, \mathbf{p}\right) + b_n\left(x_1, \ldots, x_n, \mathbf{p}\right) \cdot v
\end{aligned}$$

- Uncertain parameters $\mathbf{p} \in [\mathbf{p}]$

Outlook: Interval-Based Backstepping Control (2)

Interval-based backstepping control

$$\begin{aligned}
\dot{x}_1 &= a_1\left(x_1, \mathbf{p}\right) + b_1\left(x_1, \mathbf{p}\right) \cdot x_2 \\
\dot{x}_2 &= a_2\left(x_1, x_2, \mathbf{p}\right) + b_2\left(x_1, x_2, \mathbf{p}\right) \cdot x_3 \\
&\vdots \\
\dot{x}_n &= a_n\left(x_1, \ldots, x_n, \mathbf{p}\right) + b_n\left(x_1, \ldots, x_n, \mathbf{p}\right) \cdot v
\end{aligned}$$

Control procedure

- Successive stabilization of the dynamics for x_i, $i = 1, \ldots, n-1$
- Treatment of x_{i+1} as a virtual control signal in the equation for $\dot{x}_i$
- Stabilization using a Lyapunov function candidate, e.g. $V_i = \frac{1}{2}x_i^2$, $\dot{V}_i = x_i \dot{x}_i < 0$
- Differentiability of the virtual control signals is required up to x_n

Outlook: Interval-Based Backstepping Control (3)

Interval-based backstepping control

$$\begin{aligned}
\dot{x}_1 &= a_1(x_1, \mathbf{p}) + b_1(x_1, \mathbf{p}) \cdot x_2 \\
\dot{x}_2 &= a_2(x_1, x_2, \mathbf{p}) + b_2(x_1, x_2, \mathbf{p}) \cdot x_3 \\
&\ \vdots \\
\dot{x}_n &= a_n(x_1, \ldots, x_n, \mathbf{p}) + b_n(x_1, \ldots, x_n, \mathbf{p}) \cdot v
\end{aligned}$$

Control procedure

- Successive stabilization of the dynamics for x_i, $i = 1, \ldots, n-1$
- Treatment of x_{i+1} as a virtual control signal for the equation $\dot{x}_i$
- Variable-structure approach for control signal v is possible (including $\mathrm{sign}(\cdot)$), for earlier stages smooth or regularized controllers (e.g. $\tanh(\cdot)$)
- Finally: Proof of the overall system stability

Conclusions and Outlook on Future Work

- Control-oriented modeling of dynamic systems
- Verified parameter identification as the basis for control design
- Stabilization of the error dynamics using interval techniques
- Handling of input and state constraints (guaranteed overshoot prevention, two-sided worst-case bounds for the system output)
- Use of interval analysis in real time

Conclusions and Outlook on Future Work

- Control-oriented modeling of dynamic systems
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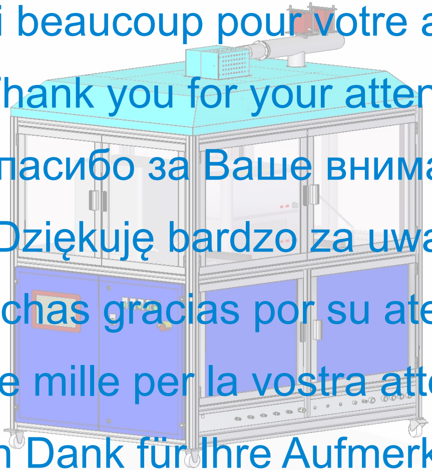
- Extension by a sensitivity-based predictive controller
- Extension by a (sensitivity-based) state and disturbance observer
- Extension by backstepping-like control procedures



Merci beaucoup pour votre attention!

Thank you for your attention!

Спасибо за Ваше внимание!

Dziękuję bardzo za uwagę!

¡Muchas gracias por su atención!

Grazie mille per la vostra attenzione!

Vielen Dank für Ihre Aufmerksamkeit!