Control of an Autonomous Underwater Vehicle under robustness constraints

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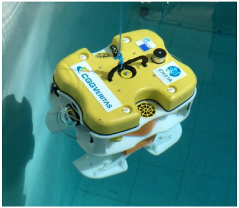
Plan

AUV CISCREA



Size	0.525m (L) 0.406m (W) 0.395m (H)
Weight in air	15.56kg (without payload and floats)
Degrees of Freedom	Surge, Sway, Heave and Yaw
Propulsion	2 vertical and 4 horizontal propellers
Speed	2 knots (Surge) and 1 knot (Sway, Heave)
Depth Rating	50m
On-board Battery	2-4 hours

AUV CISCREA model

Rigid-body dynamic:

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = \tau_{env} + \tau_{hydro} + \tau_{pro} \quad (1)$$

Hydrodynamic formulations:

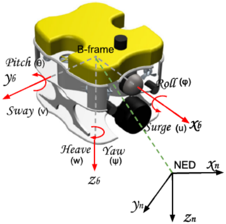
$$\tau_{hydro} = -M_A\dot{\nu} - C_A(\nu)\nu - D(|\nu|)\nu - g(\eta) \quad (2)$$

Damping:

$$D(|\nu|) = D_L + D_N|\nu|\nu \quad (3)$$

Parameter	Description
M_{RB}	AUV rigid-body mass and inertia matrix
M_A	Added mass matrix
C_{RB}	Rigid-body induced coriolis-centripetal matrix
C_A	Added mass induced coriolis-centripetal matrix
$D(\|\nu\|)$	Damping matrix
$g(\eta)$	Restoring forces and moments vector
τ_{env}	Environmental disturbances(wind,waves and currents)
τ_{hydro}	Vector of hydrodynamic forces and moments
τ_{pro}	Propeller forces and moments vector

AUV CISCREA Yaw model



We consider that there are no dependencies between the yaw dynamic and dynamics along other axis.
Resulting Yaw dynamic:

$$(I_{YRB} + I_{YA})\ddot{x} + D_{YN}|\dot{x}|\dot{x} + D_{YL}\dot{x} = K_t\tau_i \tag{4}$$

However, H_∞ synthesis requires a linear system. Thus, the CISCREA yaw model could be linearized as:

$$(I_{YRB} + I_{YA})\ddot{x} + (D_{YLA} + \delta)\dot{x} = K_t\tau_i, \tag{5}$$

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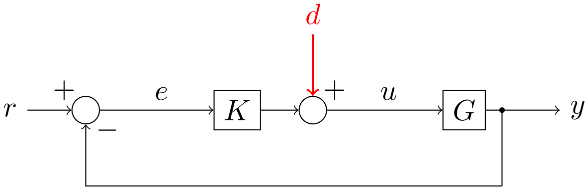
Control objectives specifications

We aim to synthesize a controller to meet the following objectives:

1. Small tracking error e.
2. External perturbation rejection.

External perturbation can be modeled as a control disturbance signal d.



H_∞ formulation

The two objectives can be formulate as H_∞ constraints:

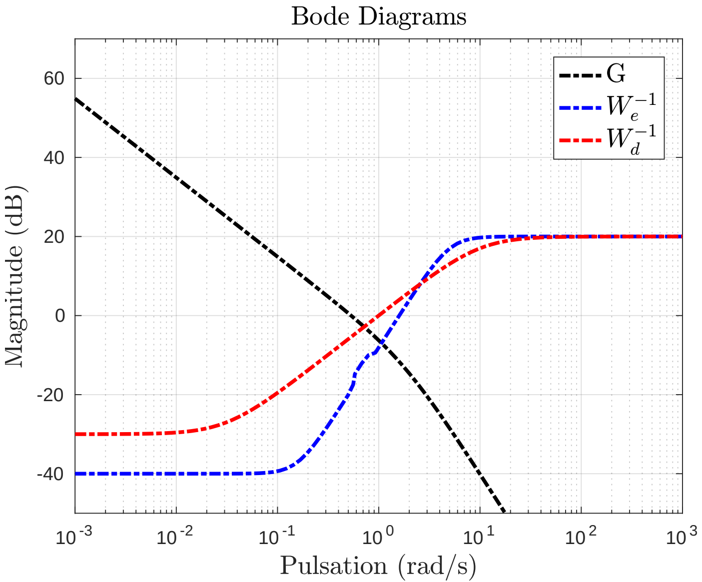
1. Small tracking error:

$$\frac{|e(i\omega)|}{|r(i\omega)|} \leq |W_e^{-1}(i\omega)|, \forall \omega \geq 0 \iff ||T_{r\to e}(i\omega)W_e(i\omega)||_\infty \leq 1$$

2. External perturbation rejection:

$$\frac{|y(i\omega)|}{|d(i\omega)|} \leq |W_y^{-1}(i\omega)|, \forall \omega \geq 0 \iff ||T_{d\to y}(i\omega)W_y(i\omega)||_\infty \leq 1$$

Bode diagrams of Weighted functions



Min Max Problem

- The controller $K(k,s)$ depends on free parameters k.
- $T_{r\to e}(k,s) = \frac{1}{1+G(s)K(k,s)}$ depends on k
- $T_{d\to y}(k,s) = \frac{G(s)}{1+G(s)K(k,s)}$ depends on k

The constraint satisfaction problem is:

$$\text{Find } k, \ \max(||T_{r\to e}(k,s)W_e(s)||_\infty, ||T_{d\to y}(k,s)W_y(s)||_\infty) \leq 1$$

- $||T(s)||_\infty = \sup_\omega |T(i\omega)|$

The Min Max problem is:

$$\min_k \sup_{\omega\geq 0}\{\max(|T_{r\to e}(k,s)W_y(s)|, |T_{d\to y}(k,s)W_y(s)|)\}$$

Solving the Min Max problem

We solve the Min Max problem with Global optimization based on interval analysis.

- Existing methods are based on local optimization. They only provide an upper bound of the objective function.
- Global optimization provides an enclosure of the objective function. It is possible to prove that the CSP is not feasible.

Uncertainties

The model of the CISCREA carries uncertainties. The controller is synthesize from a nominal model, and robustness to uncertainties must be analyzed.

- An uncertainty is represented by an interval. **p** is the vector of uncertainties.
- $G_\Delta(s, p)$, $p \in \mathbf{p}$ describe the uncertain system.
- The closed loop system is robust iff:

$$\forall p \in \mathbf{p},\ \max(||T_{\Delta r \to e}(p, s) W_e(s)||_\infty, ||T_{\Delta d \to y}(p, s) W_y(s)||_\infty) \leq 1$$

- The robustness condition can be validated with interval analysis in a reliable way.

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Controller synthesis

- PID controller: $K(k,s) = k_p + \frac{k_i}{s} + \frac{k_d s}{1+\tau s}$
- $k = (k_p, k_i, k_d, \tau)$
- CISCREA model: $G(s) = \frac{6.725}{s^2+2s}$
- Weighting functions:

$$W_e = \frac{0.1s^2 + 0.7109s + 2.527}{s^2 + 0.2248s + 0.02527}, \; W_y = \frac{0.1s + 0.9935}{s + 0.03142}$$

- k is searched in $[0,2]^4$

Controller synthesis

- Solution to the Min Max problem computed:
  $k^* = (1.987, 1.731, 0.638, 0.001)$
- $||T_{r\to e}(k^*, s)||_\infty = 0.325$
- $||T_{d\to y}(k^*, s)||_\infty = 0.154$
- $\min_{k} \sup_{\omega\geq 0}\{\max(|T_{r\to e}(k, s)W_y(s)|, |T_{d\to y}(k, s)W_y(s)|)\} \in [0.225, 0.325]$

Robustness Analysis

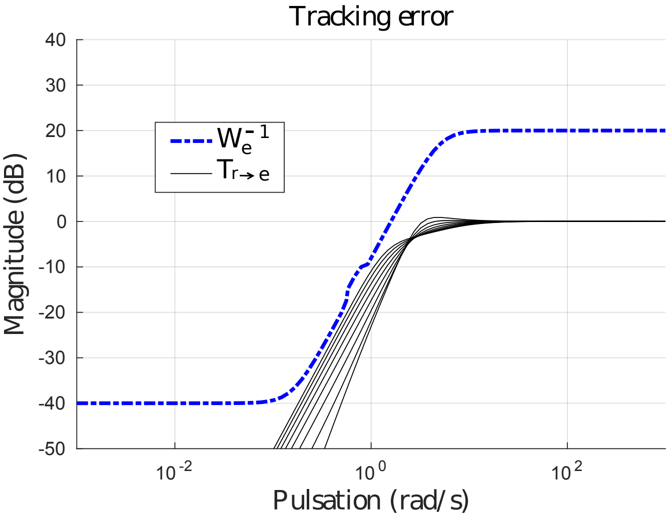
Uncertain CISCREA model:

$$G_\Delta(s,p) = \frac{6.725}{s^2 + ps}, \, p \in [0,4]$$

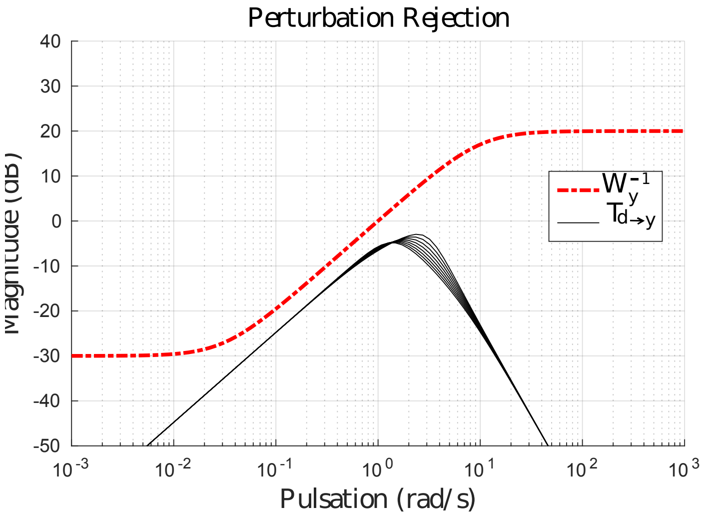
- $||T_{\Delta r \to e}(k^*, s, p)||_\infty \leq 0.82$
- $||T_{\Delta d \to y}(k^*, s, p)||_\infty \leq 0.162$

- $||T_{r \to e}(k^*, s)||_\infty = 0.325$
- $||T_{d \to y}(k^*, s)||_\infty = 0.154$

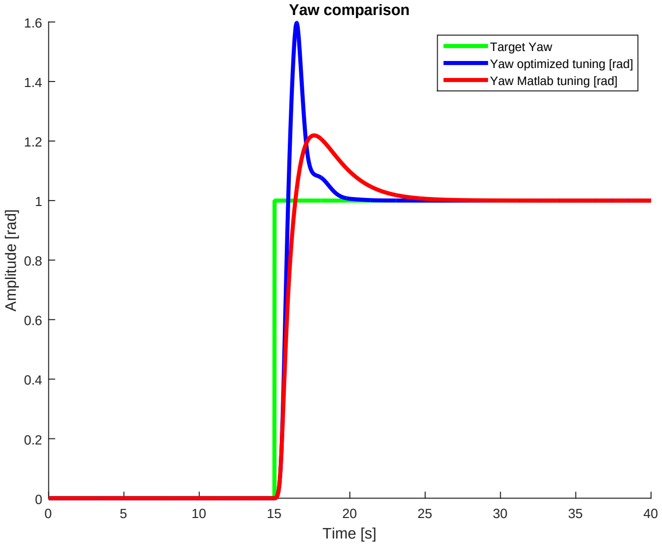
Tracking error constraint



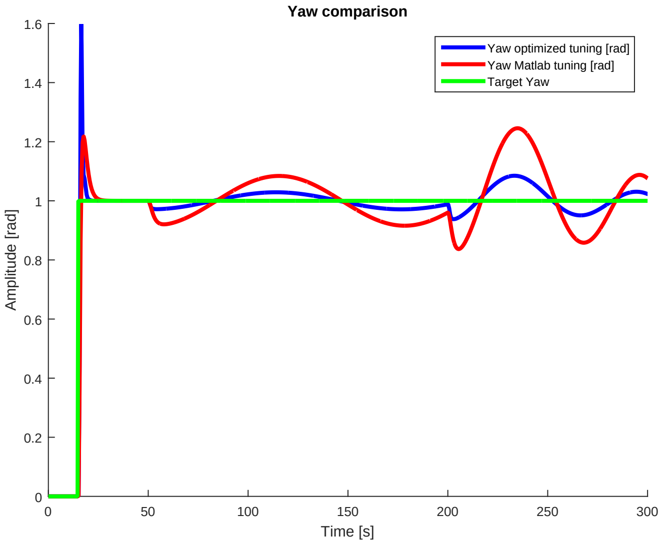
Perturbation rejection



Step response without perturbation



Step response with perturbation



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Conclusions

- Robust synthesis method.
- Robustness analysis with respect to uncertainties.
- Simulation validation.