Avoiding fake boundaries in interval analysis

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ENSTA Bretagne

Presentation

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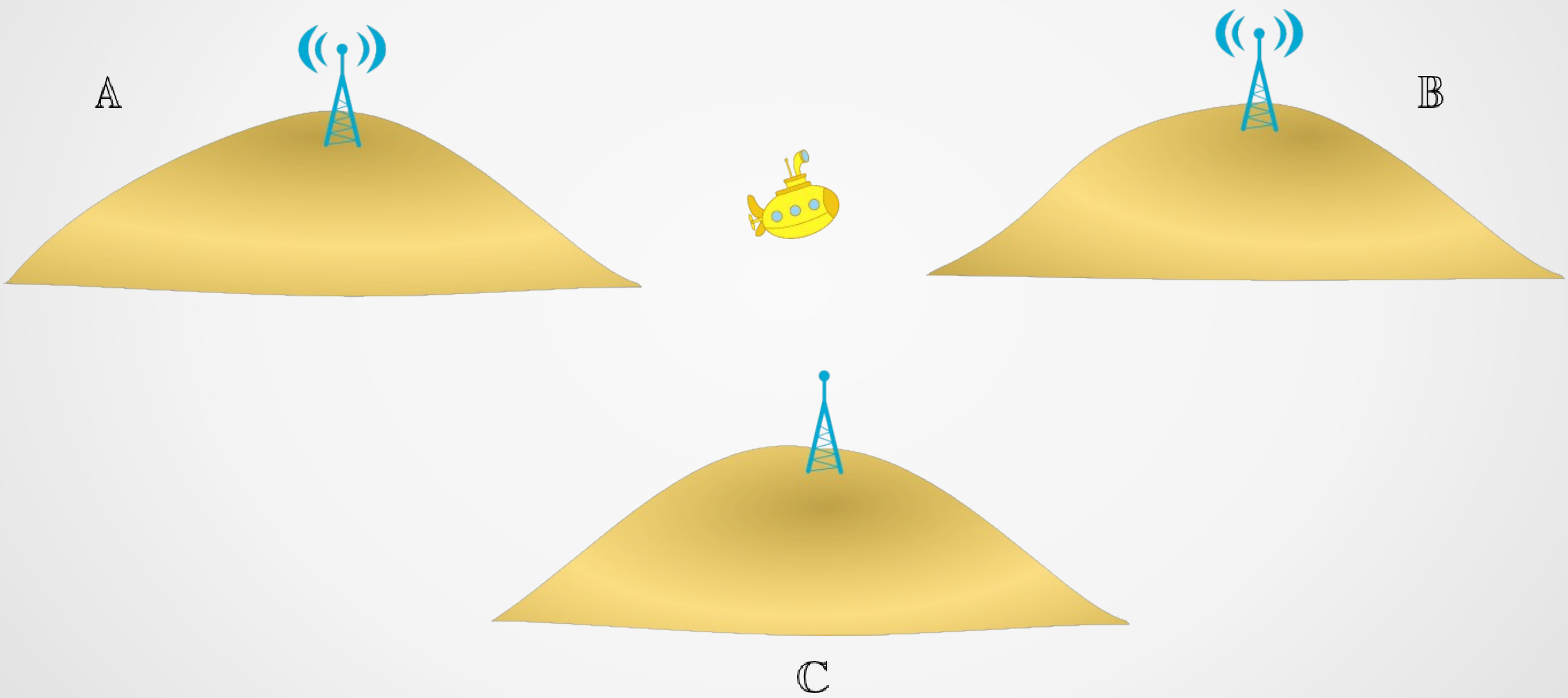
- Resume:
  - Graduate in Systems Analysis – UNIFRA - Brazil
  - Graduate in Geomatics – UFSM - Brazil
  - Master degree in computer sciences at UFRGS – Brazil
  - PhD at ENSTA Bretagne - Brest
- Advisor: Luc Jaulin

Localization Problem



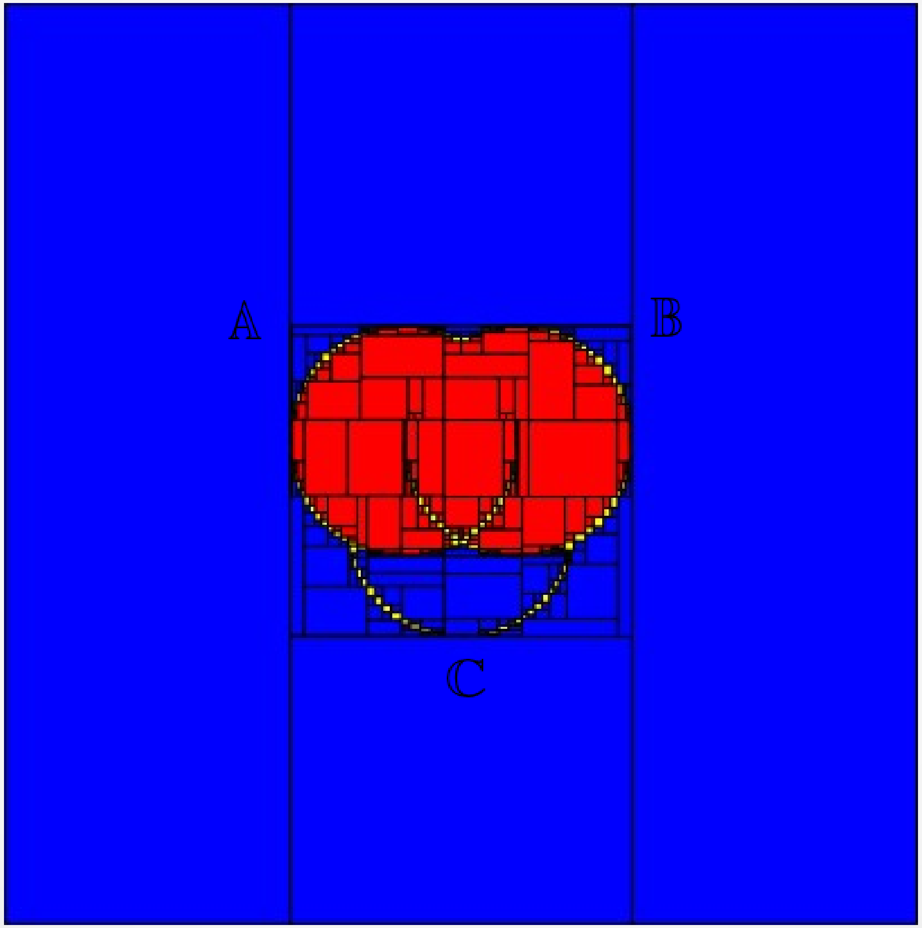
Inner contractor

$$A\overline{B}\cup\overline{A}B\cup AB\cup((A\cup B\cup C)\cap\overline{C})$$

Outer contractor

$$\overline{A}\,\overline{B}C\cup\overline{A}\,\overline{B}\,\overline{C}$$

Fake boundary



Inner contractor

$A\overline{B}\cup\overline{A}B\cup AB\cup((A\cup B\cup C)\cap\overline{C})$

Outer contractor

$\overline{A}\,\overline{B}C\cup\overline{A}\,\overline{B}\,\overline{C}$

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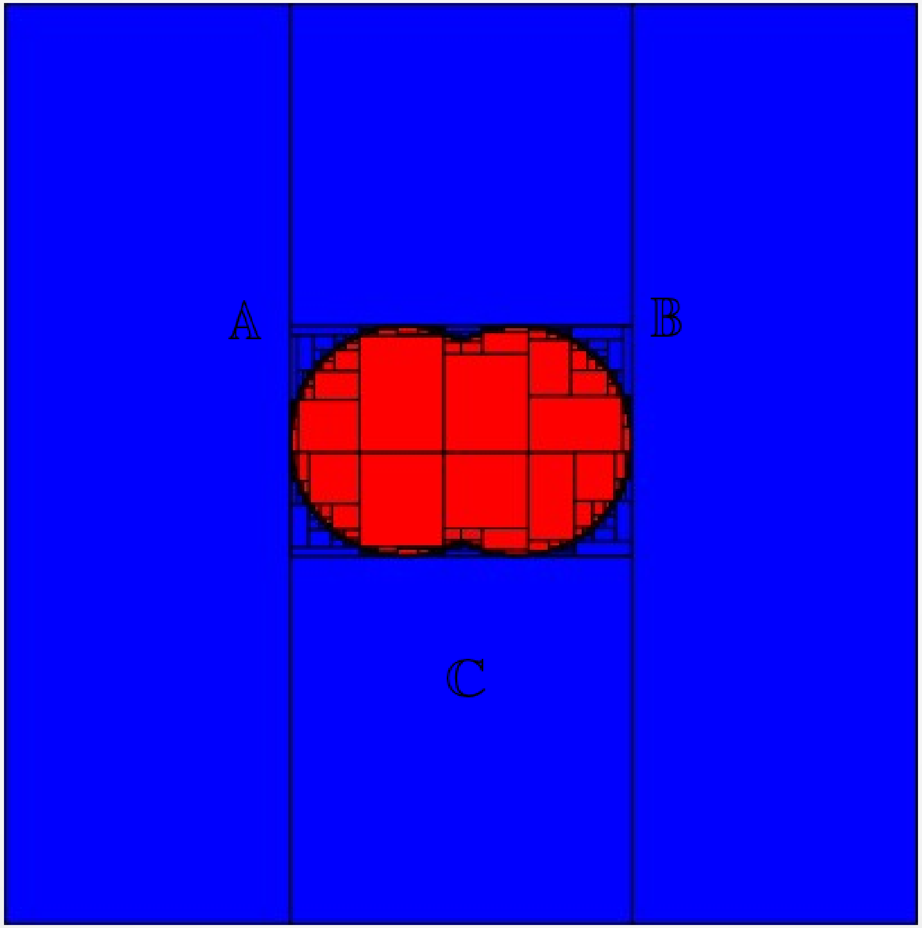
Simplification

$A\overline{B}\cup AB\cup\overline{A}B\cup((A\cup B\cup C)\cap\overline{C})$	
$A\cap(\overline{B}\cup B)\cup\overline{A}B\cup((A\cup B\cup C)\cap\overline{C})$	By distributive law over union
$A\cup\overline{A}B\cup((A\cup B\cup C)\cap\overline{C})$	By complement law and identity law
$((A\cup\overline{A})\cap(A\cup B))\cup((A\cup B\cup C)\cap\overline{C})$	By distributive law over union
$(A\cup B)\cup((A\cup B\cup C)\cap\overline{C})$	By complement law and identity law
$(A\cup B)\cup((A\cup B)\cap\overline{C})\cup C\overline{C}$	By distributive law over union
$(A\cup B)\cup((A\cup B)\cap\overline{C})$	By complement law
$A\cup B$	By absorption law

Expected set



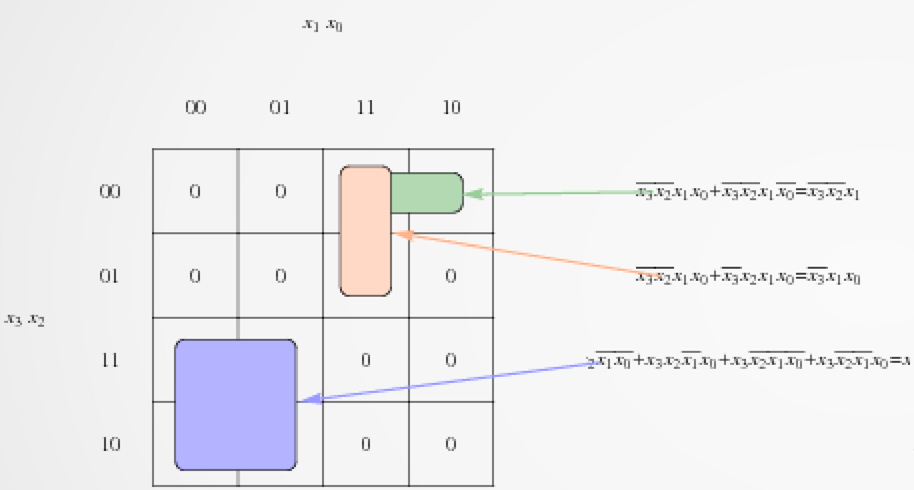
Inner contractor

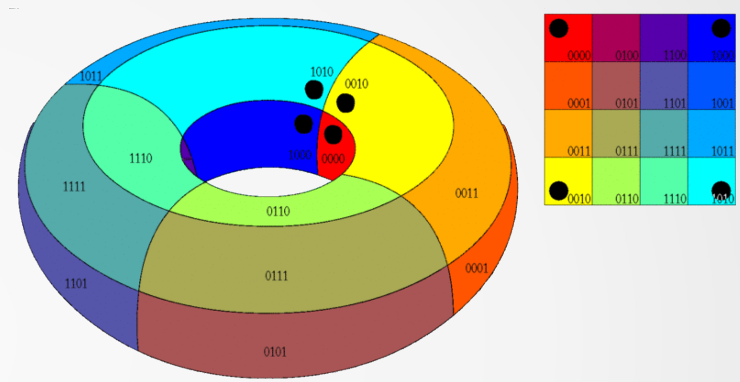
$A \cup B$

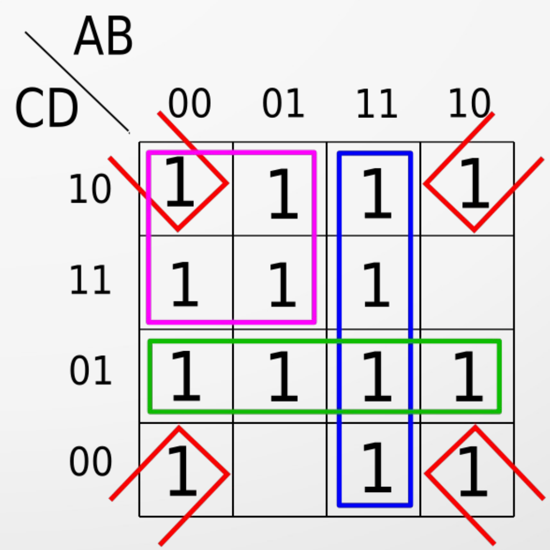
Outer contractor

$\overline{A}\,\overline{B}$

Karnaugh Map

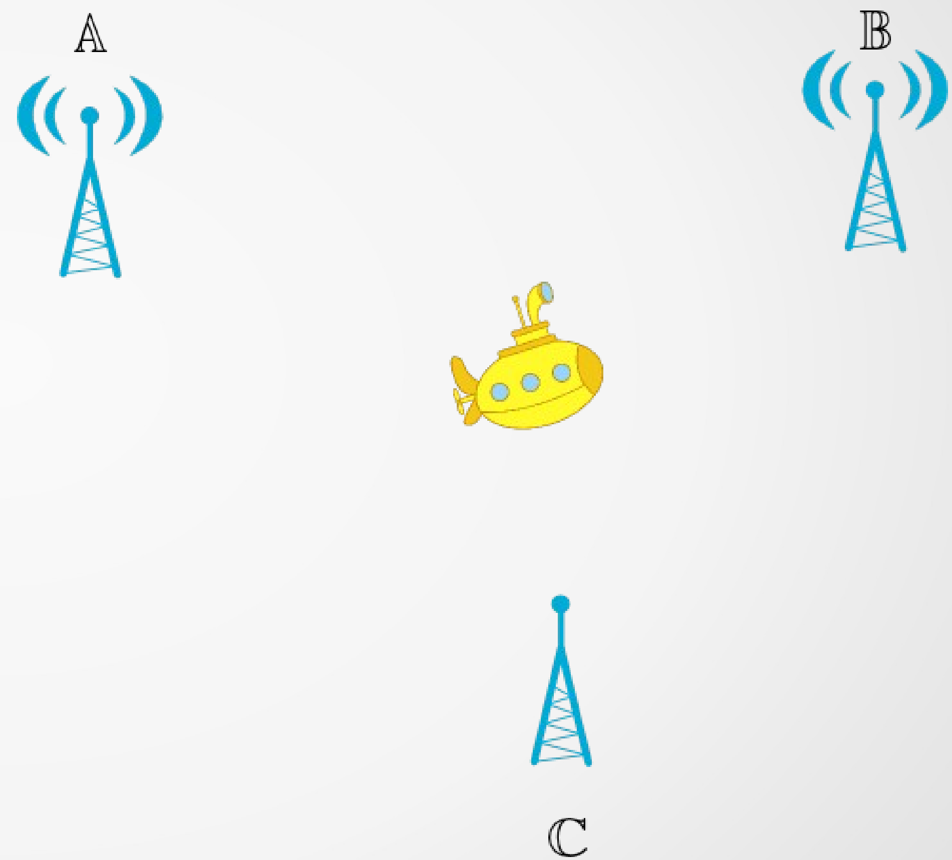






Karnaugh Map

	$\mathbb{A}\mathbb{B}$	$\mathbb{A}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\mathbb{B}$
$\mathbb{C}$	1	1	0	1
$\overline{\mathbb{C}}$	1	1	0	1



Inner contractor

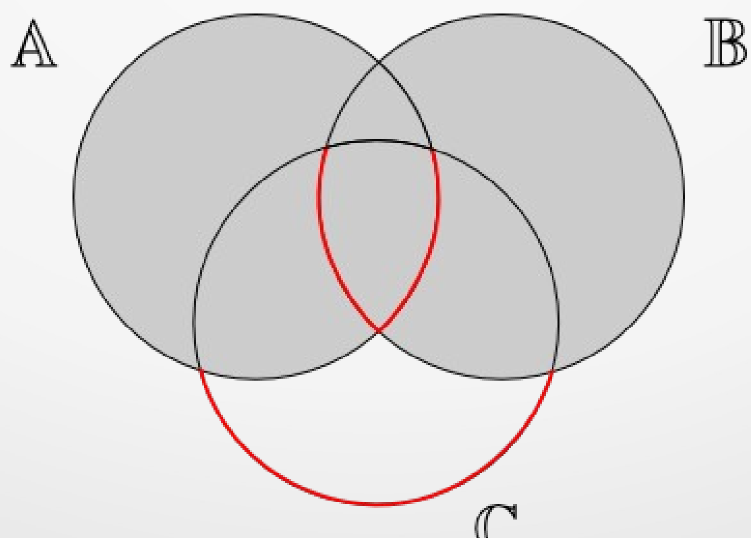
$$A\overline{B}\cup\overline{A}B\cup AB\cup((A\cup B\cup C)\cap\overline{C})$$

	$\mathbb{A}\mathbb{B}$	$\mathbb{A}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\,\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\mathbb{B}$
$\mathbb{C}$	1	1	0	1
$\overline{\mathbb{C}}$	1	1	0	1

Outer contractor

$$\overline{A}\,\overline{B}C\cup\overline{A}\,\overline{B}\,\overline{C}$$

	$\mathbb{A}\mathbb{B}$	$\mathbb{A}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\,\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\mathbb{B}$
$\mathbb{C}$	1	1	0	1
$\overline{\mathbb{C}}$	1	1	0	1



Inner contractor

$A \cup B$

	$\mathbb{A}\mathbb{B}$	$\mathbb{A}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\mathbb{B}$
$\mathbb{C}$	1	1	0	1
$\overline{\mathbb{C}}$	1	1	0	1

Outer contractor

$\overline{A}\,\overline{B}$

	$\mathbb{A}\mathbb{B}$	$\mathbb{A}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\overline{\mathbb{B}}$	$\overline{\mathbb{A}}\mathbb{B}$
$\mathbb{C}$	1	1	0	1
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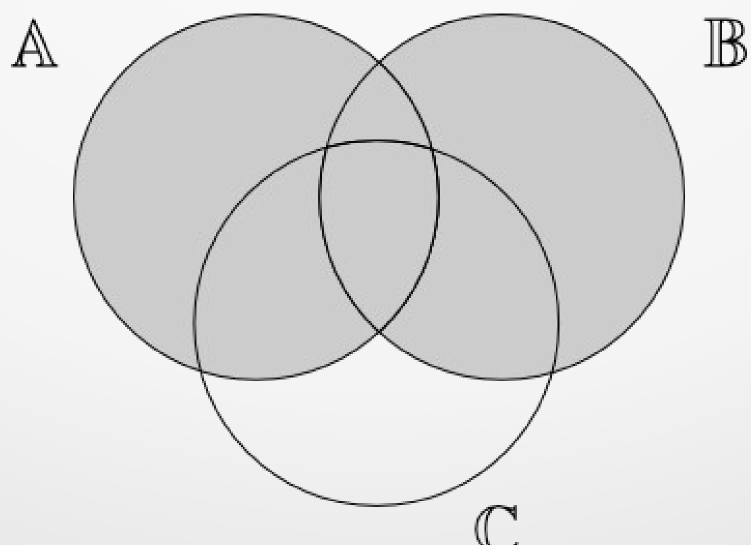
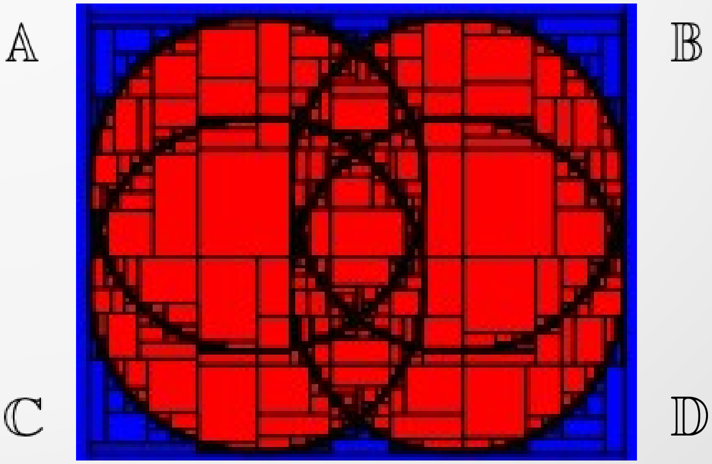


Solution

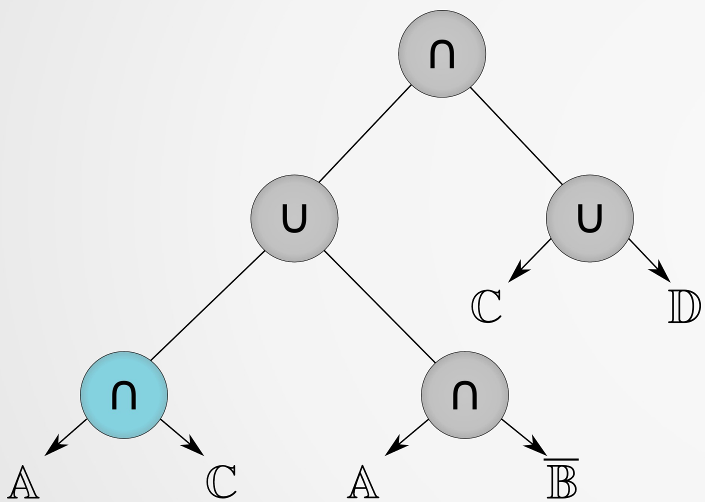
- Two parts:
  - Build Karnaugh Map
  - Extract the Minimal Equation

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



Building Karnaugh Map

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\overline{D}$	1	1	0	0
$\overline{C}D$	1	1	0	0

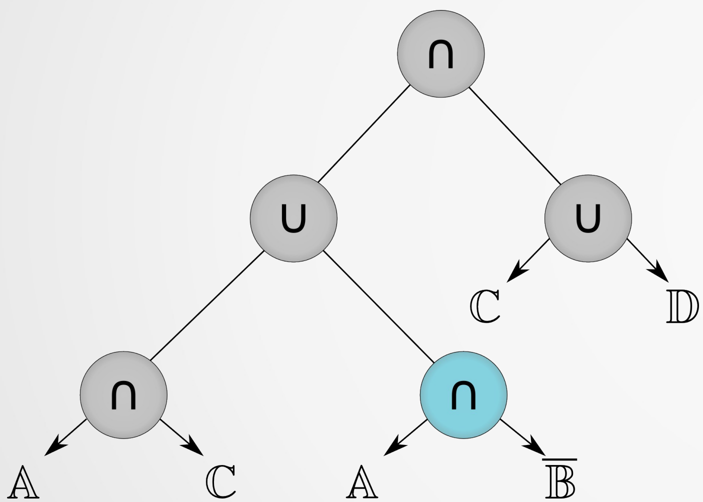
∩

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	1	1
$C\overline{D}$	1	1	1	1
$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	0	0	0

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	0	0	0

Building Karnaugh Map

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\,\overline{D}$	1	1	0	0
$\overline{C}D$	1	1	0	0

$\cap$

	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
CD	0	1	1	0
$C\overline{D}$	0	1	1	0
$\overline{C}\,\overline{D}$	0	1	1	0
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	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
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$C\overline{D}$	0	1	0	0
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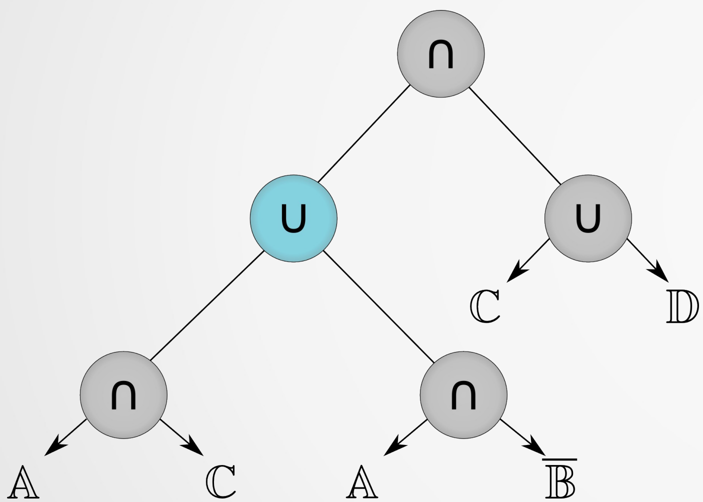
Building Karnaugh Map

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
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∪

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	0	1	0	0
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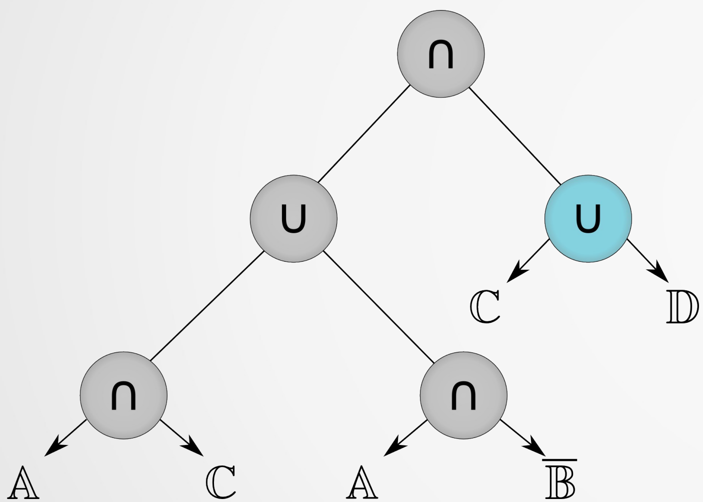
$\overline{C}D$	0	1	0	0

Building Karnaugh Map

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	1	1
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∪

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
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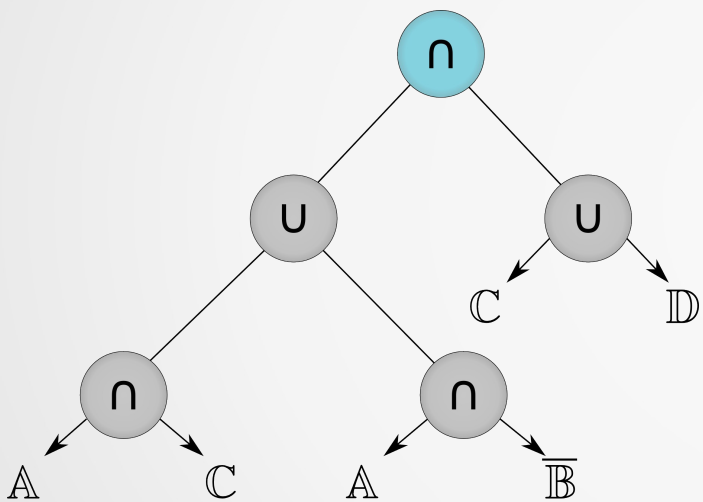
$\overline{C}D$	1	1	1	1

Building Karnaugh Map

$$(AC \cup A\overline{B}) \cap (C \cup D)$$



	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\overline{D}$	0	1	0	0
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$\cap$

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$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

Extracting the Minimal Equation

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

$\longrightarrow$

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

Extracting the Minimal Equation

	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
CD	1	1	0	0
$C\overline{D}$	1	1	0	0
$\overline{C}\,\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

→

	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
CD	1	1	0	0
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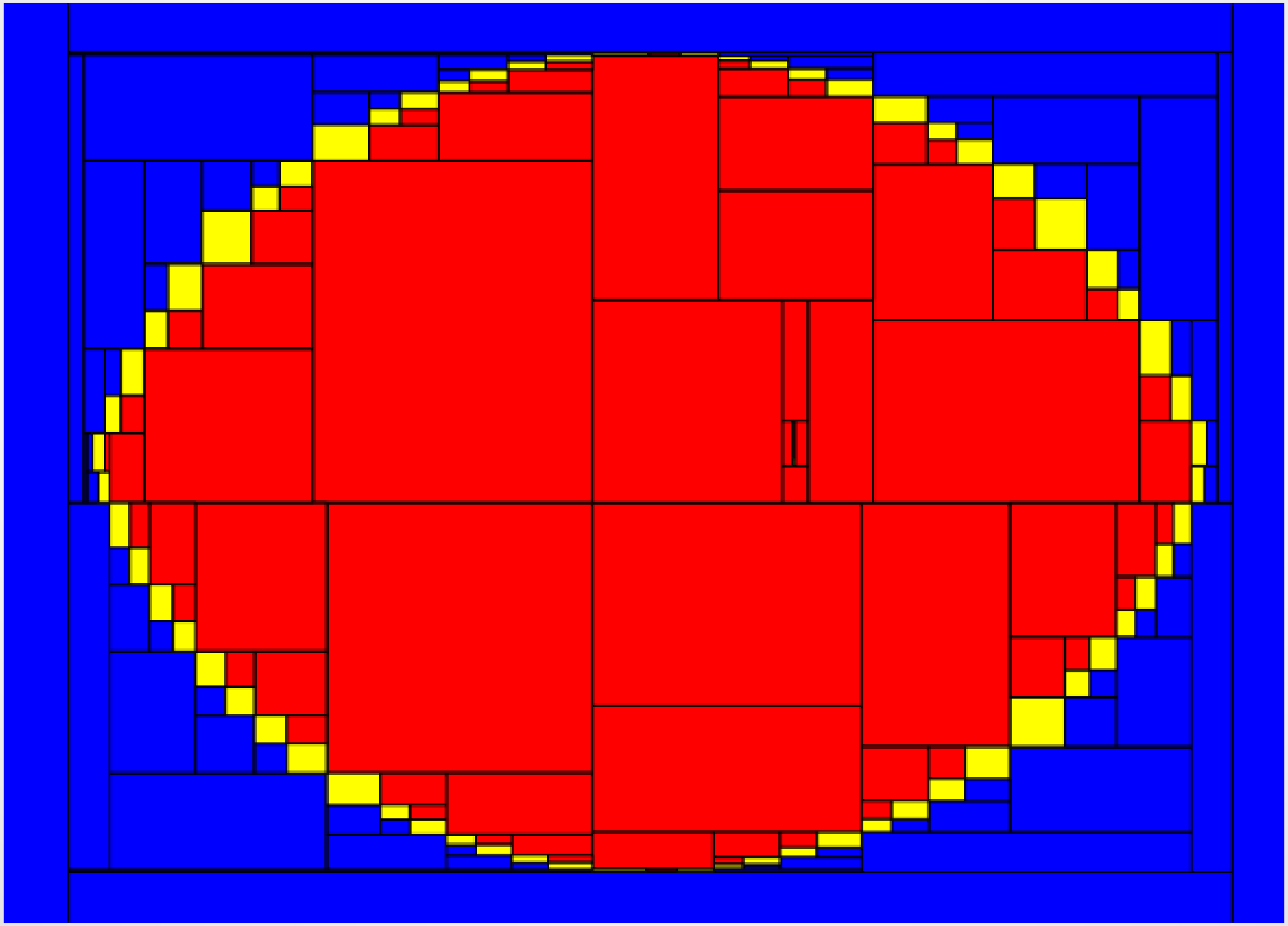
$\overline{C}\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

$$ACD \cup AC\overline{D} \cup ABC \cup A\overline{B}C$$

Extracting the Minimal Equation



Extracting the Minimal Equation

	AB	$A\overline{B}$	$\overline{A}\overline{B}$	$\overline{A}B$
CD	1	1	0	0
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Extracting the Minimal Equation

	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
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→

	AB	$A\overline{B}$	$\overline{A}\,\overline{B}$	$\overline{A}B$
CD	1	1	0	0
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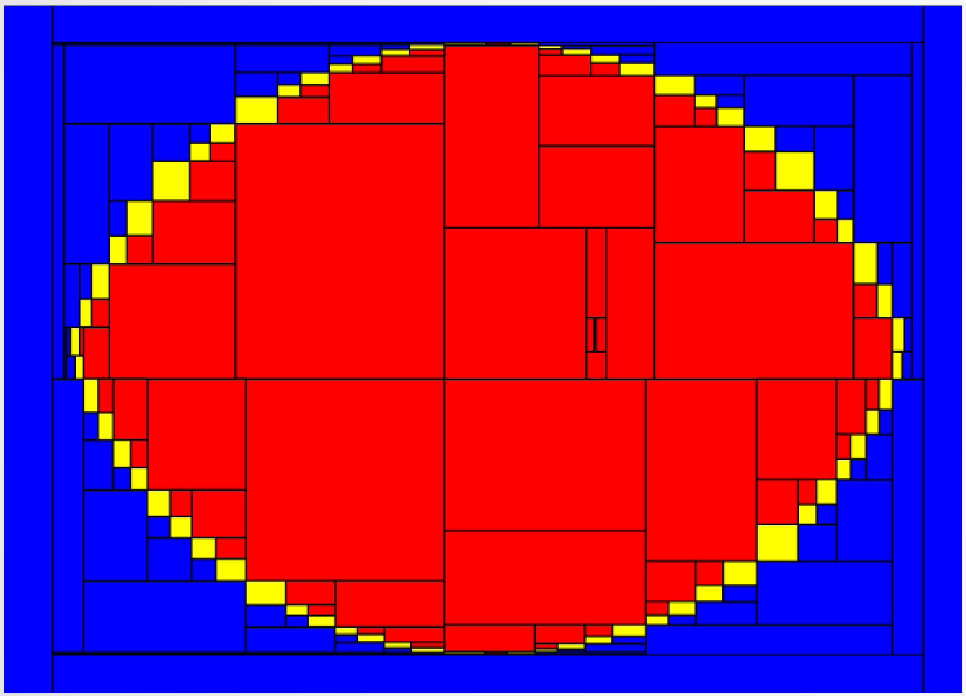
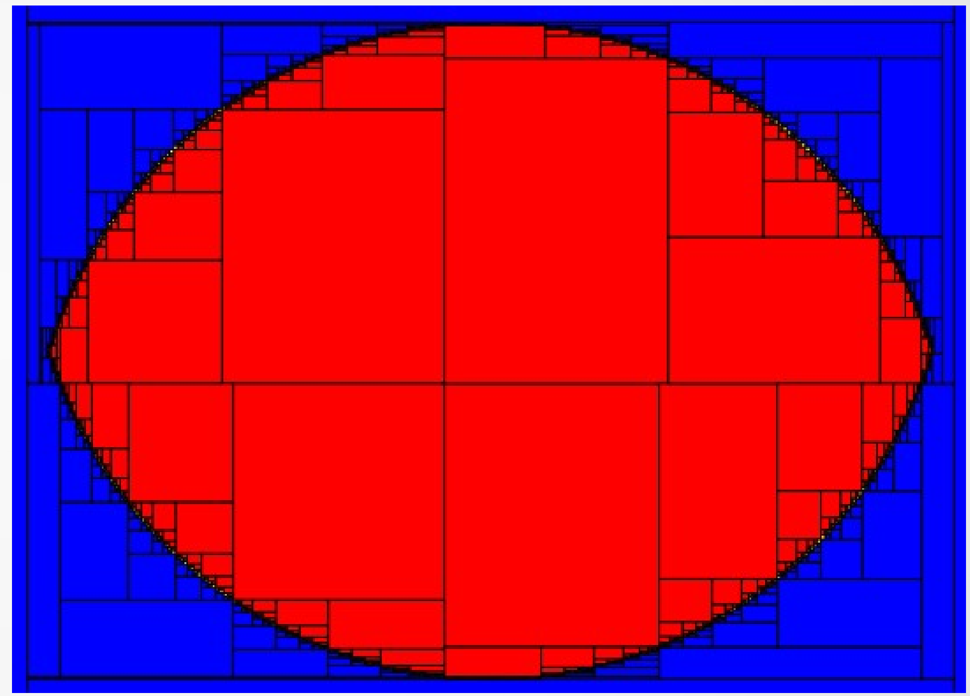
$\overline{C}\,\overline{D}$	0	0	0	0
$\overline{C}D$	0	1	0	0

$$AC \cup AD\overline{B}$$

Extracting the Minimal Equation



$A C D \cup A C \overline{D} \cup A B C \cup A \overline{B} C \cup A D \overline{B}$



$A C \cup A D \overline{B}$

Questions?

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Schvarcz

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guilherme.schvarcz@ensta-bretagne.org