

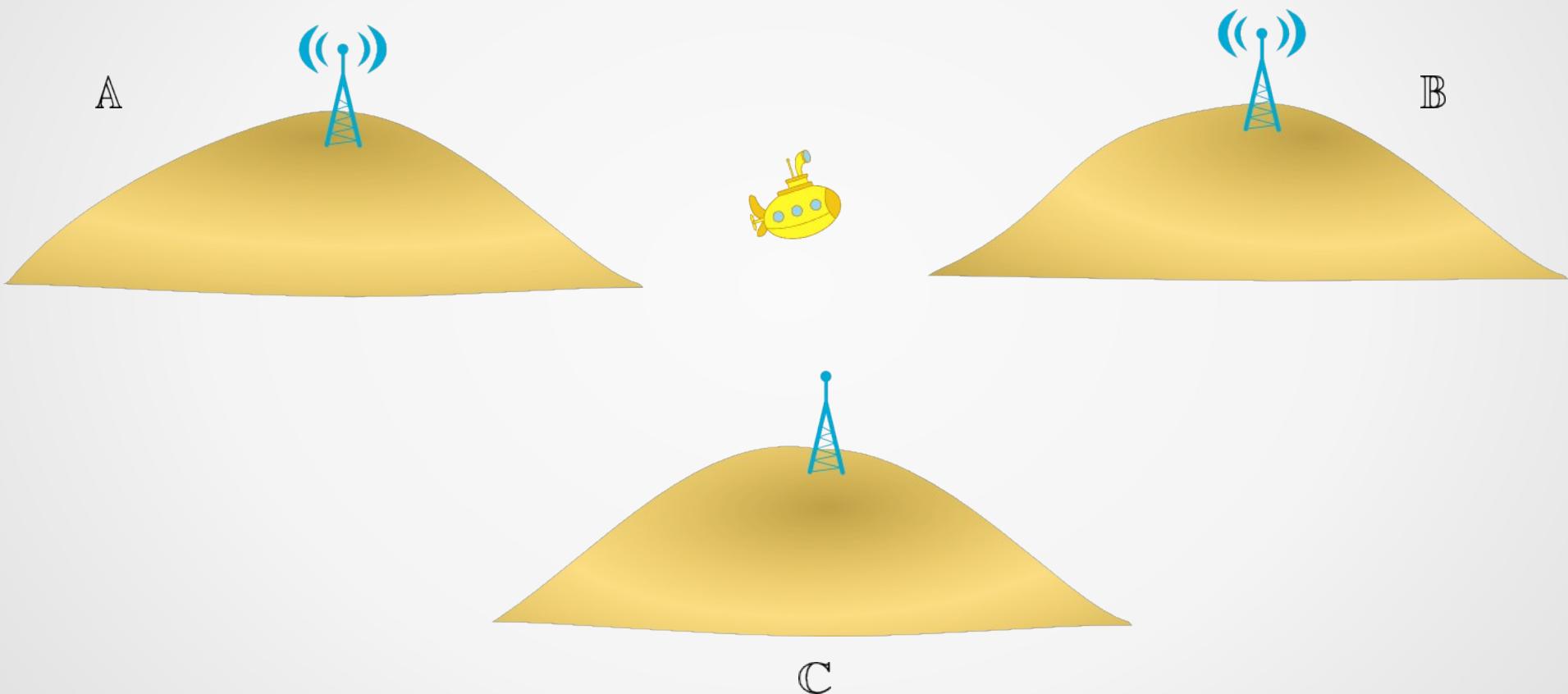
Avoiding fake boundaries in interval analysis

Guilherme Schvarcz Franco and Luc Jaulin

Presentation

- Guilherme Schvarcz Franco
- Resume:
 - Graduate in Systems Analysis – UNIFRA - Brazil
 - Graduate in Geomatics – UFSM - Brazil
 - Master degree in computer sciences at UFRGS – Brazil
 - PhD at ENSTA Bretagne - Brest
- Advisor: Luc Jaulin

Localization Problem



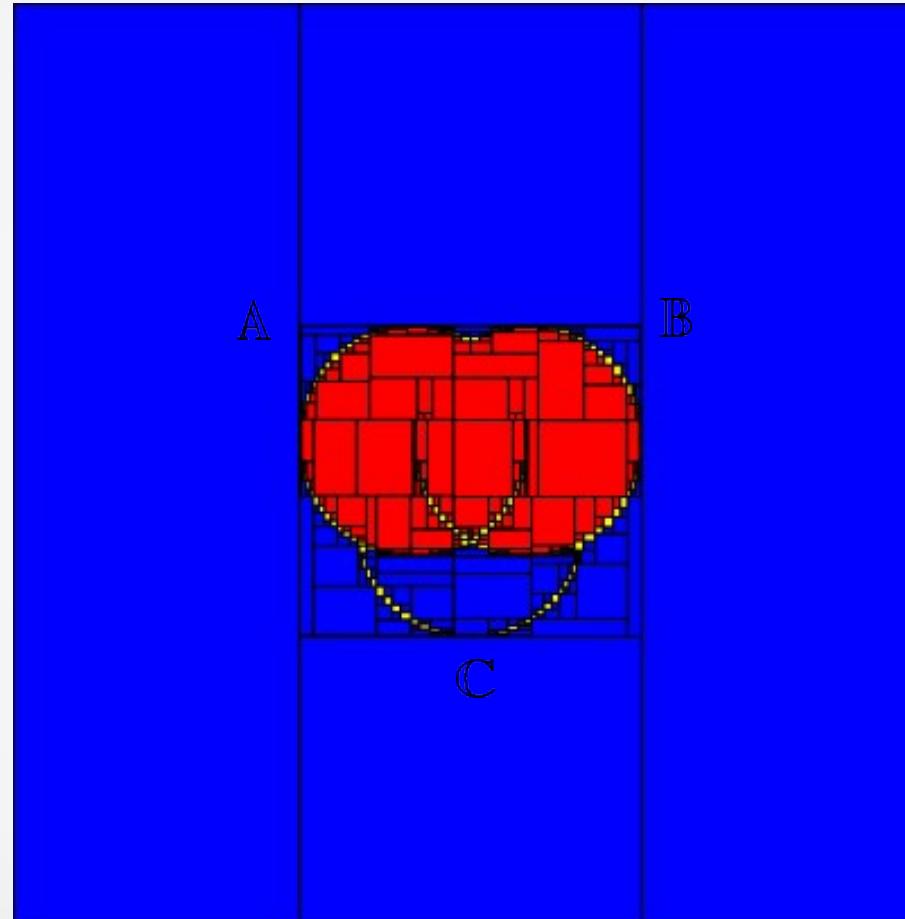
Inner contractor

$$A\bar{B} \cup \bar{A}B \cup AB \cup ((A \cup B \cup C) \cap \bar{C})$$

Outer contractor

$$\bar{A}\bar{B}C \cup A\bar{B}\bar{C}$$

Fake boundary



Inner contractor

$$A\bar{B} \cup \bar{A}B \cup AB \cup ((A \cup B \cup C) \cap \bar{C})$$

Outer contractor

$$\bar{A}\bar{B}C \cup A\bar{B}\bar{C}$$

Simplification

$$A\bar{B} \cup AB \cup \bar{A}B \cup ((A \cup B \cup C) \cap \bar{C})$$

$$A \cap (\bar{B} \cup B) \cup \bar{A}B \cup ((A \cup B \cup C) \cap \bar{C}) \quad \text{By distributive law over union}$$

$$A \cup \bar{A}B \cup ((A \cup B \cup C) \cap \bar{C}) \quad \text{By complement law and identity law}$$

$$((A \cup \bar{A}) \cap (A \cup B)) \cup ((A \cup B \cup C) \cap \bar{C}) \quad \text{By distributive law over union}$$

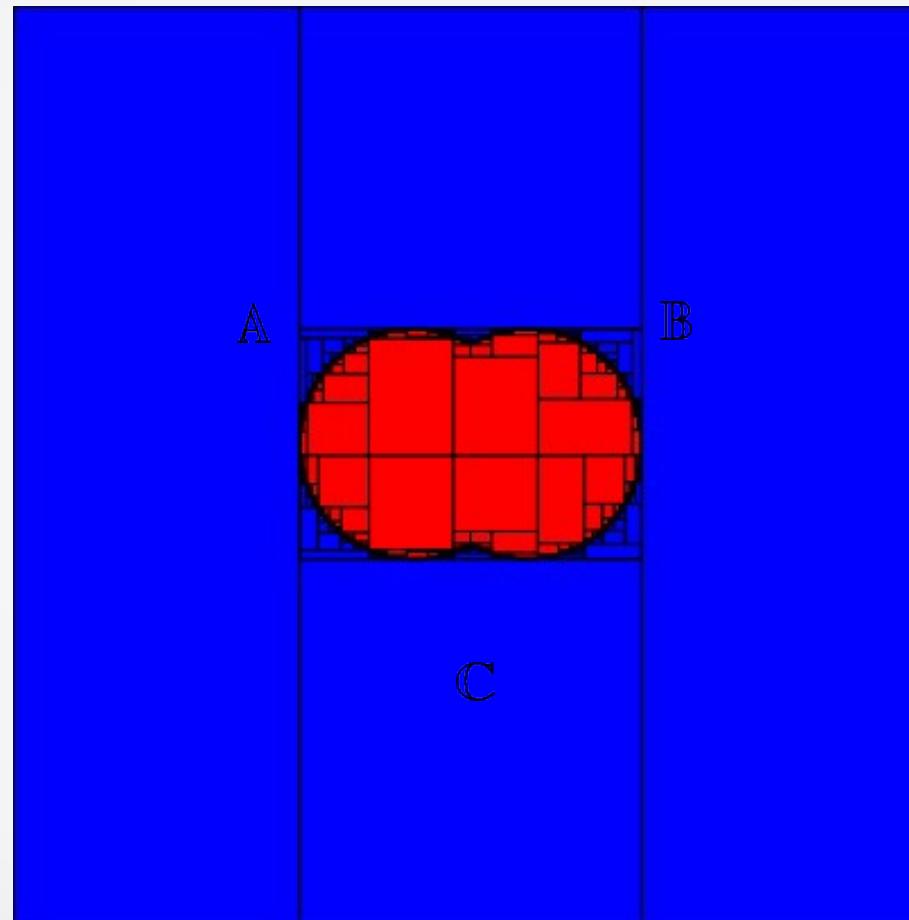
$$(A \cup B) \cup ((A \cup B \cup C) \cap \bar{C}) \quad \text{By complement law and identity law}$$

$$(A \cup B) \cup ((A \cup B) \cap \bar{C}) \cup C\bar{C} \quad \text{By distributive law over union}$$

$$(A \cup B) \cup ((A \cup B) \cap \bar{C}) \quad \text{By complement law}$$

$$A \cup B \quad \text{By absorption law}$$

Expected set



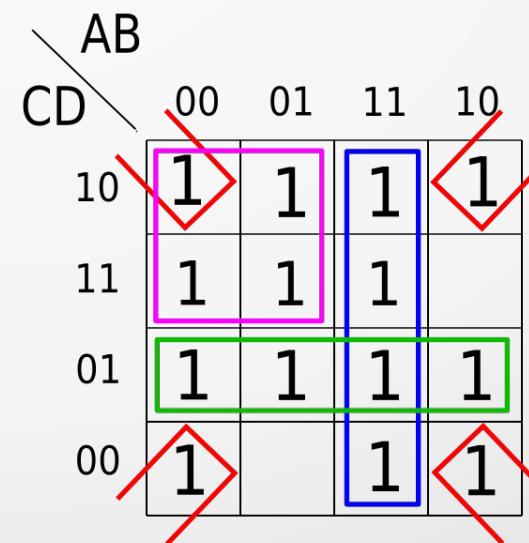
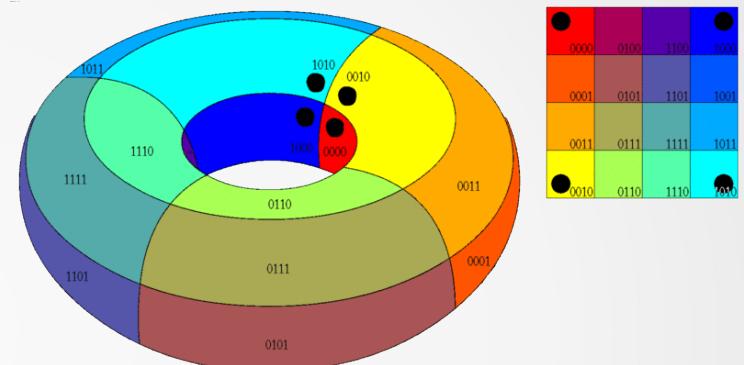
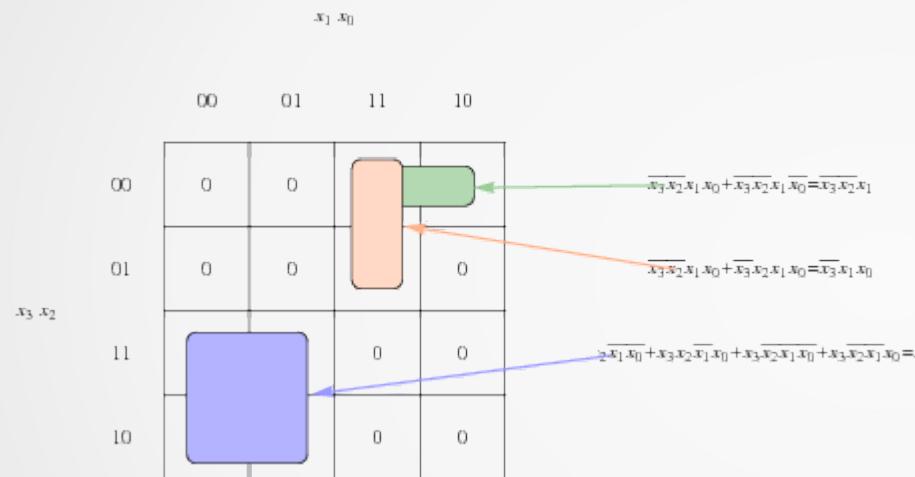
Inner contractor

$$A \cup B$$

Outer contractor

$$\overline{A} \overline{B}$$

Karnaugh Map



Karnaugh Map

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
C	1	1	0	1
\bar{C}	1	1	0	1



Inner contractor

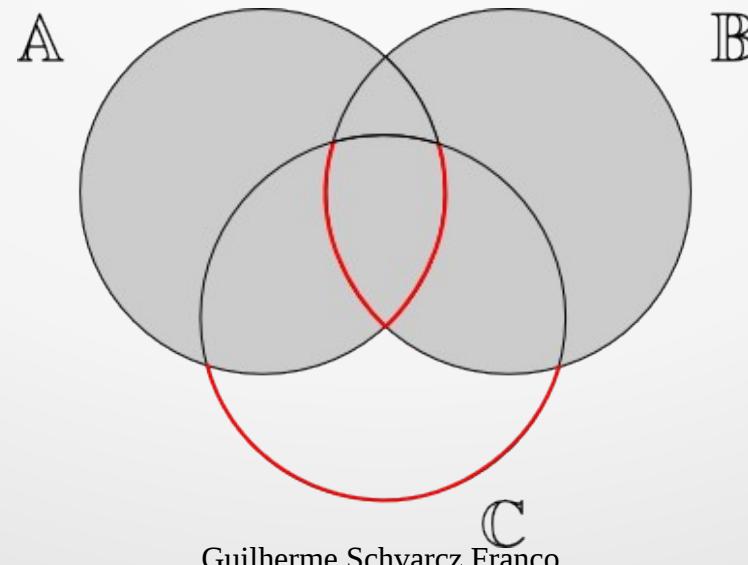
$$A\bar{B} \cup \bar{A}B \cup AB \cup ((A \cup B \cup C) \cap \bar{C})$$

	AB	$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$
C	1	1	0	1
\bar{C}	1	1	0	1

Outer contractor

$$\bar{A}\bar{B}C \cup A\bar{B}\bar{C}$$

	AB	$\bar{A}\bar{B}$	$\bar{A}B$	$A\bar{B}$
C	1	1	0	1
\bar{C}	1	1	0	1



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Inner contractor

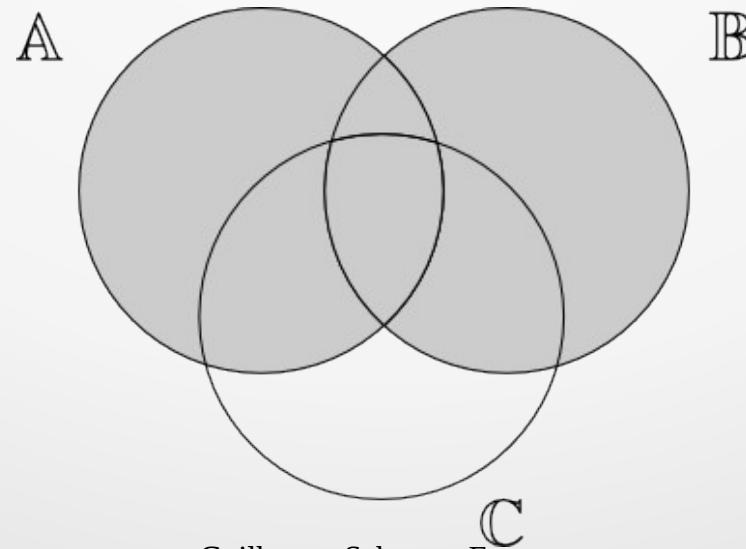
$A \cup B$

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
C	1	1	0	1
\bar{C}	1	1	0	1

Outer contractor

$\bar{A}\bar{B}$

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
C	1	1	0	1
\bar{C}	1	1	0	1

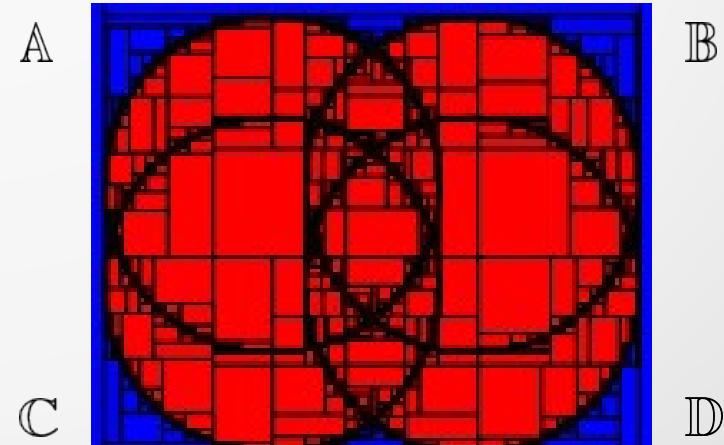


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Solution

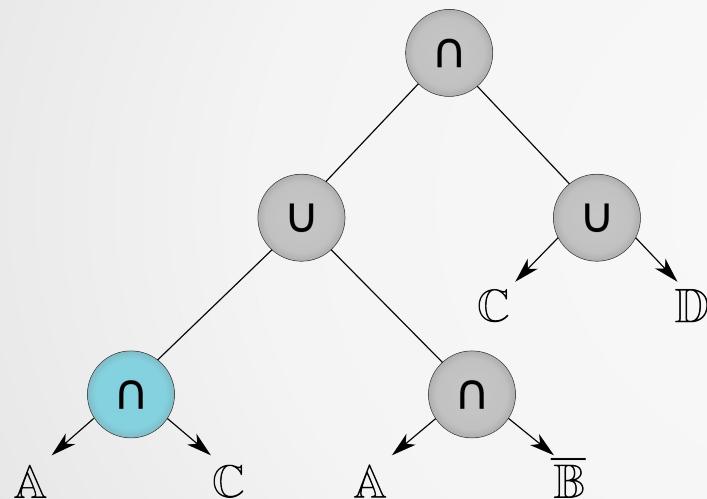
- Two parts:
 - Build Karnaugh Map
 - Extract the Minimal Equation

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



Building Karnaugh Map

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



	AB	ĀB	ĀB̄	ĀB̄̄
CD	1	1	0	0
C̄D	1	1	0	0
C̄D̄	1	1	0	0
C̄D̄̄	1	1	0	0

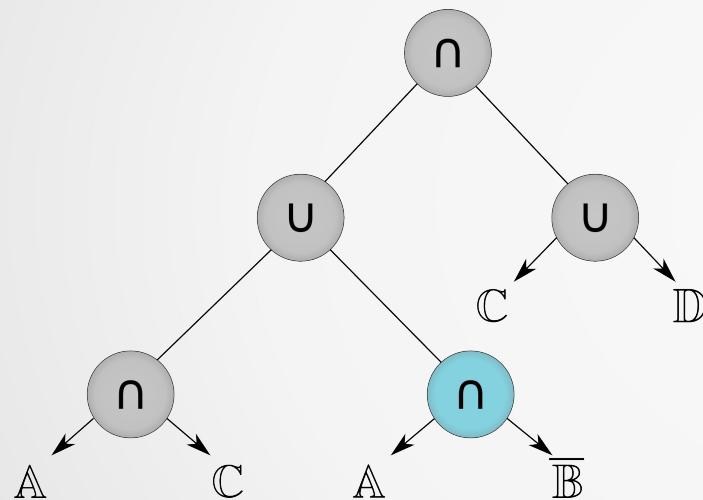
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	AB	ĀB	ĀB̄	ĀB̄̄
CD	1	1	1	1
C̄D	1	1	1	1
C̄D̄	0	0	0	0
C̄D̄̄	0	0	0	0

	AB	ĀB	ĀB̄	ĀB̄̄
CD	1	1	0	0
C̄D	1	1	0	0
C̄D̄	0	0	0	0
C̄D̄̄	0	0	0	0

Building Karnaugh Map

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



	AB	ĀB	̄AB	̄ĀB
CD	1	1	0	0
C̄D	1	1	0	0
̄C̄D	1	1	0	0
̄CD	1	1	0	0

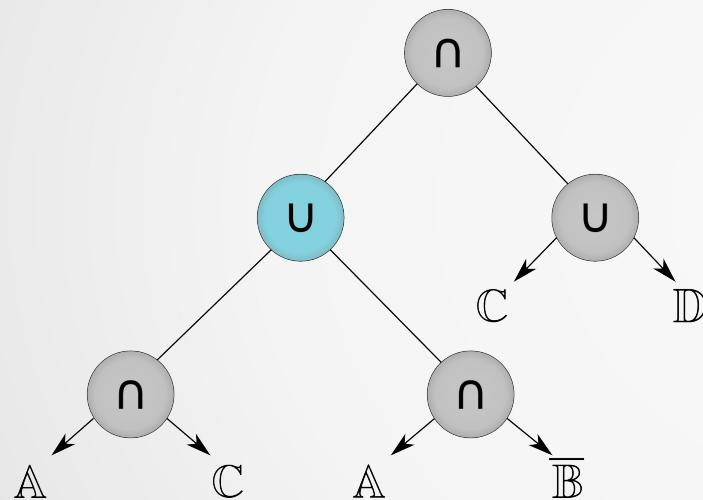
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	AB	ĀB	̄AB	̄ĀB
CD	0	1	1	0
C̄D	0	1	1	0
̄C̄D	0	1	1	0
̄CD	0	1	1	0

	AB	ĀB	̄AB	̄ĀB
CD	0	1	0	0
C̄D	0	1	0	0
̄C̄D	0	1	0	0
̄CD	0	1	0	0

Building Karnaugh Map

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



	AB	ĀB	̄AB	̄ĀB
CD	1	1	0	0
C̄D	1	1	0	0
̄C̄D	0	0	0	0
̄CD	0	0	0	0

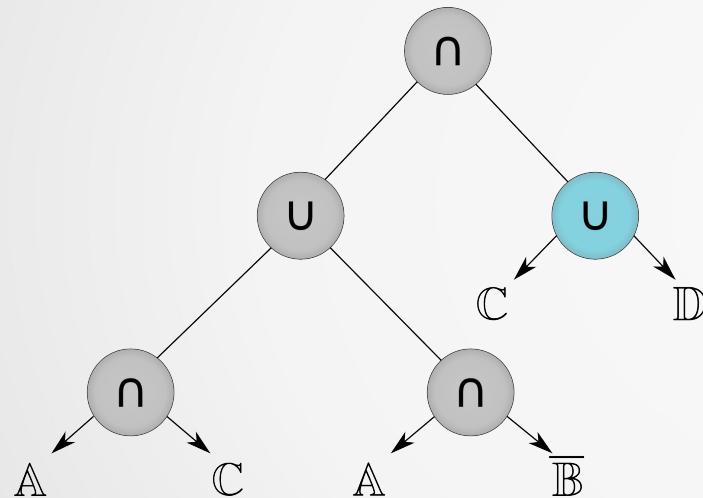
U

	AB	ĀB	̄AB	̄ĀB
CD	0	1	0	0
C̄D	0	1	0	0
̄C̄D	0	1	0	0
̄CD	0	1	0	0

	AB	ĀB	̄AB	̄ĀB
CD	1	1	0	0
C̄D	1	1	0	0
̄C̄D	0	1	0	0
̄CD	0	1	0	0

Building Karnaugh Map

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



	AB	ĀB	̄AB	̄ĀB
CD	1	1	1	1
C̄D	1	1	1	1
̄C̄D	0	0	0	0
̄C̄D	0	0	0	0

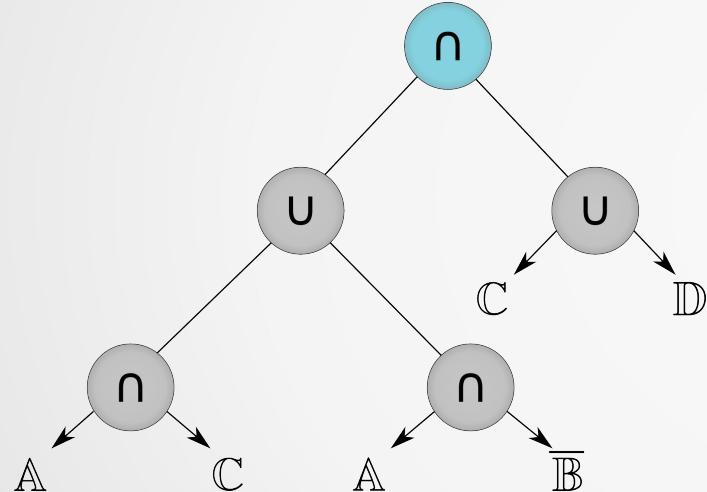
U

	AB	ĀB	̄AB	̄ĀB
CD	1	1	1	1
C̄D	0	0	0	0
̄C̄D	0	0	0	0
̄C̄D	1	1	1	1

	AB	ĀB	̄AB	̄ĀB
CD	1	1	1	1
C̄D	1	1	1	1
̄C̄D	0	0	0	0
̄C̄D	1	1	1	1

Building Karnaugh Map

$$(AC \cup A\bar{B}) \cap (C \cup D)$$



	AB	ĀB	̄AB	̄ĀB
CD	1	1	0	0
C̄D	1	1	0	0
̄C̄D	0	1	0	0
̄C̄D	0	1	0	0

∩

	AB	ĀB	̄AB	̄ĀB
CD	1	1	1	1
C̄D	1	1	1	1
̄C̄D	0	0	0	0
̄C̄D	1	1	1	1

	AB	ĀB	̄AB	̄ĀB
CD	1	1	0	0
C̄D	1	1	0	0
̄C̄D	0	0	0	0
̄C̄D	0	1	0	0

Extracting the Minimal Equation

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0



	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Extracting the Minimal Equation

The diagram illustrates the process of extracting a minimal equation from a Karnaugh map. It shows two maps side-by-side, connected by a right-pointing arrow.

Left Map (Four Variables):

	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Right Map (Three Variables):

	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

The transformation involves grouping the '1's in the first two columns of the left map into a single group, which is then mapped to the first two columns of the right map. The third column of the left map is mapped to the third column of the right map, and the fourth column is mapped to the fourth column.

Extracting the Minimal Equation

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0



	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Extracting the Minimal Equation

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0



	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Extracting the Minimal Equation

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0



	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Extracting the Minimal Equation

The diagram illustrates the simplification of a logic function using Karnaugh maps. It shows two Karnaugh maps side-by-side, separated by a right-pointing arrow.

Left Karnaugh Map (4 variables):

	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

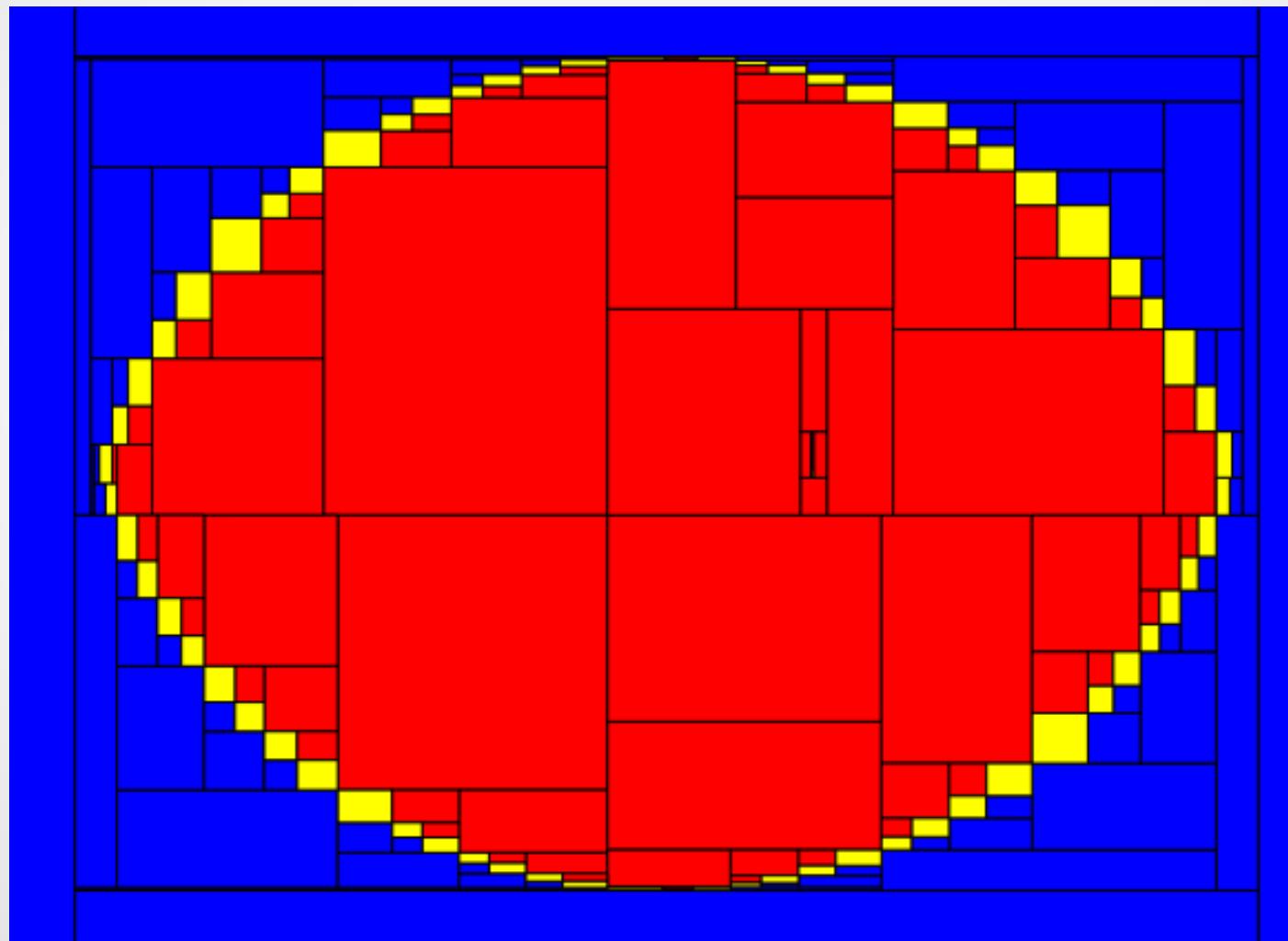
Right Karnaugh Map (3 variables):

	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

The transformation highlights the simplification of the term CD to C . In the first map, the CD cell contains a '1'. In the second map, this cell is removed, and the $C\bar{D}$ cell contains a '1' instead. A red square at the intersection of the $C\bar{D}$ row and the $A\bar{B}$ column indicates the removed term.

$$ACD \cup ACD' \cup ABC \cup A\bar{B}C$$

Extracting the Minimal Equation



Extracting the Minimal Equation

	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0



	AB	$A\bar{B}$	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
$C\bar{D}$	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Extracting the Minimal Equation

The diagram illustrates the simplification of a logic function using Karnaugh maps. It shows two Karnaugh maps side-by-side, connected by a right-pointing arrow.

Left Karnaugh Map (4 variables):

	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

Right Karnaugh Map (3 variables):

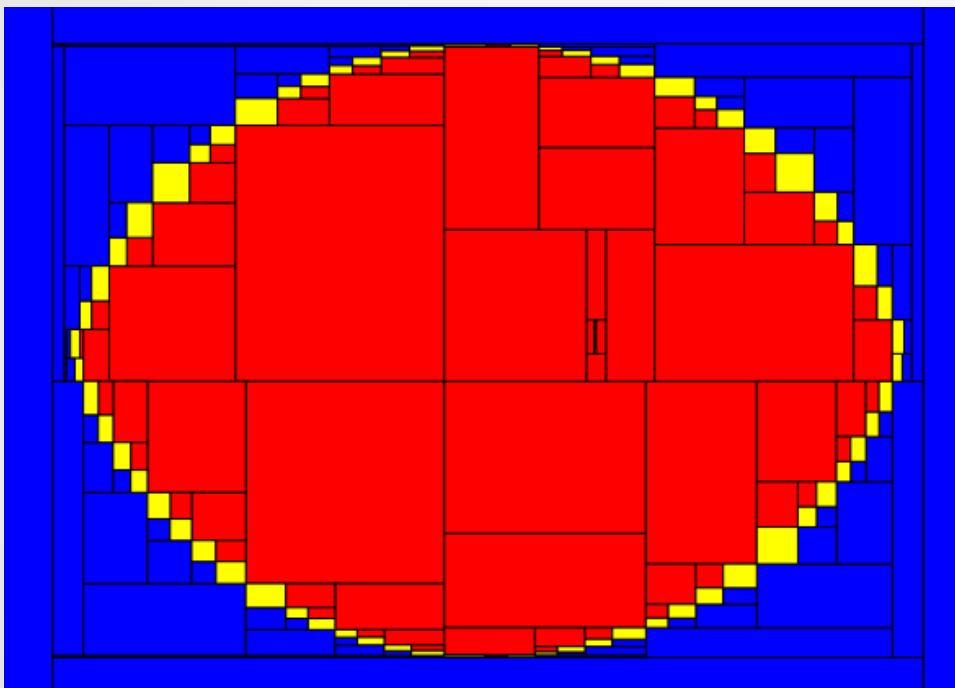
	AB	A \bar{B}	$\bar{A}B$	$\bar{A}\bar{B}$
CD	1	1	0	0
C \bar{D}	1	1	0	0
$\bar{C}\bar{D}$	0	0	0	0
$\bar{C}D$	0	1	0	0

The transformation highlights the following changes:

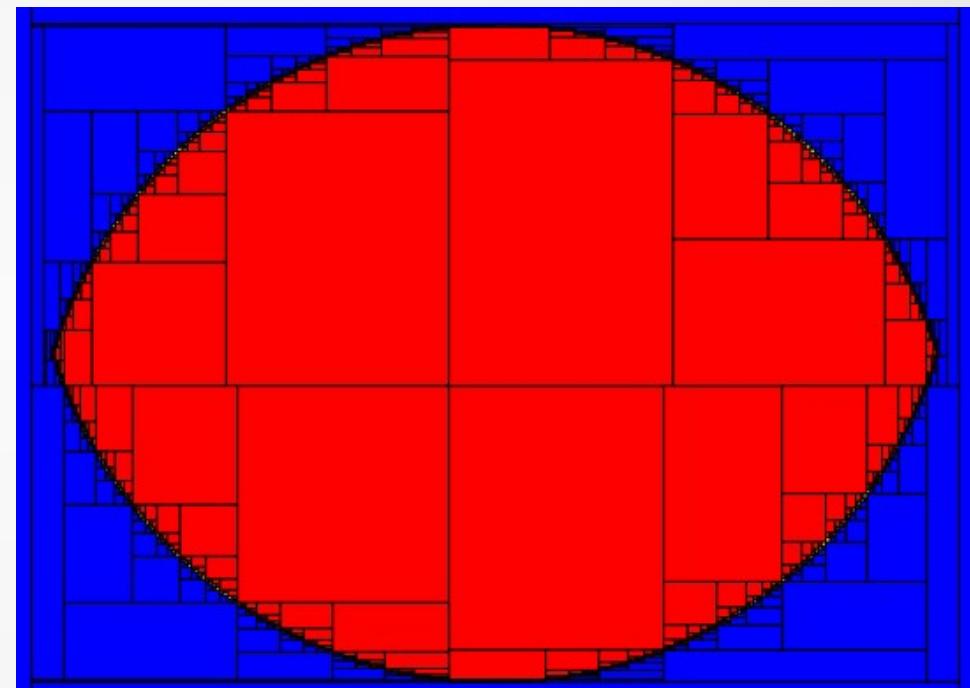
- Row 1 (CD):** The first two columns (AB, A \bar{B}) are highlighted with a blue box. The value 1 is present in both cells, indicating a minterm.
- Row 2 (C \bar{D}):** The first two columns (AB, A \bar{B}) are highlighted with a blue box. The value 1 is present in both cells, indicating a minterm.
- Row 3 ($\bar{C}\bar{D}$):** All four columns (AB, A \bar{B} , $\bar{A}B$, $\bar{A}\bar{B}$) are highlighted with a blue box, indicating that all four cells in this row are 0.
- Row 4 ($\bar{C}D$):** The first column (AB) is highlighted with a blue box. The value 0 is present in this cell, indicating it is a don't care for this row.
- Final Simplified Expression:** The expression $AC \cup AD\bar{B}$ is shown below the simplified Karnaugh map.

$$AC \cup AD\bar{B}$$

Extracting the Minimal Equation



$$ACD \cup ACD' \cup ABC \cup A\bar{B}C \cup AD\bar{B}$$



$$AC \cup AD\bar{B}$$

Questions?



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